THE RELATIONSHIP BETWEEN PERSONALITY & MENTAL ABILITY VARIABLES AND ACHIEVEMENT THROUGH PROGRAMMED INSTRUCTIONAL STYLES

ABSTRACT

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SUMMARY:

Learning effectiveness is a function of personality and mental ability variables of the learner and treatment modes. Therefore, for the optimal learning, treatment modes should be individualised, which implires that the treatment modes should be in accordance with personality and mental ability variables of the learner. Researches concerned with the interaction between treatment modes and personality and mental ability variables of the learner have been terms as ATI (Antitude-Treatment Interaction) or TTI (Trait-Treatment Interaction) studies, The present study is a modest attempt in this direction. In specific terms, the present study seeks to investigate the relationship between the following personality and mental ability variables as measured by the tools shown against each and achievement through linear and branching styles of programming and expository method of teaching.

(1) Anxiety - Measured at Sarason's General Anxiety Scale for Children (GASC).

(2) Extroversion - Indian Adaptation to Eysenck's Modsley Personality Inventory (MPI) by Kapoor and Jalota.

(3) Intelligence - Mixed Type Group Test of Intelligence by Mehrotra.

(4) Creativity - Non-Verbal Test of Creative Thinking by Baquor Mehdi.

Previous achievement in mathematics measured by junior high school marks in mathematics and intelligence, as measured above, have been employed as control variables. The present study is likely to help in assigning treatment modes to students of differing personality and mental ability variable/characteristics for optimal achievement.

Two programmes - linear and branching on 'Measure of Central Tendencies' (Statistics) suitable for high school students of Board of High School and Intermediate Education, U.P., were developed and validated for the present study. This constituted an important part of the present study. A pre-test consisting of items for measuring entry level behaviours and terminal behaviours was constructed to ensure that the students possess the required prerequisite behaviours and do not possess the terminal behaviour before starting instructions. A post-test consisting of only items measuring terminal behaviours served as a criterion measure.

PROCEDURE.

287 students studying in class IX of three Intermediate Colleges in Urban areas of U.P. constituted the subjects of the present study. The students were administered the two measures of personality (anxiety and extroversion) and the two measures of mental ability (intelligence and creativity . The pre-test was then administered to ensure that the subject possessed the required entry level behaviours

and do not possess any of the behavioure which are proposed to be taught. Three groups of students from each of the above three schools were taught separately by linear and branching styles of programming and expository methods of teaching. The post-test was administered after the instructional programme. The post-test scores served as criterion measure.

The personality and mental ability scores and criterion scores of students from three institutions taught through each of the treatment modes were pooled separately. Thus, three groups were constituted each taught by one of the treatment modes. The three groups taught by different treatment modes were divided into high and low groups on the basis of the personality and mental ability scores. Comparison were made between the high groups on any personality or mental ability variables taught by the three treatment modes to ensure that the groups were matched. Similar comparisons were also made fofor the low groups. In doing so scores of some students had to be rejected. All the 't' values were insignificant showing thereby that the differences between these groups were negligible. Comparisons were also made between the high and low groups on each variable. All the 't' values were significant beyond 0.01 level which indicated that very significant differences existed between high and low groups.

Means of post-test scores obtained by students high and low on each personality and mental ability variables taught through different treatment modes were adjusted for previous achievement in mathematics and intelligence by means of analysis of covariance. Comparisons of these adjusted means were made for different pairs of treatment modes to study the interaction between different personality and mental ability variables and achievement through different treatment modes for assigning treatment modes for optimal achievement. For this purpose, graphs of adjusted post-test means of students high and low on any personality or mental ability variable for different pairs of treatment modes were constructed. If the two treatment lines crossed and the post-test means for students either high or low on any personality and metnal ability variable differed significantly, the interaction was considered significant (disordinal). This was regarded as sufficient evidence for assigining treatment mode for optimal achievement.

CONCLUSIONS:

INTELLIGENCE-TREATMENT INTERACTION:

Table CIX presents the adjusted means and 'F' values for the difference between adjusted means of high and low intelligence students for different modes viz. linear and branching styles of programming and expository method.

-TABLE-XCIX

ADJUSTED MEANS AND 'F' VALUES FOR THE DIFFERENCE BETWEEN ADJUSTED MEANS OF HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH THREE MODES.

Intelligence	POST ACHIEVEMENT MEANS					
	Linear Style		Branching Style		Expository Method	
	Adjusted mean	'F' value	Adjusted mean	'F' Value	Adjusted mean	'F' Value
High	48.34	24.51*	55.92	124.3*	41.18	0.25
Low	43.51		33.52		38.06	

*Significant at 0.05 level.

It will be seen from the above table that 'F' values for linear as well as branching styles of programming is significant at 0.05 level for 28 degree of freedom which shows that there is significant interaction between intelligence and achievement through linear as well as branching style of programming. The high intelligence students achieved higher as compared to low intelligence students when taught through any of the above two treatment modes or styles of programming. But the 'F' values for expository method is not significant at 0.05 level for 28 degree of freedom, which shows that the interaction between intelligence and achievement through expository method is not significant. However, high intelligence students achieved slightly higher through expository method in comparison to low intelligence ones.

The above analysis do not provide any evidence for assignment of treatment modes to the high and low intelligence groups. Comparisons were therefore, made of the achievements through any of the two treatment modes for either of the intelligence groups for assignment of treatment modes to the high and low groups of intelligence.

It will be seen from the above table CIX that while the linear style is better for low intelligence students in comparison to branching style, the branching style of programming has proved to be more effective for the high intelligence students in comparison to linear style of programming. The interaction is disordinal according to Lubin (1967), but is not so according to Bracht and Glass (1968) and the present investigator as the 't' values for the differente between achievement scores of linear and branching styles of programming for low and high two intelligence groups were 1.91 and 1.50 respectively, which are not significant at 0.05 level for 28 degree of freedom. The low intelligence student may be taught more profitable by the linear style of programming in comparison to branching style and the high intelligence students by the branching style of programming in comparison to linear style, but the edevidence is not very sound. However, since the 't' value for the low intelligence students is significant at 0.1 level for 28 degree of pfreedom, such students may be taught more profitably by the linear style in comparison to branching style of programming.

It will also be seen from the table CIX that expository method is better in comparison to branching style of programming for low intelligence students, the branching style is better in comparison to expository method for high intelligence students. The interaction is disordinal according to Lubin (1967) but is not so according to the Bracht and Glass (1968), as the 't' values for the difference between achievement of branching style and expository method for high and low intelligence groups were 2.77 and 1.08 respectively, of which the 't' value of 1.08 is not significant at 0.05 level for 28 degree of freedom. However, according to the present investigator, the interaction is disordinal as the 't' value (i.e. 2.77) at one of the end (i.e. high intelligence) is significant at 0.05 level for 28 degree of freedom. The high intelligence students may be taught through branching style of programming in comparison to expository method for better achievement but the evidence is not equally sound for low intelligence students as 't' value for low intelligence students is not significant even at 0.1 level for 28 degree of freedom.

Furthermore, it will be seen from the table CIX that both high as well as low intelligence students have secured better through linear style of programming as compared to expository method. The interaction is ordinal according to Lubin (1967), Brecht and Glass (1968) and the present investigator. Alternative treatments can not be assigned on the basis of this evidence.

CREATIVITY-TREATMENT INTERACTION

Table CX presents the adjusted means and 'F' values for the difference between adjusted means of low and high creative students for different treatment modes viz. linear and branching style of programming and expository method.

TABLE-CX

ADJUSTED MEANS AND 'F' VALUES FOR THE DIFFERENCE BETWEEN ADJUSTED MEANS OF HIGH AND LOW CREATIVE STUDENTS TAUGHT THROUGH THREE MODES.

	POST ACHIEVEMENT MEANS					
Creative	Linear Style		Branching Style		Expository Method	
	Adjusted mean	'F' Value	Adjusted mean	'F' Value	Adjusted mean	'F' Value
High	37.80	55.8*	44.11	0.35	17.33	4.34*
Low	50.17		39.38		66.08	

*Significant at 0.05 level.

It will be seen from the table CX that 'F' values for linear style of programming and expository method are significant at 0.05 level for 28 degree of freedom which shows that there is a significant interaction between creativity and achievement through linear style of programming and expository method. The low creative students achieved higher as compared to high creative students when taught through any of the two treatment modes viz. linear style or expository method. But the 'F' value for branching style of programming is insignificant which shows that there is no significant interaction between creativity and achievement through branching style of programming.

The above analyses do not provide any evidence for assignment of treatment modes to the high and low creative students. Comparisons were, therefore, made of the achievements through any of the two treatment modes for either of the creativity groups for assignment of treatment modes.

It will be seen from the table CX that while the linear style of programming is better for low creative students in comparison to branching style of programming, the branching style has proved to be better for high creative students in comparison to linear style of programming. The interaction is disordinal according to Lubin (1967) but is not so according to Bracht and Glass (1968) and the present investigator as the 't' values for the difference of achievement scores between linear and branching style of programming for low and high creative groups were 1.64 and 0.94 respectively which are not significant at 0.05 level for 28 degree of freedom. The low creative students may be taught more profitably by the linear style in comparison to branching style of programming and high creative students may be taught more profitably by the branching style in comparison to linear style, but the evidence is not sound. However, since the 't' value for low creative group is significant at 0.1 level for 28 degree of freedom such students may be assigned linear style of programming in comparison to branching style of programming for maximal achievement.

It will also be seen from the table CX that expository method is better incocomparison to branching style for low creative students, the branching style is better in comparison to expository method for high creative students. The interaction is disordinal according to Lubin (1967). The 't' values for the difference between achievements of branching and expository modes for high and low creative groups were 4.05 and 3.63 respectively, both of which are the significant at 0.05 level for 28 degree of freedom. Thus the interaction is also disordinal according to Bracht and Glass (1968) and the present investigator. The low creative students may be assigned expository method in comparison to branching style and high creative students may be assigned branching style of programming in comparison to expository method for maximal achievement.

Furthermore, it will be seen from the above table CX that while the expository method is better than the linear style for low creatives, the linear style has proved to be more effective in comparison to expository method for the high creative students. The interaction is disordinal according to Lubin (1967) but is not so according to Bracht and Glass (1968) as the 't' values for high and low creative groups taught by the two modes were 3.74 and 1.95 respectively, of which 1.95 is not significant at 0.05 level for 28 degree of freedom. According to the present investigator, the interaction is disordinal as the 't' vàlue (i.e. 3.74) at one of the end (i.e. at high creative) is significant at 0.05 level for 28 degree of freedom. The high creative students may be assigned linear style in comparison to expository method for maximal achievement. But the evidence is not very sound for low creative students as the 't' value is not significant at 0.05 level. However, since 't' values (1.95) for low creative students is significant at 0.1 level, such students may be taught a little more profitably by expository method in comparison to linear style for better achievement.

ANXIETY-TREATMENT INTERACTION:

Table CXI presents the adjusted means and 'F' values for the difference between adjusted means of high and low anxiety students for three treatment modes i.e. linear and branching styles of programming and expository method.

It will be seen from the table CXI that 'F' values for linear style and expository method is significant, which shows that there is a significant interaction between anxiety

TABLE-CXI

ADJUSTED MEANS AND 'F' VALUES FOR THE DIFFERENCE BETWEEN ADJUSTED MEANS AT HIGH AND LOW ANXIETY STUDENTS TAUGHT THROUGH THREE MODES

Anxiety	POST ACHIEVEMENT MEANS					
	Linear Style		Branching Style		Expository Method	
	Adjusted mean	'F' Value	Adjusted mean	'F' Value	Adjusted mean	'F' Value
High	53.31	5.9*	36.09	0.83	84.87	9.68
Low	39.60		41.53		-4.97	

*Significant at 0.05 level.

and achievement through linear style of programming as well as expository method. The high anxiety students achieved higher as compared to low anxiety students when taught through any of the above two treatment modes. But the 'F' value for branching style is insignificant which shows that there is no significant interaction between anxiety and achievement through branching style of programming.

The above analyses do not provide any evidence for assignment of the treatment modes to the high and low anxiety students. Comparisons were, therefore, made of the achievements through any of the two treatment modes for either of the anxiety groups of assignment of modes to high and low anxiety groups.

It will be seen from the table CXI that while the linear style of programming is better in comparison to branching style for high anxiety students, the branching style has proved to be better in comparison to linear style for low anxiety students. The interaction is disordinal according to the standard of Lubin (1967) but is not as according to Bracht and Glass (1968) as the 't' values for the difference between achievement means of linear and branching style of programming at low as well as high anxiety students were 0.27 and 2.532 respectively of which 0.27 is not significant at 0.05 level for 28 degree of freedom. According to the present investigator the interaction is disordinal as the 't' value (2.532) at high anxiety level is significant at 0.05 level for 28 degree of freedom. The high anxiety students may be assigned the linear style in comparison to branching style for better achievement but the evidence is not satisfactory for low anxiety students as the 't' value of this group is not significant even at 0.1 level.

It will also be seen from the table CXI that expository method is better in comparison to branching style for high anxiety students and the branching style has proved to be better in comparison to expository method for low anxiety students for maximal achievement. The interaction is disordinal according to Lubin (1967). The 't' values for the difference between achievement of expository and branching style for high and low anxiety students were 6.5 and 6.3 respectively, which are both significant at 0.05 level for 28 degree of freedom. Thus the interaction is also disordinal according to Bracht and Glass (1968) and the present investigator. The low anxiety students may be assigned branching style in comparison to expository method and high anxiety students may be assigned expository method in comparison to branching style of programming for better achievement.

Further, it will be seen from the table CXI that while the linear style of programming is better in comparison to expository method for low anxiety students, the expository method is superior in comparison to linear style for high anxiety students. The interaction is disordinal according to Lubin (1967). The 't' value for the difference between achievement means of linear style and expository method for high and low anxiety students were 7.3 and 8.2 respectively, which are both significant at 0.05 level for 28 degree of freedom. Thus the interaction, which is disordinal as per Lubin (1967), is also disordinal as per Bracht and Glass (1968) and the present investigator. The linear style may be assigned in comparison to expository method for low anxiety students and expository method may be assigned in comparison to linear style of programming for high anxiety students for maximal achievement.

EXTROVERSION-TREATMENT INTERACTION:

Table CXII presents the adjusted means and 'F' values for the difference between adjusted means of high and low extroversion students for linear and branching styles of programming and expository method.

TABLE-CXII

ADJUSTED MEANS AND 'F' VALUES FOR THE DIFFERENCE BETWEEN ADJUSTED MEANS OF LOW AND HIGH EXTROVERSION STUDENTS TAUGHT THROUGH THREE MODES.

Extroversion	POST ACHIEVEMENT MEANS					
	Linear Style		Branching Style		Expository Method	
	Adjusted mean	'F' Value	Adjusted mean	'F' Value	Adjusted mean	'F' Value
High	43.79		2[illegible].79		36.23	
		1.24		10.76*		0.28
Low	44.34		52.91		41.33	

*Significant at 0.05 level.

It will be seen from the table CXII that the 'F' values is significant only for branching styles of programming which shows that there is a significant interaction between extroversion and achievement only through branching style of programming. The low extroversion students achieved higher as compared to high extroversion students when taught through branching style of programming. The 'F' values for linear style of programming and expository method are both insignificant which shows that there is no significant interaction between extroversion and achievement either through linear style of programming or through expository method.

The above analyses do not provide any evidence for assignment of treatment modes to the high and low extroversion students. Comparisons were, therefore made of the avhievements through any of the two treatment modes for either of the extroversion groups for assignment of treatment modes to the high and low extroversion groups.

It will be seen from the above table XXII that while the branching style is better in comparison to linear style for low extroversion students, the linear style of programming has proved to be more effective in comparison to branching style for high extroversion students. The interaction is thus disordinal according to the criteria of Lubin (1967) but is not as according to Bracht and Glass (1968) as the 't' values for the difference between achievement score of linear and branching styles of programming for low and high extroversion students were 1.04 and 2.48 respectively which 1.04 is not significant at 0.05 level for 28 degree of freedom. According to the present investigator, the interaction is disordinal as the 't' value (i.e 2.48) for one end (i.e. at high extroversion) is significant at 0.05 level for 28 degree of freedom. The high extroversion students may be assigned the linear style of programming inc comparison to branching style of programming but the evidence is not equally sound for low extroversion students.

It will also be seen from the table XII that while the branching programming is better in comparison to expository method for low extroversion students the expository method has proved to be more effective in comparison to branching style for high extroversion students. The interaction is disordinal according to Lubin (1967), but is not so according to Bracht and Glass (1968) and the present investigator as the 't' values for the difference between achievement scores of branching style and expository method for low and high extroversion students were 1.77 and 1.14 respectively, both of which are not significant at 0.05 level for 28 degree of freedom. The low extroversion students may be taught more profitably by branching style in comparison to expository method and the high extroversion students by the expository method in comparison to branching style, butthe evidence is not very trustworthy. However, since 't' value for low extroversion students is significant at 0.1 level for 28 degree of freedom, such students may be taught more profitably by branching style of programming in comparison to expository method.

Furthermore, it will be seen from the atable CXII that both high as well as low extroversion students have secure better through linear style of programming as compared to expository method. The interaction is ordinal according to the standards of Lubin (1967), Bracht and Glass (1968) and the present investigator. Alternative treatment can not be assigned on the basis of this evidence.

SUGGESTIONS.

Students with different personality and mental ability variables should be taught through instructional modes suited to them. The conclusions of the present study may be employed by the classroom teacher for this purpose. The following suggestions are being submitted in this regard.

1. Intelligence has been found to interact very significantly with linear and branching styles of programming which means that high intelligence students are likely to achieve higher as compared to low intelligence students when taught through either of these modes. But the interaction between intelligence and expository method of teaching is not significant. A comparison of students achievement through different pairs of treatment modes revealed that high intelligence students may be taught through branching style of programming and the low intelligence students through expository method of teaching for maximal achievement.

2. The interaction between creativity and achievement through linear style of programming and expository method is significant. Low creatives have been found to achieve higher as compared to high creatives when taught through these two

modes. Creativity has yielded insignificant interaction with branching style of programming. A comparison of student's achievement through different pairs of treatment modes revealed that low creatives may be taught through expository method of teaching and the high creatives through either of the programming styles employed in the study preferably the branching programme for maximal achievement.

3. General anxiety has been found to interact significantly with linear style of programming and expository method. High anxiety students have achieved better through these modes. General anxiety has not yielded significant interaction with branching style of programming. A comparison of student's achievement through different pairs of treatment modes revealed that high anxiety students may be taught through expository method and low anxiety students through either of the programming styles preferably the branching style. If the choice is between the two styles of programming, the high anxiety students may be taught through the linear style of programming and the low anxiety students through branching styles of programming.

4. The personality dimension extroversion/introversion has not yielded any interaction with either linear style of programming or expository method. However, the interaction between extroversion and branching style of programming is significant. High extroversion students achieve lower than the low extroversion students when taught through branching style of

programming. A comparison of student's achievement through different pairs of treatment modes revealed that the high extroversion students may be taught through linear style of programming and low extroversion students may be taught through branching style of programming. The evidence for high extroversion students is more dependable as compared to low extroversion students in this regard.

5. The present study has employed 'Measures of Central Tendencies' a topic of High School statistics as the subject matter of the different instructional treatment modes employed. This subject matter is highly structured and follows a regular sequence. The result of the present study may be applied with caution to other subject matter areas. The applicability of the conclusions of the present study to other areas will depend on the similarity of the structure of the subject matter of these areas and the area investigated.

6. The conclusions of the present study may be generalized to other instructional treatment modes. The treatment modes employed in the present study range from a highly structured teaching-learning situation provided by the linear programme to an unstructured teaching-learning situation provided by the expository method. The branching style seems to be intermediate between these two situations. The generalization to other treatment modes would perhaps, depend upon the degree of structured situation which a treatment mode provides.

SUGGESTIONS FOR FURTHER RESEARCH

1. The present study has employed only four (two personality and two mental ability) variables for assigning treatment modes. Variables like neuroticism, study habits, achievement motivation, cognitive styles etc. have also been found to interact significantly with some treatment modes but the studies for assignment of treatment modes to students high or low on these variables do not seem to have been attempted. Studies should, therefore, be undertaken to investigate the interaction between these variables and different treatment modes with a view to assign treatment modes for maximal achievement.

2. The present investigation has employed only three treatment modes. There are many other treatment modes which seem to require different learner characteristics for success through them. Treatment modes which are known to require different personality and mental ability variables for success through them should, therefore, be studied for interaction effects.

3. Patterns of classroom behaviours assessed through technique like 'Flander's Interaction Analysis' have been found to have different associations with achievement at different levels when different learning activities are employed. Attempts do not seem to have been made to study thei interaction of these patterns and achievement of

student's having different personality and mental ability characteristics. Attempts should, therefore, be made to study the interaction of different patterns of classroom behaviours and achievement of student possessing different personality and mental ability characteristics.

4. It is not one aspect of individuals personality that leads to success, but rather the interaction of different personality characteristics within the individual. For example, it may not be the introverts who achieve high on programmed learning task but the neurotic-introverts who may succeed best on it. Therefore, future ATI researches must take into consideration the interaction of different personality variables within the individual.

5. The present study has employed only 'Measures of Central Tendencies' a topic of high school statistics as subject matter of different treatment modes. A review of previous researches has shown t at ATI may appear in one type of subject matter and not with the other. Researches should therefore, be undertaken to investigate the interaction between different treatment modes employing a variety of subject matter areas and different learner characteristics.

6. A review of previous researches has shown that variables like age, grade and sex play an important role in interaction between learner attributes and treatment modes.

Studies should, therefore, be undertaken to investigate the interaction between these variables and different treatment modes controlling important learner characteristics.

7. The present study has employed extreme group design with analysis of covariance for studying the interaction between learner attributes and treatment modes. This technique tend to create artificial levels of a rather continuous personological variables. Regression analysis can help in the study of interaction without creating such artificial levels. However, the regression analysis can not help in the assignment of treatment modes to students high and low on the personological variables investigated. Therefore, this technique can not be employed in investigations like the present one, which aim at the assignment of treatment modes. Johnson-Neyman Technique which helps to define regions of personological variables in which, the treatment are significantly different on the criterion can be employed for this purpose when hypothesis of homogeneous regression lines in the treatment groups rejected. Studies should, therefore, be undertaken employing this technique for the assignment of treatment modes for maximal achievement.

Study of interaction between learner characteristics and treatment modes with a view to assigning treatment modes for maximal achievement, is a complex problem requiring well designed studies employing sophisticated statistical techniques. The present investigation is an humble attempt in this direction and has merely touched the fringe of the problem. The investigator is conscious of the many limitations of the study, but it is submitted in the hope that it might stimulate better and more detailed research.

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Akumar.

Anil Kumar

C O N T E N T S

LIST OF TABLES

LIST OF FIGURES

CHAPTER - I

I N T R O D U C T I O N

Origin and Selection of the Problem

Hypothesis

Procedure in Outline

Delimitations

References

ORIGIN AND SELECTION OF THE PROBLEM

The individualisation of learning has been and is still considered as one of the most important pedagogical problems. Learning effectiveness is not only a function of intellectual and personality characteristics of the learner but also of the instructional treatment mode to which the learner is exposed(Tobias and Abhramson, 1971; Cronbach, 1967; Jensen, 1967). Individualisation of learning would, therefore, consist in changing the instructional treatment modes in accordance with the intellectual and personality characteristics of the learner (Gagne, 1971; Jensen, 1967, 1968 and Glaser, 1967). Cronbach (1967) also suggests, "most adequate way open with individual differences in the schools might be to alter the instructional methods to fit the aptitude pattern of the learner". The implicit assumption is that learning results can not be explained by looking only at the intellectual and personality characteristics of the learner. The effect of learning must rather be interpreted as the results of interaction between intellectual and personality characteristics of the learner and specific features of instructional treatment modes (Heidt, 1976). Given a common set of objectives, a method may provide optimal achievement for learners with certain characteristics whereas the other may do so for learners with certain other characteristics. Consequently, a greater proportion of learners would get optimal achievement when students with differing intellectual and personality characteristics will receive instructions with instructional treatment modes suited to them. Therefore, there is a need to study the i interaction between intellectual and personality characteristics of the learner and specific features of the instructional treatment modes. Cronbach (1957) also writes,

> Psychologists should combine their interests and methods to observe experimental effects for the subjects of different characteristics and to conduct investigations to find out the relationship between experimental treatment and individual difference variables, so that maximum profit can be gained in terms of achievement.

Researches concerned with the interaction between instructional treatment modes and learner characteristics have been termed as Aptitude - Treatment Interaction (ATI) st dies. Following, Cronbach and Snow (1969), aptitude can be defined as, "Any characteristics of the individual that increases (or impairs) his probability of success in a given treatment" and treatment as, "variations in the pace and style of instruction"(p.7). Tobias (1969), however, is of the view that there is a close identification between the term 'aptitude' and 'intellectual abilities' and the definition of the ATI construct is much wider than intellectual characteristics alone, a more accurate name for research in this area would be 'Attribute-Treatment

Interaction' (ATI). More recently, Heidt (1976) and Berliner and Cohen (1973) have suggested the name as 'Trait - Treatment Interaction (T.T.I.)' for this type of research. The present investigator however, believes that the term 'trait' is generally associated with personality characteristics and thus does not seem to be sufficiently comprehensive. Tobias (1976) also writes,

> The trait term in TTI has different connotations that may be equally disfunctional. Trait usually suggests a stable predispositions not infrequently in the personality domain, relatively unchanging over short periods of time.

Therefore, the term 'attribute' in ATI research seems to very appropriate because it includes all the characteristics of the learner - personality, intellectual, motivational which have long been established as influencing achievement through different instructional treatments. The term 'Attribute-Treatment Interaction or simply ATI would, therefore, be used for this type of research in the present investigation.

The Attribute-Treatment Interaction (ATI) research is of relatively recent origin (Tobias, 1976) but has tremendous potential for improving educational practices. Quite a few ewell designed research have been conducted in the area. These researches have attempted to explore the relationship between variety of learner attributes and instructional treatments.

The attributes that have, generally, been investigated are:

(a) Intelligence Mental age - Shah, 1964; Nagar, 1970; Stern and Keisler, 1970; Kapadia, 1971.

(b) Creativity or Divergent Thinking Ability - Stolurow, 1964; Doty and Doty, 1964; Tobias, 1969.

(c) Cognitive Styles - Brown and Coop, 1970; Grieve and Davis, 1971; Douglas and Verna, 1979.

(d) Anxiety - Traweek, 1964; Flynn and Morgan, 1966; Knight and Sarsenrath, 1966, Tobias, 1973.

(e) Extraversion Introversion - Leith, 1969; Leith and Wisdom, 1970; Kapadia, 1971; Greer, 1976.

(f) Neuroticism - Socher, 1966; Crocker and others, 1976; Seddon, 1977.

(g) Achievement Motivation or Need Achievement - McKeachie, 1963; Kight and Sarsenrath, 1966, Doty and Doty, 1964.

Some attempts have also been made to study the relationship between attributes like study habit (Patel, 1978), independence (McCollongh and Vanattai, 1958), sociability (Beach, 1960) and the instructional treatment modes.

The instructional treatments generally studied are: traditional method, demonstration method, group study methods self instructional methods like various styles of programming, individually prescribed instruction and computer assisted instruction.

A review of ATI researches conducted by Buxton (1964), Bracht (1970), Kapadia (1972), Berliner and Cohen (1973) and the present investigators (Chapter II), reveals that the results of these studies are quite often contradictory and inconsistent. For example, Stolurow (1964) obtained positive relationship between scores on creativity test and achievement through programmed instruction but Gotkin and Massa (1963) concluded that less creative students make significantly greater achievement using programmed instruction than do highly creative ones. Similarly Stolurow (1964) and Bhushan and Sharma (1975) found that scores on intelligence test do not correlate significantly with achievement scores based on programmed instruction, but others have concluded contrariwise (Shay, 1961; Keisler and Stern, 1970; Nagar, 1970; Kapadia, 1972). The contradictory and inconsistent findings of the ATI studies may in part be attributed to inappropriate selection of instructional treatment modes. The instructional treatment modes employed in these studies, differ in most cases, only in minor details and may not require very different attribute patterns for learning effectiveness through them. After a review of ATI research, Tobias (1976) also writes,

> In such studies both the attribute dimension and instructional treatments have not been as carefully thought out as they might be. For example, the alternative instructional treatments, the inputs have often differed from one another in only minor details and the input differences may not have engaged different abilities and hence are unlikely to interact with aptitude dimensions.

Therefore, for satisfactory results the ATI studies should employ instructional treatments which required widely different learner attributes for success through them. Sheehan and Hambleton (1977) as also support this view. They write, "To substantiate interaction, future ATI studies will have to utilise treatments which are known to require different psychological processes".

Inconsistent result of some ATI researches may also be attributed to the subject matter variability. Different subject matter areas may require differing instructional treatments for learners possessing similar characteristics. Thus ATI results obtained for one subject matter area may not be equally applicable to another area (Nuthall, 1968; Tallmadge and Shearer, 1969; Tobias, 1969). Tallmadge and Shearer (1969), for example, obtained a triple interaction between instructional treatments, subject matters and learner attributes. Thus learner for whom an inductive method was optimal for one subject matter achieved better through a deductive approach in a different subject matter area. These results suggest that ATI's may be highly specific and may vary with subject matter areas.

The lack of consistent relationship may also be attributed to inadequate research design of these studies. Variables other than those under investigation may have effected achievement, which should have been controlled for dependable conclusion. Most of the ATI studies thus seem to suffer from improper selection of instructional treatment modes, unsuitable subject matter areas and inadequate controls. For dependable results, future ATI studies should, therefore, take care of these shortcomings. The present investigation is an attempt in this direction.

A variety of learner attributes, can be employed for ATI research. A review of related studies reveals that attributes such as intelligence or mental age (Keisler and Stern, 1970; Kapadia, 1972), creativity or divergent thinking ability (Rechard, Jason and Marwin, 1969; Tobias, 1969), anxiety (Leith, 1969; Tobias and Williamson, 1968) and introversion/extroversion (Leith, 1969; Leith and Wisdom, 1970; Greer, 1976) are important for achievement. The present study, therefore, proposes to employ mental ability variables like intelligence and creativity or divergent thinking ability and personality variables like anxiety and introversion/extroversion for studying interaction effects. The achievement prior to instruction, which has come out as very important variable which influences achievement after instruction in most of the investigations (Tobias, 1976) is proposed to be employed as a control variable in the present study.

A review of researches also shows that significant interaction effect have not been revealed in number of studies because they have employed instructional treatment modes which may not require the learner attributes employed for success in them. The present study therefore, proposes to employ the treatment modes which seem to require different mental ability and personality variables - intelligence, creativity, anxiety and extroversion / introversion - employed in the present study. There is a strong rationale for interaction between these learner attributes and achievement via programmed instruction. A number of investigaters such as Kight and Sarsenrath (1966), O'Reilly and Ripple (1967) and Leith (1969) have presented strong arguments for expecting anxious and introvert subjects to profit more from programmed instruction as compared to less anxious and extrovert subjects. Programmed format removes personal evaluation from the learning situation, reduces the difficulty of the subject matter, and decreases the amount of uncertainity a pupil experiences making it eminently suitable for anxious and introvert subjects.

There is sufficient evidence to show that different programmed instructional styles require different intelligence and creativity levels for success through them (Reed and Hayman, 1962; Doty and Doty, 1964; Kapadia, 1972; Singh, 1977; Agarwal, 1978). The present study, therefore, proposes to employ programmed instruction - linear and branching styles and expository method for this purpose.

Tallmadge and Shearer's (1969) observation, that a particular attribute or pattern of attributes is not equally important in the instruction of different subject matter area seems to be important. A variety of subject matter can be employed for investigating the ATI effects. In studies like the present one it may not be possible to employ more than one such subject matter areas because of limitations of time and resources. The subject matter areas selected for this purpose should suit the instructional treatment modes and the learner characteristics employed in the study. Statistics because of its well defined and sequential steps seems to be eminently suitable for programmed learning styles. Studies employing statistics and mathematicoes as the subject matter areas have shown sufficient interaction with learner attributes proposed to be employed in the present study. The present study therefore, proposes to employ statistics or more specifically "measures of central tendencies" as the subject matter area.

In specific terms the present study proposes to investigate the relationship between personality variables like anxiety and extraversion/introversion and mental ability variables like intelligence and creativity and achievement through linear and branching styles of programming and expository method of teaching, "measures of central tendencies" a topic of high school statistics course has been employed as subject matter area. The achievement prior to instruction and intelligence is proposed to be employed as a control variables.

HYPOTHESES:

In order to investigate the relationship the study proposes to test the following hypotheses:

1. The achievement of high intelligence students will differ significantly from that of low intelligence students, employing either linear or branching style of programming or expository method of teaching.
2. The achievement of high creative students will differ significantly from that of low creative ones employing either linear or branching styles of programming or expository method of teaching.
3. The achievement of high anxiety students will differ significantly from that of low anxiety students,

employing either linear or branching styles of programming or expository method of teaching.

4. The achievement of extrovert students will differ significantly from that of introvert students employing either linear or branching style of programming or expository method of teaching.

PROCEDURE IN OUTLINE:

1. Two programmes - linear and branching on measures of central tendencies (statistics) suitable for high school students of the board of high school and intermediate education, U.P., were developed and validated. A pretest consisting of items for measuring entry level and terminal behaviours and a post test consisting of only items measuring terminal behaviours were also constructed.

2. Suitable measures of the personality and mental ability variables proposed to be employed in the present study were selected. These measures are:

   (a) Mixed Type Group Test of Intelligence by P.N.Mehrotra.

   (b) Verbal Test of Creative Thinking by Baqar Mehdi.

   (c) Hindi Version of Eshynok's Modsley Personality Inventory by S.D. Kapoor and S.Jalota for measurement of extraversion.

   (d) Sarason's General Anxiety Scale for Children.

3. Previous achievement in maths as measured by Junior

Highschool marks in mathematics and intelligence are proposed to be employed as control variables. The marks obtained by the students comprising the present study in mathematics at the Junior Highschool stage were collected.

4. The sample of the study consisted of 287 students studying in class IXth of three urban Intermediate Colleges of Uttar Pradesh. The different measures of personality and mental ability were administered to these students. Pretest was then administered to ensure that the subjects possessed the required entry level behaviours and do not possess any of the behaviours which are proposed to be taught.

5. Three groups of students from each of the above three schools were taught separately by linear and branching style of programming and expository methods of teaching. The post test was administered after the instructional programme. The post test scores served as criterion measure.

6. The personality and mental ability and the criterion scores of the students from three institutions taught through each of the treatment modes were pooled separately. Thus three groups were constituted. Each taught by one of the treatment modes. Each group was than divided into

high and low groups on the basis of personality and mental ability scores.

7. Comparisons were made between the high as well as low groups on any personality or mental ability variable taught by the three treatment modes to ensure that the groups were matched. Comparisons were also made between the groups high or low on each variable to find that significant differences existed.

8. Means of criterion scores obtained by students high and low on each personality and mental ability variable taught through three treatment modes were adjusted for previous achievement in mathematics and intelligence by means of analysis of covariance. Comparisons of these adjusted means were made for different pairs of treatment modes for assigning treatment modes for optimal achievement.

DELIMITATIONS:

In view of the paucity of time and resources, the study has been delimited as follows:

1. The study is confined to the male students studying in IXth class of three urban Intermediate colleges of western Uttar Pradesh. The extent to which the findings may be considered applicable to other situation will depend upon the similarity between the sample under study.

2. A variety of personality and mental ability variables may be employed for studying interaction effects. The present study has employed only two personality (anxiety and extr version) and two mental ability variables (intelligence and creativity) for this purpose as they were considered suitable for the instructional treatment modes selected for the present study.

3. A variety of instructional treatment modes can be employed for studying interaction effects. The present study has employed only linear and branching styles of programming and expository method of teaching as instructional treatment modes because they seem to require different mental ability and personality variables - intelligence, creativity anxiety and extra-version - for success through them.

4. The present study has attempted to investigate the interaction between some measures of mental ability and personality and three instructional treatment modes in only one subject matter area i.e. statistics. This area was selected because it was considered suitable for the instructional treatment modes and the learner attributes proposed to be investigated in the present study.

5. A variety of statistical treatments have been employed in related research. They includes - compution of

coefficient of correlation, partial correlation, analysis of variance and analysis of covariance. The present study has employed analysis of covariance for this purpose, specially because it helps to eliminate statistically the effects of extranious variables which might influence interaction. This technique also allows the comparison of extreme groups on any personality or mental ability variable so that alternative instructional treatment modes may be assigned for optimal achievement.

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CHAPTER - II

REVIEW OF STUDIES

Interaction between Intelligence and Treatment Modes

Interaction between Creativity and Treatment Modes

Interaction between Cognitive Styles and Treatment Modes

Interaction between Anxiety and Treatment Modes

Interaction between Extroversion and Treatment Modes

Interaction between Achievement Motivation/Need Achievement and Treatment Modes

Interaction between Neuroticism and Treatment Modes

Interaction between Other Variables and Treatment Modes

References

It is well recognised that achievement is a function of both learner characteristics and treatment modes. A learner may achieve low with one instructional treatment, but may achieve quite high when another instructional method is employed. In recent years quite a few studies have been conducted to investigate the interaction between the different personality and mental ability variables and instructional treatment modes. These studies have been called as Aptitude-Treatment Interaction or Attribute-Treatment Interaction (Tobias, 1976), ATI or Trait-Treatment Interaction, TTI (Berliner and Cohen, 1973). These studies have attempted to investigate the interaction between quite a large number of personality and mental ability variables and a variety of instructional treatment modes. Bracht (1970), Tobias (1973) and Berliner and Cohen (1973) have presented exhaustic reviews of these studies. According to Bracht (1970) more than 95 studies had been attempted upto 1969 alone. Sufficient research work has been attempted in this area since the year 1969 and according to the estimates of the present investigator, around one hundred and twenty studies have been completed till the year of writing this review. Therefore, it would not be possible to review all the studies conducted in the area. However, an attempt has been made to review the representative studies. This review has been presented in the following pages.

INTERACTION BETWEEN INTELLIGENCE AND TREATMENT MODES:

It is a well known fact that intelligence is the single most important factor contributing to scholastic achievement. Therefore, Cronbach and Snow (1969) recommend the development of alternative treatments on the basis of general mental ability or intelligence. In view of the importance attached to intelligence, some educational psychologists have attempted to individualise instruction on the basis of intelligence or mental ability alone. In the following pages review of studies concerned with the interaction between intelligence and various treatment modes has been presented. Some investigators have sought to investigate the relationship between intelligence and single treatment methods while others have attempted to investigate the interaction between intelligence and two or more than two treatment methods.

(A) Intelligence and Single Treatment:

Silberman (1961) conducted an empirical study to investigate the interaction between intelligence and one treatment method. The method consisted of a programme with multiple choice answers. He concluded that intelligence and achievement through the above programme were significantly related. The sample of study was high school students. Limbert and others (1962) observed that intelligence was significantly related to immediate achievement through linear programmed instruction. But Glaser and Reynolds (1962) while investigating the learning variables in programmed instruction concluded that intelligence does not predict learning from a linear programme.

Stolurow (1964) with a small group of 20 highly intelligent, + 13 years old students found that the scores on intelligence or mental age did not significantly correlate with any of the score based upon self-instructional material in logic, mathematics, or statistics. But Evans (1965) obtained a positive and significant correlation between intelligence and achievement through a programmed material on the same material.

Davis and Leith (1967) conducted a study to establish where, when and how a programme functions at optimal efficiency and obtained a value of Pearson's coefficient of correlation of +0.495 ($p < .001$) between intelligence and achievement through 'Clearway Programmed Book Logarithms', devised by J. Hartlay (1964). The significant correlation shows that high intelligence group achieved better than low intelligence group.

O'Reilly's (1960) tried to explore the relationship of some personolôgical variables including verbal intelligence and achievement through a linear programme. The sample consisted of 120 boys and 122 girls of sixth grade.

The regression analysis of the data indicated that verbal intelligence contributed to achievement significantly. The result also revealed that only verbal intelligence contributed significantly to error rate on programme.

Bhushan (1973) attempted to study the relationship between different personality and mental ability variables including intelligence and achievement through a linear programme. His sample consisted of 42 (27 males and 15 females) B.Ed. students teachers. The analysis revealed that the post test scores were significantly and positively correlated with intelligence.

Sansanwal (1978) conducted a study to find out the relationship between achievement through programmed learning and intelligence. Measures of academic motivation and English language comprehension were employed as controls. The 24 B.Ed. and 43 M.Sc. students formed the sample of the study. The achievement of students through programmed learning was found to be significantly related with intelligence.

A review of the studies involving interaction between intelligence and single treatment mode reveals that quite a few of them have succeeded in establishing significant interaction between them. They show that students of high intelligence achieve high through the treatment mode investigated and vice versa. Bhushan, 1973; Sansanwal, 1978; O'Reilly; 1969; Davis and Leith, 1967; Evans, 1965; Limbert and others, 1962; Silberman, 1961). Some have, however, failed to reveal any interaction between intelligence and treatment modes employed by them (Stolurow, 1964; Glaser and Reynolds, 1962). The lack of interaction obtained in the above studies may be due to the treatment modes employed by them and the range of intelligence of the subjects of the study. Better interaction may be obtained by employing more than one treatment modes. Bracht (1970) also support this view. He writes that "Research with alternative treatment which rely differentially on general ability may provide evidence for the occurance of ATI".

Studies have also been conducted to investigate relationship between intelligence and more than one treatment modes. A review of such studies has been presented in the following paragraphs.

(B) Intelligence and Two or More Treatment Modes:

McConnell (1934) in a study to compare the relative effectiveness of discovery and authoritarian method of teaching obtained a higher correlation between mental age and achievement through authoritarian method as compared to achievement through the discovery method. The sample of his study was six hundred and fifty three pupils of second grade arithematic from fifteen different schools.

Guetzkow and others (1954) in an experimental comparison of recitation (A-V) discussion and tutorial methods found that different levels of intelligence did not call for different methods of teaching. Haigh and Schmidt (1956) also obtained insignificant interaction between intelligence and teaching through teacher centered and group centered methods of teaching.

Ward (1956) in an experiment to assess the comparative effectiveness of group study and lecture-demonstration methods in physical science instruction found that high mental ability group achieved more from group method whereas students with low ability benefitted more from lecture - demonstration, method of teaching.

Calvin, Hoffman and Harden (1957) attempted to investigate the interaction between intelligence and achievement on problem solving behaviour through authoritarian and permissive classroom climates. It was found that low intelligence group achieved significantly higher through authoritarian climate whereas high intelligence group achieved higher through permissive climate.

Porter (1959) carried out an experiment in the teaching of spelling to elementary school children. In the experiment one group was taught via machine teaching (experimental group) and the other through conventional method (control group). The analysis of data revealed that the coefficient of correlation between intelligence and achievement through machine teaching was - 0.128, which is now significant whereas that between intelligence and achievement through conventional method was +0.343, which is significant at 0.05 level. But Hatch and Flint (1962) concluded that intelligence is significantly related to achievement through both programmed and conventional teaching methods. Kay (1961) in a study to compare the relative effectiveness of discovery and direct instruction method of teaching for students with different levels of intelligence did not obtain any significant interaction between the above mentioned teaching methods and intelligence. The experiment was performed with 117 students of class ninth on a task of '0-1 inch micrometer caliper'.

Reed and Hayman (1962) in an experiment involving use of "Eng.2600" an automated instruction text found that high ability group achieved significantly higher from programmed learning method whereas the low ability group achieved higher from classroom instruction.

Hutchinson (1963) designed an experiment to study the relationship between mental age and two methods of teaching (authoritarian and creative teaching) of Highschool students. The control group was taught through authoritarion teaching and another experimental group was taught through creative teaching method. The result indicated that among the control subjects there was a significant correlation between mental age and achievement gain. In contrast there was no correlation between mental age and achievement gain for the experimental group.

Wittrock (1963) conducted a study with 80 elementary school children. The experimental group was taught 'A kinetic molecular theory programme' by responding aloud whereas the same programme without overtly responding was administered to control group. The analysis of variance revealed a significant ($p < .05$) interaction between mental age and response mode. The analysis further revealed that overt response mode was more effective with the below medium mental age students than above medium mental age students.

Shah (1964) developed a programme on equation solving for class sixth in an English medium school in Delhi and evaluated it against conventional lecture method for achievement of students of different ability (intelligence) levels. The analysis of covariance showed that although programmed learning was more effective as compared to conventional teaching method, but there was no significant interaction between treatment and ability (intelligence) measures. Foord (1964) in a study comparing the effectiveness of programmed instruction (deductive approach) and programmed instruction (inductive approach) for achievement by students of different I.Q.'s concluded that no significant interaction existed between the method and level of intelligence.

Hartlay (1965) in a study of 68 secondary school girls found that the linear programme does not correlate with intelligence as measured by Heim's AH-4 intelligence test, whereas branching programme correlate significantly with intelligence. The result further revealed that low ability students were equally profited by linear and branching programmes but the students with higher ability shows a better mark of achievement from branching programme in comparison to linear one.

Desai (1966) attempted to study the comparative merit of programmed and conventional methods of teaching for students with different levels of intelligence. The sample of the study consisted of 80 students studying in class ninth. The result revealed that although programmed learning method was more effective than the conventional one, but intelligence, as measured by Desai's Intelligence Test did not interact significantly with either programmed or conventional method of teaching.

Rai (1907) conducted an exhaustive study to compare two methods of teaching viz. lecture-cum-demonstration and question-answer-cum-demonstration. His sample consisted of 40 boys of class eighth. After the teaching for twentyone periods of 40 minutes each, he concluded that high intelligence students profit more with lecture-cum-demonstration method of teaching as compared to question-answer-cum-demonstration method of teaching. The low intelligence students show no significant difference in achievement gain by the above two methods.

In an experiment conducted to study the relationship of different intelligence levels and different methods of teaching. Katz (1967) failed to obtain any evidence to support the proposition that students with different levels of intelligence should receive different instruction for optimal achievement. Similarly Gagne and Wiegand (1968) failed to obtain a significant interaction between mental ability and instructional modes. The study deals with learning to concrete rules by fourth graders.

A study carried out by Jamieson and others (1969) with 96 female and 88 male students of postgraduate level for compairing the relative efficiency of the programmed learning A-V Lectuers and lectures method of teaching for academic achievement concluded that programmed learning was the most superior method but failed to obtain any evidence regarding interaction between intelligence and the methods employed.

Ripple, Millman and Glock (1969) attempted to study the relationship between selected characteristics of the students including intelligence and relative degree of success through programmed versus conventionally structured method for teaching vocabulary (word meaning and usage). The sample consisted of 1100 students of grade eighth from 22 schools. Four way analysis of variance revealed no significant interaction of intelligence with any of the treatment modes.

Keisler and Stern (1970) conducted a study with 82 children from second and third graders to investigate the relationship between mental age and achievement by two different teaching strategies - single hypothesis strategy and multiple hypothesis strategy. The single hypothesis strategy is analogous to Bruner's conservative focussing whereas the multiple hypothesis strategy is analogous to Bruner's scranning. heir results revealed that children of high mental age profit more from multiple strategy whereas children of lower mental age profit more from single strategy. In another study Stern and Keisler (1967) confirm the above findings.

Koran (1971) conducted a study at the University of Texas with 167 upper division students enrolled in an introductory course in teacher education to study the intersection of abilities (induction, reasoning, verbal comprehension and memory) and two treatment methods (viz.inductive and deductive programming). A 2X2 analysis of variance revealed that on the criteria of time, both inductive and deductive treatment methods were almost equally effective. The analysis further concluded that induction, verbal comprehension and reasoning score interacted significantly with above two treatment methods.

Nagar (1971) tried to compare the achievement through two instructional methods - Harbartian method and linear programme at three different levels of intelligence employing stratified sample of 207 boys and 143 girls of class ninth. He concluded that intelligence and methods of teaching did not interset significantly.

Kapadia (1971) computed coefficient of correlation between intelligence, as measured by Desai and Bhatt Group Test, and achievement through linear and branching programmes after partialling out the effect of anxiety, self sufficiency and introversion/extraversion. He obtained an significant and positive relationship between intelligence and achievement through the above two methods.

Pandaya (1974) carried out a study to determine the relationship between intelligence and achievement through programmed learning and conventional method by a group of Vth class students. The topic was 'Light'. Achievement motivation was used as a control varibble. The study revealed that the students of different mental ability levels profit more from programmed method as compared to conventional method.

Ayman (1974) in a study to compare the relative effectiveness of 'modern approach' (Nuffield approach) and 'traditional approach' on children's understanding of taxonomy at two levels of intelligence found that an interaction between intelligence and teaching methods existed.

The 'modern approach' favoured above than average intelligence students whereas the 'traditional approach' was favoured by below average intelligence students. Ryman (1977) in an another study with 96 first year of secondary education students confirm the above interaction effect. He also concluded that the use of 'modern approach' was particularly unsuitable for girls of below average ability.

Bhushan and Sharma (1975) conducted a study comparing three instructional strategies - traditional, programmed instruction and programmed instruction followed by lecture - at three different levels of intelligence. The result showed that achievement through programmed instruction (linear) followed by lecture was the highest of all the three treatments at all the intelligence levels. Out of remaining two treatments, programmed instruction was found to be superior to the lecture method for all the three intelligence groups. Thus levels of intelligence showed no interaction with any of the methods included in the study. The subject of the study were B.Ed. students and subject matter was rank correlation.

Patel (1975) attempted to study the effectiveness of programmed learning and conventional teaching in geometry for standard IXth for different variables including intelligence. Programmed learning proved to be more effective than conventional for different levels of intelligence.

Govinda (1976) developed a programmed text on testing and technique of evaluation for B.Ed. students and attempted to study the relationship between achievement through programmed and structured lecture method and attitude, intelligence, academic motivation and English language comprehension. The matched group design with 35 students in experimental and 34 students in control group was adopted for conducting the experiment. The study did not reveal any interaction between intelligence and the two treatment mode employed.

Sodhi (1977) attempted to study the relationship between different variables including intelligence and achievement through linear programme and lecture method on a group of 155 students of class IXth. The study revealed no interaction between intelligence and achievement through the above modes.

Singh (1977) attempted to study the relative effectiveness of three instructional methods - linear and branching programming and conventional methods for teaching arithematic to VIIIth class students with different levels of intelligence. Significance of difference between means of scores obtained by the students of different groups was ascertained by computing 't' values. It was found that branching programme was specially suited for high and medium intelligence groups and the low intelligence group benefited more by the linear programme. The branching programme was superior than the linear for both high and medium intelligence groups. Branching programme was also superior than conventional for high intelligence group but not for medium intelligence group. Linear programme was inferior for both high and medium intelligence groups but better than conventional method for low intelligence group.

Bhushan and Tyagi (1977) conducted an empirical study to investigate the interaction between intelligence and treatment methods. The two treatment methods consisted of a linear programme in geography with two response modes - constructed response and discriminant response mode. The sample was of sixty students of class eighth. It was found that the two response modes interacted significantly with intelligence. Although the average and above average intelligence groups showed no significant difference on the measure of achievement gain, but the achievement gain of below average intelligence group was significantly lower than the other two groups in both the treatments.

Sheehan and Hambleton (1977) in a study to adopt instruction to individual differences found that intelligence showed no significant interaction with the instructional modes employed. The instructional modes employed were teacher directed, media, reading and programmed instruction.

Agarwal (1978) conducted a study to investigate the interaction between intelligence (verbal and non-verbal) and achievement through three treatment modes - linear programme, branching programme and expository method of teaching. The sample of the study consisted of 125 students of class eighth and the subject matter under study was 'arithematic'. The partial correlation of the intelligence (after partialling out the effect of creativity and anxiety) with the achievement through all the three treatment modes revealed that intelligence plays a very important part for achievement through branching programme and is least important for expository method. The correlations further revealed that intelligence seems to be more important for linear programme than for expository method.

Gupta (1978) attempted to investigate the relationship between intelligence and analytic - synthetic method and Narration - Explanation method of teaching. His sample consisted of 110 students of VIIIth and IXth classes. Previous achievement was used as control variables. The analysis of covariance revealed that high intelligence class VIIIth students did not differ significantly with respect to their achievement through the two modes under study, whereas the analytic-synthetic method was found to be more effective than narration-explanation method to the low intelligence students of VIII class. For class IXth, the analytic-synthetic method proved to be significantly better than the narration-explanation method for low as well as high intelligence groups.

Hermann and Hinckeman (1978) attempted to study the relative effectiveness of inductive and deductive approach of programming for students with different levels of I.Q. The only difference in the two treatments was the relative placement of rules and examples. The study revealed that intelligence did not interact significantly with either inductive or deductive programming approach.

The review of above mentioned studies clearly indicates that some of them have obtained significant interaction between intelligence and treatment modes (McConnell, 1934; Ward, 1956; Calvin and others, 1957; Porter, 1959; Hatch and Flint, 1962; Reed and Hayman, 1962; Hutchinson, 1963; Wittrock, 1963; Hartlay, 1965; Rai, 1967; Stern and Keisler, 1967; Keisler and Stern, 1970; Koran, 1971; Kapadia, 1972; Ryman, 1974and 1977; Singh, 1977; Bhushan and Tyagi, 1977; Gupta, 1978; Agarwal, 1978). The above mentioned studies clearly indicate that intelligence or a measure of mental abilities plays a crucial role in assigning treatment mode. The different intelligence levels require different treatment modes for optimal gains. However, there are some studies which have obtained statistically insignificant relationship between intelligence and treatment mode (Guetzkov and others, 1954; Haigh and Schmidt, 1956; Ray, 1961; Shah, 1964; Foord, 1964; Desai, 1966; Katz, 1967; Gagne and Wiegand, 1968; Jamieson and others, 1969; Ripple and others, 1969; Nagar, 1971; Pandaya, 1974; Patel, 1975; Bhushan and Sharma, 1975; Govinda, 1976; Sodhi, 1977; Sheehan and Hambleton, 1977 and Herman and Hicksman, 1978). The insignificant finding could have been due to poor design and small sample. Desai (1966), Stolurow (1964) and Govinda (1976) have employed samples of even less than 100 subjects. The lack of significant interaction could also be due to minor differences in treatment modes employed by them. For example, Bhushan and Sharma (1975) employed programmed learning and programmed learning followed by lecture method, which are obviously not very different and hence an interaction can not be expected.

INTERACTION BETWEEN CREATIVITY AND TREATMENT MODES:

The studies reviewed in the preceding paragraphs have shown that intelligence is quite an important variable for selecting treatment modes for optimal gain, but some investigators feel that additional advantage can be gained if besides intelligence some other mental ability variables are also used to select the instructional treatment modes (Stolurow, 1964). The studies in the area suggest that creativity or divergent thinking ability may interact with treatment modes. Quite a few studies have attempted to explore the relationship between creativity and different treatment modes. A review of such studies has been presented in the following pages.

Some of the investigators have studied the interaction of creativity and single treatment modes while the others have investigated the interaction between creativity and two or more treatment modes.

(A) Creativity and Single Treatment Mode:

Stolurow (1962) conducted a study with 20 High-school students ranging in age from 13 to 15 and in I.Q. from 112 to 157. He used self instructional material in logic, mathematics and statistics. These students were scored for originality with the help of three Guilford's Tests and low TAT cards with instructions to make up a story. The originality scores obtained from Guilford's test correlated significantly with achievement on the statistics unit while the originality score obtained from the T.A.T. test correlated significantly with mathematics final examination marks.

Gotkin and Massa (1963) carried out a study to establish the relationship between creativity and achievement through programmed learning. 42 highly intelligent fourth and fifth grade pupils received about 30 minutes per day of programmed instruction in language art for a period of about two months. The post achievement test revealed that among the fifth grade pupils it was the less creative who made significantly greater achievement gain. The relationship between the measure of creativity and post test achievement was -0.34 and -0.29, respectively for IVth and Vth grade. The study also revealed that most of the highly creative children expressed a strong dislike for programmed instruction, while almost all of those in the lower half of the distribution liked programmed instruction.

Doty and Doty (1964) in a study to establish the effectiveness of programmed instruction in relation to some student characteristics including creativity found a significant (+)ve correlation between creativity and achievement through programmed instruction. The sample of their study was 100 college undergraduates of age 18-19 years.

O' Reilly's (1969) study aimed at determining the importance of creativity for achievement through programmed learning. The result showed that creativity did not contributed significantly to achievement through programmed instruction. Welcome (1973) in his study reported that high creative childrens of class seventh, eighth and ninth grade achieved well under individualised learning situation.

A review of studies involving the interaction between creativity and single treatment mode are very few. However, most of the studies have succeeded in establishing significant interaction between them (Stolurow, 1962; Gotkin and Massa, 1963; Doty and Doty, 1964 and Welcome, 1973). The study of O'Reilly (1969) did not obtain any significant interaction between creativity and the treatment mode employed.

(B) Creativity and Two or more Treatment Modes:

The above studies have employed single treatment modes. Some studies have also attempted to investigate the interaction of creativity with two or more treatment modes. In the following pages the review of such studies has been presented.

Hutchinson (1963) conducted a study to investigate the interaction between creativity and achievement through authoritarian and creative method of teaching. With a sample of highschool students he obtained a significant relationship between measures of creativity and achievement in an experimental group taught through creative teaching method whereas no significant relationship was obtained for the control group taught through authoritarian method of teaching.

MacDonald and Raths (1964) using the Minnesota Tests of Creative Thinking (Torrance, 1962a) divided 81 children in the intermediate grades into three groups (high, middle and low creativity) with a view to adopt twelve different curricular tasks to their learning styles. In a carefully controlled experiment the above mentioned three creative groups were given a series of twelve fifteen minute curricular tasks. The twelve tasks were (i) three frustating tasks one each in the areas of ideas, numbers and words, (ii) three closed tasks in which every student was required to respond alike to be successful, (iii) three open tasks and (iv) three passive tasks which required the children to only listen to the material. The study revealed that high creative thinking ability children were significantly more productive on the frustating task than the other two groups. The children low in creative thinking ability were significantly low on the frustating task than the other two groups. The low creative thinking ability group was significantly less productive than the other two groups on the 'open' task. There were statistically no significant differences among the three groups on the closed and permissive tasks. Thus it appears that children who were low in creativity tend to dislike frustating curricular tasks and to frustating tasks. Those high in creativity tend to dislike closed assignments. The results thus suggest that pupils of varying level of creativity should be taught by varying procedures for optimal gains.

In an extensive study conducted to investigate interaction between two levels of creativity (high and low), two response modes (constructed response and reading) and two kinds of material (familiar and technical), Tobias(1969) found that creative students achieved higher under all conditions, thereby, indicating absence of any significant interaction between creativity and treatment modes employed. The creativity score was calculated by Remote Association Test, which conceptualises creativity from associationists point of view. The study, however, reveals significant interaction between subject matter and response mode. The constructed response mode group achieved more on technical material than on familiar subject. The subject of the study were 100 students recruited primarily from educational psychology class with a mean age of 20.5 years.

Ripple, Millman and Glock (1969) in a study with junior school children found no interaction between certain student characteristics including creativity and achievement through programmed instruction and conventionally structured learning task. The subject matter was meaning, and usage of 24 English language words.

Crocker and others (1976) attempted to study the interaction between verbal and figural creativity and structured and unstructured teaching strategies. Programmed instruction constituted structural strategy, and expository method an unstructured strategy. The sample of the study was 120 sixth grade students. The result showed no statistically significant interaction of either structured or unstructured teaching strategies with creativity.

Agarwal (1978) conducted a study with 125 students of class eighth to investigate the relationship between creativity and achievement through three treatment methods - linear programme, branching programme and expository method. The partial correlations, after partialling out the effect of anxiety revealed that creativity has insignificant relation with achievement through either linear or branching. It yields a slightly better relation with achievement through expository method. The r is +.34 (S.E. = .15) which is significant at .05 level.

A review of studies investigating the interaction between creativity and achievement through different treatment modes reveals that some of them have obtained insignificant interaction between the two (Tobias, 1969; Ripple and others, 1969; Crocker and others, 1970). This seems to have occured due to the fact that in most of these studies relevant subject variables have not been controlled adequately and/or the independent variables had not been defined objectively (Tobias, 1969). However, there have been some studies which have obtained significant interaction (Hutchinson, 1963; McDonald and Ruthe, 1964; Doty and Doty, 1964; Melcome, 1973; Agarwal, 1978). It would be noted that these are some of the well designed studies and conducted by fairly well known researchers. It would, therefore, we safe to conclude that an interaction does exist between creativity and treatment modes specially when one of the modes studied either seems to dampen or promote creative potential of the learner. Torrance also writes, "Children with differentially developed mental abilities profit differentially from different methods of instruction".

INTERACTION BETWEEN COGNITIVE STYLE AND TREATMENT MODES:

Recently some workers in the area have attempted to study the interaction between cognitive style and instructional treatment modes. Cognitive styles are likely to play an important role in determining success through instructional treatment. For example, field independent student may achieve higher with unstructured instructional methods of teaching and field dependent students may achieve higher with teacher dominated method. Similarly, the students with analytic and non-analytic cognitive styles may differ in their achievement employing such instructional treatments as inductive and deductive method of teaching. A review of such studies has been presented in the following pages.

Brown and Coop (1970) conducted a study to investigate the interaction between cognitive style and achievement through teacher structured approach and independent problem solving approach of teaching. The sample consisted of 170 undergraduate educational psychology students, out of which 40 most analytic and 40 most non-analytic students were identified on the basis of Siegel Cognitive style Test. The study revealed an insignificant interaction between cognitive style and treatment mode employed. Grieve and Davis (1971) also did not obtain any significant interaction between analytic and non-analytic cognitive style and method of instruction. The sample of their study was IXth grade students of geography.

Nelson (1972) conducted a study to find out the effects of analytic global (non-analytic) and reflectivity-impulsivity cognitive styles on the acquisition of geometry concepts presented through emphasis or no emphasis and discovery or expository lessons. The study revealed that neither type of instruction was more effective either for analytic or for global subjects.

Grippin and Ohnmacht (1977) carriedout a study to find the interaction between field independence and dogmatism (cognitive style variables) and achievement on a programmed learning task with and without strong prompts. The study did not reveal any significant interaction between the cognitive style variable and treatment modes employed. The subjects were 47 undergraduate students and the subject matter was Russian vocabulary lesson.

Thornell (1977) attempted to study the relationship the analytic and non-analytic dimensions of cognitive styles and two instructional strategies varying in the degree of written guidance provided to the learner. The subject matter was mathematical symmetry. The study did not obtain any interaction between cognitive styles and treatment modes employed. Similarly Douglas and Verna (1979) conducted a study to find out the interaction between field dependence-independence cognitive style and achievement through low guidance or a high guidance treatment modes. The subjects were 61 prospective elementary school teachers and the subject matter was 'Net works' in mathematics. The study revealed no significant interaction between cognitive style and treatment modes employed.

Douglas and Kahle (1978) attempted to identify the possible interaction between cognitive styles. Field dependence and field independence of the students and deductive and inductive sequence of instruction. The subjects of their study were 627 (312 females and 315 males) biology students from six midwestern Highschools. The subject matter was Mendelian genetics. The control and treatment groups were equalised on the basis of derived I.Q. The study revealed that the interaction of cognitive style and instructional sequence was not significant. However, when only those students found in the tails of the field dependence-independence continuum were considered, the interaction reached significance at .001 level. The nature of the treatment aptitude interaction was such that field independence subjects experienced greater success with the inductive method and the field dependence subjects experienced greater success with deductive method.

Douglas and Verma (1979) attempted to study the interaction between field independence cognitive style and achievement through discovery and expository treatment modes. The subjects were 46 prospective elementary school teachers and the subject matter was numeration system in mathematics. Multiple regression analysis revealed a significant disordinal interaction. The field independent students achieve most in a discovery treatment, and field dependent students learn best in expository instruction. Mcleod and others (1978) in a similar type of study and on the similar type of mathematics content also found a significant interaction between field independence and instructional treatment modes.

A perusal of the study reviewed above shows that the studies investigating interaction between analytic and non-analytic cognitive styles and different instructional modes have not shown any interaction (Brown and Coop, 1970; Grieve and Davis, 1977; Nelson, 1972; Grippin and Ohnmacht, 1977; Thornell, 1977; Douglas and Verma, 1979) while the studies employing field independence-dependence and different instructional modes have generally present evidence of significant interaction (Douglas and Kahle, 1978; Douglas and Verma, 1979 and Mcleod and others, 1978). However, the study by Douglas and Verma (1979) employing field dependence independence as cognitive style and programmed learning with and without prompt as treatment mode has not shown any interaction. This perhaps is due to the fact that the instructional treatments differ very insignificantly and the difference does not seems to have any relationship with the cognitive styles employed.

INTERACTION BETWEEN ANXIETY AND TREATMENT MODES:

Anxiety is a highly unpleasant and painful state of tension which the individual does any thing to get rid off. Anxiety is for the personality what pain is for the body. The importance of anxiety as a powerful influence in our present day civilisation is acknowledged in all quarters. Since the effectiveness of different treatment modes varies with individual differences in personality, hence there is a possibility of an interaction between anxiety and treatment modes (Sarason, Mandler and Chaighall, 1952; Flander, 1951). In the following pages a review of such studies has been presented. Some studies have sought to investigate the interaction between anxiety and single treatment mode while the others have sought to investigate the interaction between anxiety and two or more treatment modes.

(A) Anxiety and Single Treatment Mode:

Kight and Sarsenrath (1966) investigated the relationship between anxiety and achievement through linear programme on test construction. The study revealed that high anxiety group worked faster and made fewer programme errors in comparison to low anxiety group. There were, however, no achievement differences between the low and high anxiety groups. The sample of the study was 139 undergraduates.

O'Reilly and Ripple (1967) attempted to study the interaction between anxiety and learning through linear programme dealing with longitude and latitude. His subjects were sixth grade students. The study revealed that there was a correlation of -0.53 between achievement and test anxiety, measured by Sarason's Test Anxiety Scale for children, thereby showing that low anxiety subjects achieve higher than the high anxiety subjects with linear programmed instruction method. But Davis and Leith (1967) in a similar type of study on a 'Clear way Programmed Book Logarithms' devised by J.Hartlay (1964) obtained insignificant relationship (r = +0.106) between achievement through programmed task and anxiety. Similarly a study by Gangopadhyaya(1971) revealed no significant differences in achievement of lower and higher anxiety group on a constructed response programme.

Most of the studies reviewed above (Kight and Sassenrath, 1966; Davis and Leith, 1967; Gangopadhyaya, 1971) show that anxiety does not play any significant part in learning through programmed method. However, one of the studies (O'Reilly and Ripple, 1967) did obtain a significant negative relationship showing thereby that subjects with a low level of anxiety are likely to achieve better. Thus there is a need for further researcher, for dependable conclusions.

(B) Anxiety and Two or More Treatment Modes:

Some research workers have attempted to investigate the interaction between anxiety and two or more treatment modes.

Grime and Allinsmith (1961) after a study of the relationship between anxiety and structured (programmed) and unstructured method (traditional) of teaching concluded that anxious students performed better through structured method (programmed learning) than unstructured method (traditional). Traweek (1964) in a study with fourth grade learners also concluded that learners with high level of anxiety achieved more in a programmed instructional treatment than would have been expected if they had been taught by traditional method. Similarly Leith (1969) in a study to assess the effectiveness of programmed learning - complete guidance and guided discovery treatment reported that high anxiety group scored significantly higher than the low anxiety group learning under complete guidance treatment while the reverse was true for guided discovery treatment.

Flynn and Morgan (1966) tried to explore the effectiveness of programmed instruction in relationship to anxiety and other variables on an introductory unit of vector geometry. Two sets of students, equated on the basis of intelligence and achievement prior to instruction were instructed through programmed instructional method and teacher instructional method separately on a uniform subject matter. A 2 X 3 analysis of variance revealed that the interaction bwbetween anxiety and instructional modes was not significant. Similarly, Lache (1967) in a study of interaction between anxiety and two response modes found no statistically significant interaction between the two on a vocabulary programme for eighth grade students. The study by Tobias and Williamson (1968) also does not provide any evidence for interaction between anxiety and achievement through programmed instruction. The investigators hypothesised that anxious students would profit more from a constructed response programme than they would by reading the programme with response blanks filled in. The subjects of the study were 110 undergraduate students in educational psychology course. The students are randomly assigned to one of the three response mode conditions - constructed response mode, reading the programme with the response blanks filled in and constructed response without reinforcement. A ninty frame linear programme on binary numbers developed by Silberman and Alter (1961) was used for the study.

An extensive study conducted with anxiety and two treatment methods - traditional and programmed learning was that of Ripple, Milliman and Glock (1969). The analysis of variance revealed no statistically significant interaction between anxiety and treatment methods employed.

Tobias and Abramson (1971) in a study to investigate the interaction among anxiety, response modes and familiarity of subject matter on achievement from programmed instruction found with the help of multiple linear regression analysis that there was no statistically significant relationship between anxiety and achievement either through constructed response modes, with or without reinforcement or to a reading condition. The sample of their study was 144 students recruited primarily from educational psychology course.

Kapadia (1972) investigated the relationship between some personality variables including anxiety and achievement through linear and branching programmed instructional styles.

The sample of this study consisted of 525 students of class eighth. Both immediate and retention score were employed as a measure of achievement. The partial correlation between each of the personality variable (partialling out the remaining) and each of the achievement variable showed that anxiety was negatively related to achievement (immediate and retention) on linear programme but there was no significant relationship between anxiety and the two achievement measures on branching programmes.

Dowaliby, Fred and Schumer (1973) in a study with junior college students enrolled in two separate sections of an introductory psychology course arrived at a conclusion that while the teacher centered method (lecture method) optimizing learning for high anxious group, the student centered method (discussion method) resulted in higher examination performance for low anxious group. The result showed overall significant disordinal interaction.

The effect of mode of responding to computer assisted instruction, and anxiety on achievement were studied by Tobias (1973). The subject consisted of 121 students recruited from the general psychology course. These students were randomly assigned to the instructional programme under neutral or distraction conditions and to either a constructed response mode with feedback or a reading condition. The multiple linear regression analysis of the data revealed no significant interaction between anxiety and response mode.

Papey, Costello, Hedl and Spielberger (1975) attempted to analyse the learner characteristics of state and trait anxiety in determining the effectiveness of individualized multiage classroom (I M P) instruction and traditional instruction. The first grade pupils with low trait anxiety performed better in IMP situation than in the traditional situation, but the second graders with low trait anxiety performed lower in I.M.P. situation than in the traditional situation. Thus the relation between trait anxiety and achievement through IMP and traditional situation seems to be influenced by the chronoligical age of the learner.

Trown and Leith (1975) in a study to investigate the personality treatment interaction found that out of the four factors studied (sex, ability, anxiety and extraversion) studies, only the anxiety yielded significant interaction.The study reveals that students with a lower level of anxiety may be assigned to the learner centered (exploratory) method and those at the higher level of anxiety to the teacher centered (supportive) method of teaching for the optimum achievement.

Sheehan and Hambleton (1977) conducted a study with 265 students of ninth grade science employing teacher directed, media, reading and programmed instructional treatments. Johnson-Nayman analysis (Pathroff, 1964) of the data revealed that out of the many learner variables included in the study a disordinal interaction existed only with test anxiety. The analysis further revealed that students with low level of anxiety profited more through programmed instruction whereas those with high level of anxiety gained more through reading as compared with others.

Patel (1978) in a study comparing auto-instructional programmes for class eigth and traditional classroom teaching for class eighth found that it was high anxious group who achieved better on a programme of Geography when taught through auto-instructional programme in comparison to traditional method.

Hermann and Hincksman (1978) attempted to study the relative effectiveness of inductive and deductive approach of programming for students with different levels of trait anxiety. The samples used were 299 IXth grade advanced level science students of both the sexes. The subject matter chosen for the study was stoichiometry. The result suggested that for high school chemistry, there was no support for the preposition that students with differing level of trait anxiety should receive instruction with different treatment modes. But Sakmyser(1974) in a similar study using inductive and deductive programmed instruction materials on chemical equilibrium for high school students obtained a significant interaction between trait anxiety and achievement.

The foregoing review of the studies investigating the interaction between anxiety and treatment mode reveals that in most of the studies low anxiety students achieved higher when taught through structured method of teaching like programmed instruction (Kapadia, 1972; Papey and others, 1975; Sheehan and Hambleton, 1977; O'Reilly, 1967), student centered method (Dowaliby and others, 1973), exploratory (Trown and Leith, 1975) and guided discovery (Leith,1969) but high anxiety students seems to profit more from teacher centered (Dowaliby and others, 1973), supportive method (Trown and Leith, 1975) and complete guidance method (Leith, 1969). Some studies have also reported significant and positive relationship between anxiety and programmed instruction (Grime and Allinsmith, 1961; Traweek, 1964; Sakmyser, 1974; Patel, 1978).

However, there have been some studies in which an interaction between anxiety and treatment method is not revealed (Flynn and Morgan, 1966; Lache, 1967; Tobias and Williamson, 1968; Ripple and others, 1960; Tobias and Abhramson, 1971; Hermann and Hincksman, 1978). A number of reasons have been put forward for this state of affairs.

efficiency in relation to some learner characteristics like extroversion, anxiety and I.Q. found that the coefficient of Pearson correlation was -0.182 for extroversion, which implies that extroverts are not likely to succeed with programmed materials.

Leith (1969) attempted to investigate the interaction between introversion/extroversion and three treatment modes (viz. discovery, guided discovery and complete guidance). He divided the introverts and extroverts into low and high anxiety groups. He did not obtain any significant interaction between the four groups classified on the basis of introversion-extroversion and anxiety (Introversion-high anxiety, Introversion-low anxiety, extroversion-high anxiety and extroversion-low anxiety). His subjects were 64 children of age 10-11 years and the task was set of spatial problems with five principles. But in a study with 78 female adults aged over eighteen years, Leith (1969) obtained a statistically significant interaction of extroversion/introversion and mode of teaching. The study further reveals that extroverts were better in achievement from discovery method than complete guidance method.

Shadbolt and Leith (1969) conducted an experiment to investigate the interaction between extroversion/introversion and two different self instructional programmed text i.e. reception and discovery learning. His subjects were 211 first year college students aged between 18 and 20. The study revealed a significant interaction on both the immediate and retention achievement scores. The result implies that extroverts achieved better from discovery method in comparison to reception method and vice versa.

Brown (1970) tried to explore the interaction between teaching strategies and extroversion/introversion of early secondary school children. The subject matter was 'definition and elementary algebra of vectors'. The teaching strategies differed mainly in the presentation of rules before the examples and rules after examples. The study revealed that introverts achieved better when rules were presented before examples and extroverts excelled when rules were presented after examples. The overall interaction was significant not only for immediate achievement but also for retention and transfer scores of achievement.

Kapadia (1972) attempted to investigate the interaction between linear and branching programmed instructional styles and introversion/extroversion. He obtained insignificant interaction between introversion/extroversion and immediate and retention scores on the two programming styles (linear and branching). Brown and Leith (1975) obtained a insignificant interaction between extroversion/introversion and two teaching strategies - teacher centered and student centered. The sample of the study consisted of 432 junior school children.

Greer (1978) attempted to examine the relationship between extroversion/introversion as measured by Junior High School Personality Inventory and achievement through programmed learning and conventional teaching methods. The subjects were 84 eight years old children and the programme used was a Fractious programme published by Internation Tutor Machines. The study revealed a significant value coefficient of correlation between extroversion and attainment but this correlation coefficient learned out to be insignificant when partial correlation between extroversion and attainment was calculated, holding I.Q. as a constant. This clearly shows that the apparent relationship between extroversion and attainment through programmed instruction is largely a function of I.Q.

A review of studies investigating the interaction between extroversion/introversion and treatment modes reveals that a fairly large number of the studies have succeeded in establishing a statistically significant interaction (Davis, and Leith, 1967; Leith and Wisdom, 1969; Shadbolt and Leith, 1969; Trown, 1970). The overall generalisation that can be inferred from these studies is that introverts are likely to profit from a structured learning situation e.g. programmed learning whereas extroverts profit more from a unstructured learning situation viz. discovery and discussion methods. Some studies have, however, obtained insignificant interaction between extroversion/introversion and different treatment modes (Leith, 1969; Trown and Leith, 1975, Kapadia, 1972; Greer, 1974). The insignificant interaction obtained in the above studies may in part be attributed to the age level of the subjects investigated by them. This contention is based on the studies of Leith (196) Shadbold and Leith (1969). The other reasons for insignificant of interaction may be due to difficulty level of the task and subject matter familiarity.

Most of the studies investigating the interaction between personality variables and treatment modes have employed anxiety and extroversion/introversion as personality variables. This seems fairly logical becuase the variables are likely to effect achievement to a fair degree. However, some studies have also employed other personality variables like achievement motivation or need achievement, neuroticism, study habit, self sufficiency and others.

INTERACTION BETWEEN ACHIEVEMENT MOTIVATION/NEED ACHIEVEMENT AND TREATMENT MODES:

Atkinson and Reitman (1956) conducted a study to investigate the interaction between motive strength and achievement through achievement orientation treatment and multi-incentive treatment. He found that students with low motive strength achieved better with the achievement orientation treatment in comparison to multi-incentive treatment, in which students with high motive strength turn out to be high achiever.

Koeing and McKeachie (1959) conducted a study to show the interaction between need achievement and two treatment methods i.e. independent study and lecture method. The study revealed that women high in need for achievement preferred independent study to lecture method.

Doty and Doty (1964) investigated the interaction between few personality characteristics including achievement motivation and achievement through programmed instruction. The sample comprised of 100 college undergraduates. The achievement motivation as measured by Edward's Personal Preference Schedule, showed insignificant interaction. The subject matter was 1507 frames programme on physiological psychology. Similarly Sheehan and Hambleton (1977) performed a study with four treatment methods viz. teacher directed, media reading and programmed instruction. With a test of Junior Index of Motivation, the study did not reveal any significant interaction of achievement motivation to treatment methods used in the study.

Wight and Sarsenrath (1966) in a study with 139 undergraduate pupils showed that high achievement motivated students learned more efficiently through programmed learning than low achievement motivated students on all the three criteria of immediate, retention and transfer learning scores.

Pandaya (1974) carried out a study to determine the relationship between achievement motivation and the amount of learning occured through programmed learning on a group of Xth class students on a topic of 'light'. The analysis of variance and covariance of the data revealed that the programmed learning situation was more challenging to low motivated pupils than to highly motivated pupils. Govinda (1976) while investigating the effectiveness of programmed learning against structured lecture method found that academic motivation had no definite effect on achievement through either programmed or lecture method.

Kuruvilla (1977) conducted an investigation to study the relationship between achievement motivation and students achievement through different styles of programming (linear overt, branching skip and response prompt).

The population under study consisted of 301 VIIIth class students from a english medium school. The analysis revealed that there was no significant relationship between academic motivation and performance of student after post test.

Sansanwal (1978) conducted a study to find out the relationship between achievement through programmed learning and academic motivation and also compared the achievement of students of high, average and low academic motivation by taking intelligence and english language comprehension as covariates on a sample of 24 B.Ed. and 43 M.Sc. students. The analysis revealed that the achievement of students through programmed learning was not significantly related with academic motivation.

Patel (1978) conducted a study on a subject unit of geography for class VIII and compare the achievement of pupils having high and poor need achievement and learning through programmed instruction and traditional way of teaching. The study revealed that pupils with high need achievement proved themselves superior in achievement through programmed instruction in comparison to traditional one.

The above review of studies investigating the interaction between achievement motivation and different treatment methods clearly shows that this variable has played very significant role in achievement specially through auto-instructional methods like programmed learning in most of the studies (Atkinson and Reitman, 1956; Koeing and Mc Keachie, 1959; Kight and Sersenrath, 1966; Pandaya, 1974; Patel, 1978). However, some studies (Doty and Doty, 1964; Sheehan and Hambleton, 1977; Govinda, 1976; Kuruvilla, 1977, Sansanwal, 1978) have failed to arrive at such a significant relationship. This perhaps seems to be due to the measures of motivation adopted by them.

INTERACTION BETWEEN NEUROTICISM AND TREATMENT MODES:

Crocker and others (1976) conducted a study to investigate the relationship between neuroticism, as measured by 'Eysenck Jr. Personality Inventory' and achievement through structured and unstructured modes of teaching science process activities. The unstructured mode was defined as one in which the teacher controlled only the area of investigation and the supply of apparatus whereas in structured mode the teacher also controlled the specific arrangements of apparatus, the specific variables of investigation, the number and type of measurement to make and analysis procedures. The sample of the study was 120 sixth grade pupils. The analysis revealed a significant interaction between neuroticism and treatment mode.

Seddon (1977) conducted a follow up study to investigate the interaction between neuroticism and achievement through a self instructional chemistry programme. The sample of the study was 741 students of different age. The result showed a insignificant interaction between neuroticism and achievement through the self instructional programme. Greer (1978) attempted to examine the relationship between neuroticism as measured by 'Junior Eysenck Personality Inventory' and achievement through programmed learning and conventional method. The subjects were 84 eight year old children and the programme used was a 'Fraction' programme published by International Tutor Machine. This study also revealed a insignificant interaction between neuroticism and achievement through programmed or conventional method of teaching.

From the above review of studies it can be very easily inferred that the study of Crooker and others (1976) revealed a significant interaction whereas the others (Greer, 1978; Seddon, 1977) have failed to note a significant interaction between neuroticism and treatment methods employed.

INTERACTION BETWEEN OTHER VARIABLES AND TREATMENT MODES:

Wispe (1951) conducted a study to find out the relationship between independence and achievement through permissive and directive teaching methods in a introductory psychology course. He concluded that inependent group of students achieved more from permissive teaching in comparison to directive method of teaching.

McCollough and Vanattai (1958) in a study of investigating the interaction between rigid and in need of social supports students and achievement through independent study method found that independent study method facilitate in more achievement gain of less rigid and less in need of social support students in comparison to those who were high on these characteristics.

Beach (1960) studied the relationship between sociability and achievement through four different kinds of learning situations - a lecture, discussion group with instructor, discussion without instructor and an independent study group. The study revealed that highly social group performed better through discussion method whereas students who were not social, performed better through lecture method.

Doty and Doty (1964) conducted a study with 100 undergraduate students to investigate the relationship between student characteristics including social need and achievement through programmed instructional method. The study revealed a significant correlation between social need and achievement through programmed instructional method when effect of grade point average (a measure of previous achievement) was partialled out.

Kapadia (1972) in a study with 525 students of standard VIII found that score of self sufficiency and achievement on both types of programmes (linear and branching) did not correlate significantly.

Patel (1975) attempted to investigate the effectiveness of auto-instructional programme and conventional method in geometry for students belonging to different socio-economic levels. The study did not reveal any interaction between socio-economic status and learning through the two instructional treatment modes employed.

Sheehan and Hambleton (1977) in a study to adopt instructional method according to the different student characteristics in an individualised science programme found that different levels of study habit did not call for different instructional method. The instructional methods were teacher directed, media, reading and programmed instruction. But Patel (1978) concluded contrariwise. He found that pupils with good study habit achieved better when taught through programmed instructional approach in comparison to traditional method. The subject matter under study was geometry of class VIIIth.

A review of studies investigating the interaction between different personality, mental ability and other variables reveals that these studies have not been able to arrive at well defined conclusions. Their findings are inconsistent and contradictory. This state of affairs may be due to a variety of factors including small and poor sample, defective tools, lack of controls and variety of treatment modes employed. Therefore, studies with better designs and appropriate treatment modes be attempted for dependable and reliable conclusion.

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72957

CHAPTER - III

DESIGN OF THE STUDY

Personality Variables and Their Tools for Measurement

Mental Ability Variables and Their Tools for Measurement

Treatment Modes

Method of Analysis - Statistical

References

A review of previous researches presented in Chapter II has shown that while some studies give sufficient evidence of interaction between personality and mental ability variables and different treatment modes, other have failed to do so. The failure to obtain evidence of interaction may be due to inclusion of those personality and mental ability variables that do not seem to require different treatment modes or the treatment modes employed differ only in minor details so as to provide unsatisfactory evidence of interaction. Tobias (1976) is also of this view. After a review of ATI researches, he writes about the studies which have failed to provide any evidence of interaction,

> ....in such studies, both the attribute dimensions and the treatments have not been as carefully thought out as they might be. For example, the alternative instructional treatments, the inputs, have often differed from one another in only minor details ..... the input differences may not have engaged different abilities and hence are unlikely to interact with the aptitude dimension".

The present study has attempted to include those personality and mental ability variables which seems to require different treatment modes. The treatment modes have also been selected keeping in view the fact that they seems to require sufficiently different personality and mental ability variables for success through them. The review has also shown that lack of the evidence of interaction may also be due to non-employment of suitable control variables. Therefore, suitable control variables have also been incorporated in the design of this study. The present study thus seeks to investigate the interaction between personality variables (Anxiety and Extroversion) and mental ability variables (Intelligence and creativity) and three treatment modes namely, linear and branching styles of programming and expository method of teaching. Intelligence and previous achievement in the subject (mathematics) have been employed as control variables. The rationale for including these variables has been presented in the following pages.

PERSONALITY VARIABLES:

There is both theoretical and empirical support for the position that learning effectiveness is a function of the interaction of treatment modes and personality (Greer, 1977). Leith (1968) while emphasizing the importance of personality variables for assigning treatment modes write, "Learning and personality are inter-related. Whether one method or another of arranging and presenting task will help or hinder depends on the pupil's personality". Accordingly, the incorporation of personality variables seems important for determining the treatment mode best suited for an individual. The review of related researches (Chapter II) has shown that personality variable such as anxiety, extroversion-introversion, neuroticism, confidence, self sufficiency, achievement motivation, have been studied for interaction effects. Sufficient evidence of interaction has, however, been obtained for the personality variables; anxiety, extroversion-introversion, neuroticism and achievement motivation etc. Obviously,it is not possible to employ all these variables in a single research study. Ordinarily, it becomes very difficult to cope with data, if more than three or four variables are employed. Since the present investigation proposes to study the interaction effects of personality as well as mental ability variables and also to include intelligence and previous achievement in the subject as control variables, it was not considered feasible to include more than two personality variables in the present design. Extroversion-introversion and anxiety have been found to be fairly important for assigning treatment modes. Therefore, these personality variables have been employed in the present study. The rationale of inclusion of these personality variables is presented below.

(i) Anxiety:

Anxiety has emerged (Chapt.II) as an important variable determining achievement. Different levels of anxiety call for different treatment modes for optimal achievement. Grimes and Allinsmith (1961), Traweek (1964), Leith (1969), Brown and Leith (1975), Patel (1978) and others after exhaustive investigations concluded that anxious students performed better through structured or supportive methods of teaching like programmed instructions and teacher centred methods than unstructured or exploratory method of teaching. Perhaps the comparative ease with which the linear programme can be attempted and constant reinforcements incorporated in it, make it suitable for high anxiety students. This view is also supported by Tobias (1973). He writes,

> The reduced difficulty of programmes, their tighter organisation and the reduction of students uncertainity while studying a programme would suggest that these instructional modes ought to be especially beneficial for the performance of high anxiety individuals.

Carr (1962) also quotes Porter who says, "Learners who are quite anxious..... seems to be one who profit a great deal from machine instruction (programmed instruction)". According to him,

> It seems reasonable to suppose that since the machine instruction (programmed instruction) provides for many reinforcements, the learner's degree of anxiety might be reduced...... The result should be an increase in rate of learning.

Thus it can be safely inferred that structured learning situation like programmed instruction is suited to the highly anxious students whereas the unstructured learning situation like expository method is more suited to low anxiety students.

(ii) Extroversion/Introversion:

The review of related studies (Chapter II) shows that extroversion/introversion seems to play an important role in assigning treatment modes for optimal achievement. Leith and Brown (1970) after a study of extroversion/introversion and treatment modes also concluded that these children require different treatment modes for optimal achievement. The research studies by Davis and Leith (1968), Leith (1969) and others show that introverts arel likely to be profit from a structured learning situation which programmed instruction provides whereas extroverts profit more from a unstructured learning situation like expository method of instruction. After a review of a number of ATI studies, Leith (1973) concludes, "introverts are likely to be more successful when they follow a carefully sequenced, high prompted structure of learning" and extroverts, "appear to learn better in less formal discovery situation". This according to him seems due to the fact that extroverts have a greater tolerance for ambiguity and lack of structure in the teaching situation, whereas introverts are more inclined to be responsive to unambiguous and clearly structured situation. Greer (1977) also points out the fact,

> "Studies in personality have indicated that the extroverted child, who is outgoing and social, would learn best rhothrough contact with other people, his teacher, his peer group; and that the

> introverted child, the withdrawn, the emotionally disturbed, would be happiest with the individualised learning situation which programmed learning sets up. So perhaps programmed learning is more suitable learning situation for children who display traits of introversion than for those who are extroverted".

Thus it seems logical that structured and unstructured teaching modes should be assigned for introverts and extroverts respectively for maximum achievement.

The linear programme employed in the present study provides a structured, carefully sequenced and highly prompted learning situation and as such it is likely to suit the high anxiety and introverted students. This is also supported by Leith (1964). He pointed out, "We might suppose that introverted and perhaps anxious pupil will do well with linear programme." Expository method as proposed in the present study is a unstructured and unprompted learning situation, therefore, it is likely to suit the low anxiety and extroverted students. The branching programme provides a fairly structured but comparatively less prompted learning situation as compared to linear programme and thus it may suited the average persons on the introversion-extroversion and high and low anxiety dimension. Therefore, these personality dimensions were considered fairly important for assigning the treatment modes employed in the present study.

TOOLS FOR MEASUREMENT:

(1) Indian Adaptation of Eysenck's Modsley Personality Inventory:

Eysenck's Modsley Personality Inventory has been adopted by Kapoor and Jalota for Indian students. It is a questinnaire designed to measure the two factors of personality which have been found to account for most of the variance in the personality domain - Extroversion and neuroticism. The Extroversion (E) scale represents the continuum of social extroversion-introversion. High scores on the scale reflect sociability, outgoingness and carefreeness. The neuroticism scale reflects emotional instability, anxiety proneness and the tendency to develop neurotic symptoms under stress. This M.P.I. is suitable for normal and abnormal persons. This inventory can be used as a group or an individual test for persons of ages 15+. The M.P.I. consists of two scales - Long scale and short scale. Items from serial number 1 to 12 given on the front page of the test booklet make the short scale, while all the 48 items of the booklet constitute the long scale. Although no time limit is enforced in the testing, the short scale takes about 3 to 5 minutes. The long scale takes about 15 to 20 minutes. Each of these items is answerable by a tick mark '   ' in one of the boxes.

The test gives a maximum score of 48 on Extroversion and also 48 on neuroticism. The 48 items of the test booklet are distributed among the two personality dimensions and scored as shown in the following Table No.1.

TABLE I

DISTRIBUTION AND SCORING OF ITEMS OF TEST BOOKLET AMONG THE TWO PERSONALITY DIMENSIONS.

Dimension		Item No. in the booklet	No.of items	Max. Raw Score	Correct Response
Introverson	Long Scale	1,4,5,8,9,12, 20,26,28,32, 34,38,42,44, 46,48.	16	32	Yes
		14,16,18,22, 24,30,36,40	8	16	No
	Short Scale	1,4,5,8,9,12	6	12	Yes
Neuroticism	Long Scale	2,3,6,7,10,11, 13,15,17,19, 21,23,25,27, 29,31,33,35, 37,39,41,43, 45,47.	24	48	Yes
	Short Scale	2,3,6,7,10, 11.	6	12	Yes

Thus, there are 24 items for the assessment of introversion/extroversion and 24 items for neuroticism on the long scale. Each item is either an exclusive measure of introversion/extroversion or neuroticism. Each item is designed to elucidate three alternative responses namely, Yes, No and Undecided. Each item yields a score of either 0, 1 or 2. A scoring key has been provided for this purpose. The total composite extroversion or neuroticism score is simply the sum of raw scores on all the 24 items in the two dimensions. For the full scale the mean neuroticism score for the male and female groups combined was 23.2 with a S.D. of 10; this corresponds with English Norms of 19.9, S.D. 11 (Sysenck, 1959). For the extroversion scale the mean combined score was 27.8, S.D. 6.2, this compares with English Norms of 24.9, S.D. 9.7. There were no difference of any significance between males and females. The correlation between N and E for the long scale was -0.223 which is in good agreement with the English Norms. The data suggest that this Indian version of the M.P.I. gives results no essentially different from those obtained with original version in England.

In the present study the extroversion/introversion items of Esynck's Modsley Personality Inventory (Hindi adoptation by Kapoor and Jalota) have been employed as a measure of extroversion/introversion dimension of personality.

(ii) Sarason's General Anxiety Scale for Children (GASC):

Sarason's general anxiety scale for children(GASC) is a questionnaire designed to measure the anxiety level of school going children. The whole questionnaire has fourty five items in all relating to general anxiety of the children. Various areas from which the items have, generally, been chosen for the scale are the following:

(1) Health, physical appearance and injury.

(2) Afraid from animals and strange things.

(3) Success or failure in work.

(4) Worries regarding family members.

(5) Social relations and social approval.

(6) Worries regarding the future.

(7) Afraid in loneliness.

The response 'Yes' to any question is the indication for the presence of anxiety whereas the 'No' is the indication for the absence of anxiety.

The split-half reliability of the scale was found to be 0.79. Validity data was obtained by correlating the scores obtained on this scale with Sinha's anxiety scale. The validity coefficient comes out to be 0.73.

MENTAL ABILITY VARIABLES:

Since different individuals manifest different profiles of learning ability, the sucess of a given treatment mode depends on the extent to which it is accommodated to mental abilities of the learner. Torrance (1965) after a review of his own and few other important studies concluded that students with differing mental abilities profit differently with different treatment modes. He writes,

> There are now many convergent lines of research that make it clear that when a teacher changes methods of instruction or the nature of instructional materials that children with different kinds of mental abilities become the star learners and nonlearners.

Thus mental ability variables seem to be important for assigning treatment modes for optimal learning. The review of related researches (Chapter II) shows that intelligence and creativity have emerged as important mental ability variables for interaction effects. Therefore, these mental ability variables have been employed in the present study. The rationale for including these mental ability variables is given in the following pages.

(i) Intelligence:

Intelligence has been found to play prominent role in learning through different treatment modes (Chapter II). Feisler and Stern (1967) writes, "Mental maturity (Intelligence) should be considered in deciding what types of differential instruction to use in teaching certain skills". Hartlay (1965), Singh (1977) and others have concluded that linear programme is most suitable for low intelligence pupils in comparison to branching programme, which is best suited for high intelligence ones. This seems quite logical because for low intelligence pupils, as Leith (1968) points out, "Learning is facilitated by small steps, careful sequencing, cueing, immediate confirmation, low error rate and so on which linear programme provides". The average intelligence pupils achieved higher with expository method of teaching in comparison to linear and branching styles of programming (Singh, 1977). Thus it can be concluded that high intelligent pupils require a less structured and open situation whereas low intelligence pupils require a thoroughly structured one.

(ii) Creativity:

Creativity has been found to play a significant role in achievement through different treatment modes. Torrance (1965) after critically evaluating the MacDonald and Rathre (1964) study pointed out, "Pupils of varying levels of creative thinking ability..... are possibly best taught by varying procedures". Gotkin and Massa (1963) reported that for highly intelligent pupils who had worked through programme (linear), it was less creative students who made significantly greater achievement in comparison to high creatives. Torrance quotes a personal communication from Gotkin and Massa which says that in interviews conducted with the children they found that the highly creative childrens expressed a strong dislike to linear programme while low creative children liked linear programme. This seems fairly logical. Torrance (1965) also wrotes,

> Highly creative children dislike this way of learning (linear programme) and failed to learn very much from it because it was contrary to their preferred or best way of learning. They dislike being forced

> to make only tiny leaps because they are accustomed to making longer mental leaps and checking, testing and correcting their errors. The less creative childrens, however, prefer the safety of the tiny mental leaps and the authoritative feedback of their errors".

linear programme as compared to branching places stringent restrictions on independent and creative thinking and has been accused of curving original thinking and creativity (Torrance, 1965). The highly creative learners have been found to profit more from branching style of programming (Kapadia, 1972) whereas average ones achieves higher from expository method. Thus, it can be safely concluded that high creative pupils requires a less structured and open situation whereas low creatives requires a rigid and structured situation.

It can, therefore, inferred that highly structured and rigidly followed sequence situation like linear programme employed in the present study is more suitable for low intelligence and low creativity pupils. The branching programme, which is less structured and provide fairly independent learning sequence is more suitable for high intelligence and high creativity pupils. The expository method, a less independent than branching programme, employed in the present study may suit the average pupils on both intelligence and creativity dimensions of mental ability.

Therefore, these mental ability variables were considered fairly important for assigning the treatment modes employed in the present study.

TOOLS FOR MEASUREMENT:

(1) Mehrotra's Mixed Type Group Test of Intelligence:

This is a standard tool for measurement of intelligence of school going children of age ranging from 11 to 17 years. This test has been prepared in a spiral-omnibus form providing selective form of items for both parts of the test - verbal and non-verbal on the lines of Wechsler-Bellevue scale of Intelligence. Each of the verbal and non-verbal parts consisted of 5 sub-tests each. They are as follows:

Verbal Test	Non-Verbal Test
1. Analogy Test	1. Analogy Test
2. Number Series Test	2. Arrangement Test
3. Classification Test	3. Classification Test
4. Vocabulary Test	4. Digit Symbol Test
5. Reasoning Test	5. Part Fitting Test

All these sub-tests are mostly saturated with 'g' factor. Test consists of items of varying difficulty levels, high discrimination power and with positive and high correlation with the total test scores. In each test only one correct answer is to be selected for one question from the five alternatives given under each question. A score of one is awarded for each correct answer. No marks are deducted for wrong answer. Reliability of test was computed through various methods. They are presented below:

Methods	Verbal	Non-Verbal	Full
Split-Half	0.91	0.81	0.88
Test-Retest	0.89	0.82	0.86
Kudar-Richardson	0.90	0.80	0.85

Validity of the test was obtained by correlating the scores obtained on the test and teacher's ratings. It was found to be 0.87 for full test, 0.86 for verbal test and 0.72 for non-verbal test. Correlation between the scores on this test and examination marks were also obtained. These correlations were 0.39, 0.36 and 0.3, respectively.

(ii) Baquar Mehdi's Verbal Test of Creative Thinking:

This test battery is meant to identify creative talent at all stages of education except pre-primary and primary. The test is based on the Guilford's concept of divergent thinking ability. The divergent thinking abilities included are fluency, flexibility and originality. The whole test consists of the following four sub-tests:

1. Consequence Test
2. Unusual uses Test
3. Similarity Test - Relationship Test
4. Product Improvement Test.

The test-retest reliabilities of the factor scores and the total creativity scores are as below:

Fluency	Flexibility	Originality	Total Creativity
0.945	0.921	0.896	0.959

Inter-scorer reliabilities for the factor scores were found to range from 0.653 to 0.981.

The validity coefficient against the teacher ratings for each factor are given in the following manner:

Fluency	Flexibility	Originality	Total Creativity
0.40	0.32	0.34	0.39

All correlations are significant beyond 0.01 level.

CONTROL VARIABLES:

The review of researches presented in Chapter II shows that although some researchers have employed fairly important personality and mental ability variables but have failed to establish any interaction with treatment modes employed by them. This may partly be attributed to the lack of controls in their research designs. Variables other than those under study may have affected post-achievement, which should be controlled for dependable and reliable conclusion. It is, therefore, necessary to employ controls in the study. The present study employes previous achievement in the subject under study i.e. mathematics and intelligence as two control variables. The rationale for inclusion of these variables is given in the following pages

(i) Previous Achievement in the Subject:

The ultimate objective of a well designed study is to control all the factors, other than those under study which are likely to affect the post-test scores. Thus, the variables like different aspects of personality, mental ability, S.E. ., achievement motivation should be controlled for dependable conclusions. Obviously all the measures can not be controlled in a study like the present one. Previous achievement in the subject is not only due to learners' mental ability but is also influenced by his personality, study habits, achievement motivation etc. Therefore, it was considered justified that this measure if employed as a control would perhaps take care of all the variables other than those under study. Therefore, previous achievement has been employed for this purpose in the present study. Flynn and organ (196), Desai (1966), Noble (1967), Rai (1967), Khushdil (1968) and Patel (1978) have also employed previous achievement as a control variable in their studies. It was also considered that for more meaningful and dependable results, previous achievement in the subject matter area under study should be controlled because achievement in different subject matter areas is affected by quite a different sets of personality and motivational factors. For example, achievement in language is largely influenced by the socio-economic status of the family of the learners whereas the achievement in subjects like mathematics is influenced by such personality variables as persistence, study habits etc. Most of the research workers who have employed this variable as a control in their studies have used previous achievement in the subject matter area under investigation for this purpose. Desai (1966) while investigating the relative merits of programmed learning and traditional approach in the teaching of Gujarati compared the two sections on previous achievement in Gujarati language. Khushdil (1968) while comparing the integrated and traditional method in teaching of social science to class VII, matched the groups on the scores of previous achievement in social sciences. Rai (1967) also matched the control and experimental groups on the basis of previous achievement in general science for comparing two different methods in general science. In the present study, therefore, previous achievement in mathematics has been employed as a control variable.

(ii) Intelligence:

Intelligence is considered as a single most important factor determining academic achievement.

Monroe (1950) while emphasising the fact, writes,

> .....it is a truism to say that intelligence is of significance in most school situations.

Dhaliwal (1971) is also of the same view. He says,

> Intelligence is the single important factor accounting for variation in academic achievement, that it plays a major role in causing difference among individuals regarding their academic achievement.

Banreti - Fuchs (1978) very recently conducted a fairly exhaustive study to find out the relationship between intelligence and achievement in reading and mathematics of 275 boys and 263 girls. He obtains the value of coefficient of correlation for boys 0.82 and that for girls as 0.8?. This shows that achievement is highly predictable on the basis of intelligence. Dhaliwal (1971) also obtained a coefficient product moment correlation of 0.523 between intelligence and cademic achievement. Hatch and Flint (1962) also found intelligence as the most important variable for prediction of attainment. Other investigators, Vidhu, 1968; Butcher, 1968; Srivastava, 1969; Jha, 1970; Asrammir and Shain, 1970; Sinha, 1970; Desai et al., 1970; Sinha, 1972; Mohan et al., 1975; Contractor, 1975; Bhushan and Sharma, 1976; Singh, 1977; Seth, 1978 and many others have also obtained significant and positive relationship between intelligence and academic achievement. Therefore, this variable has also been employed as control variable.

TREATMENT MODES:

Quite a few researches reviewed (Chapter II) have employed single treatment mode for the purpose of establishing the interaction. Evidence of a negative or positive relationship in such studies does show that the treatment mode employed may be more suited for persons securing high or low on the trait studied, but does not provide any evidence for assigning alternative treatment mode for the students who have failed to achieve high through that treatment. Tobias (1969) also writes,

> Studies of this kind are most seriously weakened by the fact that the instructional treatment is not varied. When only one instructional mode has been employed little can be inferred about ATI's since the relationship might well differ with another instructional method.

So there is a need to employ more than one treatment mode in order to assign treatment modes to students of differing personality and mental abilities. Some investigators have employed more than one treatment modes and have mostly obtained significant interactions with them. The insignificant interaction obtained by some of them may be attributed to the fact that the treatments employed by them differed in such minor details so as not to warrant any interaction. Therefore, the treatment modes employed in any study should differ sufficient for any interaction to occur. Sheehan and Hambleton (1977) also point out, "To substantiate interactions, future ATI studies will have to utilise treatments which are known to require different psychological processes". They further add, "Until an approach such as this is taken, teachers will not be provided with empirical guidelines for matching instructional alternatives to student aptitudes".

The present study, therefore, proposes to employ such treatment modes which are known to require different psychological processes and different aptitude pattern. The discussion presented in the preceeding pages clearly shows that linear and branching style of programming and expository method of teaching require quite different personality and mental ability variables for sucess through them. In the present study, therefore, linear and branching styles of programming and expository method of teaching have been employed. Measures of central tendencies, a topic of high school statistics formed the subject matter of these instructional, treatment modes. The rationale for selecting this topic has already been presented in Chapter I (Introduction). A brief description of these treatment modes has been presented in the following pages.

(i) Linear Programme:

The linear method of programming was developed by Skinner, B.F. on the basis of 'Operant conditioning Theory of Learning'. The learner possesses a repertoire of responses - some of which are more likely to be made in a given situation than others - and these are of different degrees of complexity. In teaching some thing new to the learner it is possible, by reinforcing successive approximations to the response to be acquired, to lead him from a response already possessed, through a sequences of small progressive steps, to the new response. Only those approximations which lead to this goal are reinforced. ith a human learner an immediate knowledge of results or an informative after effect is satisfying enough. Immediate feedback of information after a response has been made is an important feature of linear programming. The linear matter are designed to cause the student to emit the behaviours defined as the subject matter to be learned, piece by piece, rewarding each instance where in the student emits the desired behaviour. Linear programme by using various prompts make no explicit provision for errors by the student, since errors are, by linear theory, simply irrelevent to the learning process.

The format of linear programme is simple. A linear programme consists in presenting the subject matter in small segments called frames. Each frame provides a very short preparation of new materials. Following a frame, the student is desired to emit a response. He then compares his written response to the correct response given. If his written response matches the correct response, he feels, thereby rewarded, the act is thus 'learned'. A linear programme is a single path sequence in which all students read and respond to the same material. Figure 1 shows a diagramatic representation of a linear programme The large circle is a frame which includes stimulus or context, enrichment materials, prompt to facilitate correct response and response. The small circle represents the correct response. The learner after reading the frame responds to the question and compares it with the correct response. In this way the student proceeds from one frame to the next and thus completes the programme.

Frame 1 — R-1 — Frame 2 — R-2 — Frame 3 — R-3 — Frame 4

Fig.1: Diagramatical representation of a linear programme.

A linear programme on measures of central tendency was developed and validated by the present investigator. The development of this programme has been described in Chapter IV.

(ii) Branching Programme:

The branching method of programming was developed by Crowder, . This method of programming makes no assumption about the nature of learning theory or process. It is not a theory about how education should be conducted rather it is a technique for preparing a written materials that will allow teaching to be more effectively and efficiently implemented.

The branching or intrinsic programming may be explained in this way: The student is given the material to be learned in small logical units. Each unit explain a concept, giving as many examples as possible. Immediately after he has read and diagested one of these units he is given a short multiple choice test on it. The results of the tests are used to determine what next unit of information shall be presented to the student. The learner chooses one of the response. If he is wrong, he will be so informed, given a reason and offered further choices or presented with the original frame. Thus, theoretically a programme could be prepared which diagnosis errors and provides remedial instruction. If a pupil chooses correctly he may bypass all these frames or 'branches' and proceed quickly through the programme. In branching programme, the short test or questions serve primarily a diagnostic purpose and appropriate remedies are provided in the different branches. Figure 2 shows a diagramatic representation of a branching programme. The circle is a frame. In this diagram the main sequence is (if no errors were made) is frame 1,2 and 3. Depending upon the errors in answering the multiple choice question, branched along various branching frames, such as frame No.24, 25, 26 and 27. In this way the student proceeds frome one frame to the next and thus completes the whole programme.

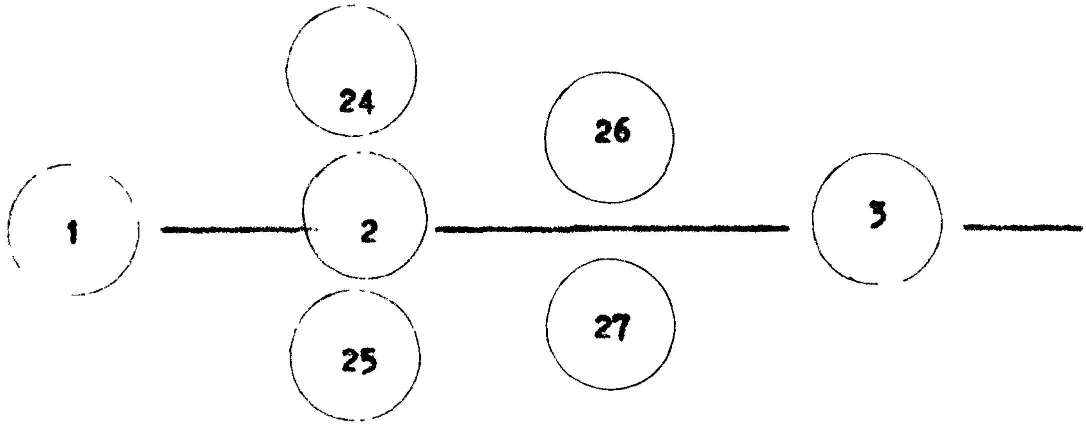


Fig.2: Diagramatic representation of a branching rogramme.

A branching programme on Measures of Central Tendencies was developed and validated by the present investigator. The development of this programme has been described in Chapter IV.

EXPOSITORY METHOD:

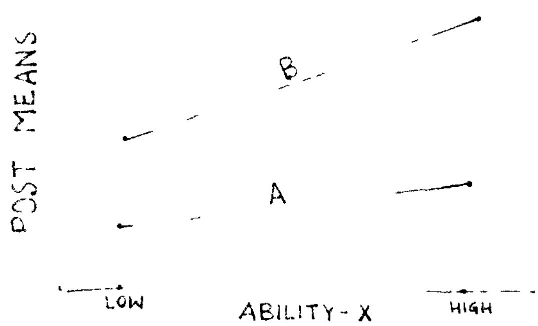
The expository method involves the presentation of the matter in such a way as to be intelligible to the pupil. The facts and the explanation of the facts are for the teacher the data of exposition. As a expositor, the business of a teacher is so to present his facts that they shall carry with them the explanation that appeals to him as satisfactory. The expository method as employed in the present study consisted a presentation of a concept verbally to the students at the first instance. It was elaborated further by the help of suitable examples i.e. an inductive deductive approach was adopted. The teacher employed no audio-visual aid, only chalk and black-board was employed pertaining to a concept. The students were allowed to put questions freely if they failed to understand a particular concept. The whole sequence was presented in the form of an expository text which has been presented in Appendix B.

METHOD OF ANALYSIS: STATISTICAL

A review of related researches presented in Chapter II reveals that a variety of 'statistical techniques' have been employed for investigating the interaction between learner characteristics and various treatment modes. In several studies the interaction has been studied by correlational and/or regressional analyses (Kapadia, 1972; Agarwal, 1976; Porter, 1962; Agarawal, 1978; Sheehan and Hambleton, 1977). They have computed simple Pearson's Product Moment Coefficient of correlation when investigating relationship between a single personological variable and two or more treatment modes. Sometimes other personological variables were considered important to influence the relationship between the variables under study and two treatment modes. For example, while studying the interaction of intelligence and two treatment modes, 'study habit' could be an important variable influencing criterion scores. The influence of such variable should be controlled or partialled out. Some researchers have, therefore, employed coefficient of partial correlation for this purpose. If the regression lines, based on the coefficient of correlation, between the measures of the personological variable and criterion test scores for the different treatment modes do not cross, the interaction is called ordinal and if these lines cross, the interaction has been termed as disordinal. Ordinal interaction is of little value for assigning treatment modes to students of differing levels of ability and a disordinal interaction help in assigning alternative treatment modes to such students. To determine the utility of significant interaction (disordinal interaction), Johnson-Neyman Analysis (Johnson and Neyman,1936; Johnson and Jackson, 1959) has been performed on each pair of treatment groups within each interaction. This technique helps to determine points within the range of observed scores on either sides of interacting score (or point) on the aptitude scale, where the predicted criterion scores are significantly different. The region between these two points is called 'region of non-significance'. On the basis of this region of non-significance the student can be assigned to either of the two treatment modes. However, an experience with this technique showed that in most cases the region of non-significance are so wide as to be of little practical value for assigning different treatment modes (Pattnoff,1964).

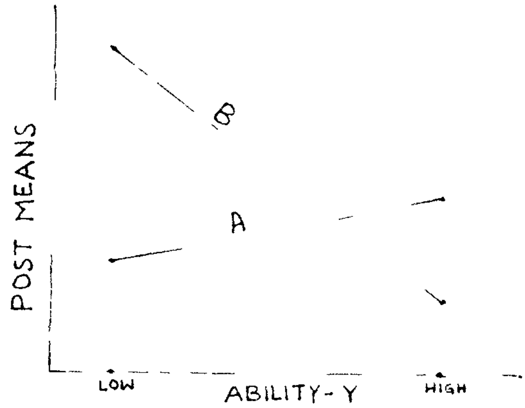
ost of the ATI studies have employed treatment by level factorial design with analysis of variance. In these studies means of criterion scores obtained by students 'high' or 'low' on a personological variable have been compared for studying interaction. The significance of interaction has been ascertained by computing 't' values. Lubin (1961) made a distinction between two types of significant interaction. With reference to graph of means, a significant interaction effect is called ordinal when the treatment lines do not cross (Fig.3) and disordinal when the treatment lines do cross (Fig.4). ome investigators interpret the crossing of treatment lines i.e. disordinal interaction as evidence for ATI i.e. students should be assigned differently to alternative treatments to obtain optimal educational payoff. racht and Glass (1968) have, however, recently modified Lubin's distinction by specifying that an interaction effect is operationally defined as disordinal only when the differences between alternative treatments at two levels of a personological variable are significantly different. For example, the interaction shown in Fig.4, considered disordinal on the basis of Lubin's criteria (1961), is not so according to Bracht and Glass (1968). In this figure, treatment A is significantly better for low ability (L) students but treatment B is not significantly better for high ability (Y) students. Thus, the interaction in Fig.4

is not regarded as disordinal by Bracht and Glass because of the non-significant treatment difference in the high ability group. The standard set by Bracht and Glass (1968) as evidence of disordinal interaction seems to be too high. In a review of ATI researches conducted by Bracht and Glass (1968) only 5 out of 90 studies could be considered disordinal on this standard. The present investigator while considering the idea of disordinal interaction as fairly sufficient evidence for assigning treatment modes feel that if the difference between the alternative treatment is significantly different even at one of the levels, then the evidence is sufficient for the purpose of ATI. Therefore, the studies in which the treatment lines cross and the difference between the two treatments at any one of the levels is significantly different, have been regarded as sufficient evidence for assigning alternative treatment modes.



SIGNIFICANT INTERACTION

Fig.3: Ordinal Interaction



SIGNIFICANT INTERACTION

Fig.4: Disordinal Interaction

As pointed out earlier very often the ATI studies require that one or more personological variables be incorporated in the research design in addition to the variable under study for the purposes of controls. A very common procedure of eliminating the effect of such control variables which may be presumed to be related to the variable under study consists in matching groups on individual to individual basis on the control variable. This is a fairly difficult procedure even when one control variable is involved. This type of matching for more than one variable quite often than not, results in a very drastic reduction of sample, randering it unrepresentative. Walker and Lev (1953) point out the following objection to this procedure:

(1) It is usually very laborious. The search for closely matched cases often takes a very long time and the research worker quite properly feels that his energy could be better spent on something else.

(2) ome of the cases, sometimes an alarmingly large number have to be eliminated because no match can be found. hus sample size is reduced and reliability sacrificed.

(3) Very often the cases finally retained at the conclusion of matching process are not representative of either of the original populations.

The present investigator proposes to employ one or two variables for the purposes of control. Therefore, it is likely to be very difficult to match the experimental and control groups. Analysis of covariance can help in this regard. Garrett (1971) also writes,

> Covariance analysis is especially useful to experimental psychologists when for various reasons it is impossible or quite difficult to equate control and experimental groups at the start, a situation which often obtains in actual experiments. Through covariance analysis one is able to effect adjustments in final or terminal scores which will allow for differences in some initial variables.

The present study has employed analysis of covariance for studying the interaction of variables (Intelligence, creativity, anxiety and extroversion) and three treatment modes such as linear and branching programming and traditional method employing previous achievement and intelligence as control variables. The analysis would yield means on criterion measures for students high and low on the ability variables duely adjusted for the effect of control variables.

The flow chart of the procedure, employed in the present study, for testing an A.T.I. in a treatment by level factorial design can be presented in Fig.5.

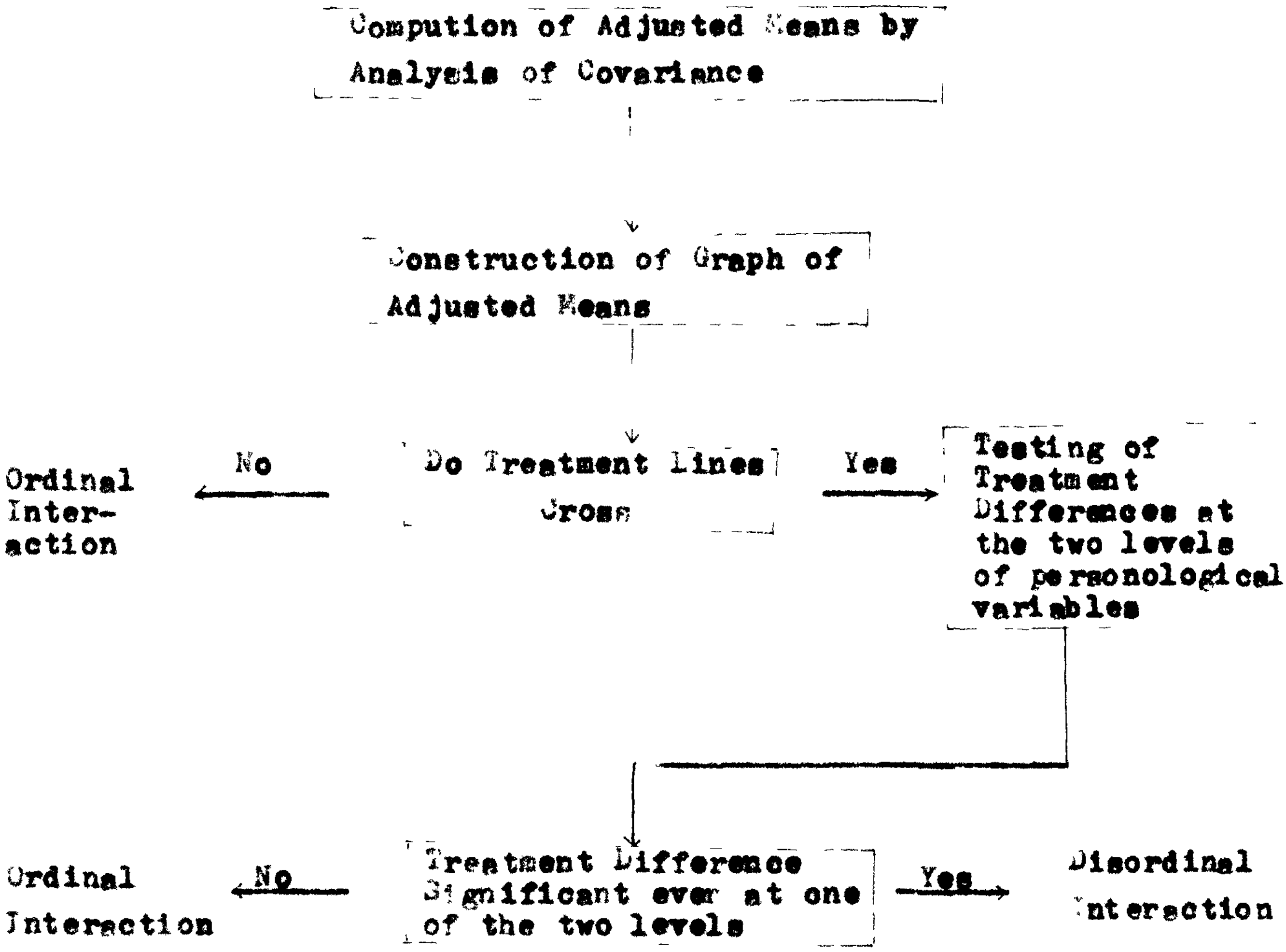


Fig.5: Flow chart showing the statistical procedure for testing ATI.

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CHAPTER - IV

DEVELOPMENT AND VALIDATION OF PROGRAMMES

A. Development and Validation of Linear Programme

Preparation

Writing the Programme

Tryout and Revision

B. Development and Validation of Branching Programme

Preparation

Writing the Programme

Tryout and Revision

References

The present study seeks to investigate the interaction between some personality and mental ability and variables and achievement t rough three instructional treatment modes namely, linear and branching styles of programming and expository method of teaching. A linear and a branching programme on a'measures of central tendencies' - a topic of hig school statistics were constructed for this purpose. The development of these programmes is presented in the following pages.

Development and validation of a programme, linear or branching, is a fairly complex excercise. It consists of the following major steps:

(A) PREPARATION - It includes selection of the topic, writing of the content outline, describing the task in explicit terms, analysing the task and constructing of the tests of entery level and terminal behaviours.

(B) WRITING THE PROGRAMME - The writing of the programme includes presenting the material in frames active responding, providing for confirmation or correction of responses, using prompts to guide student response and careful sequencing of the frames.

(C) TRYOUT AND REVISION - The tryout and revision consists of writing the original draft, editing the draft and tryout and revision of the edited draft.

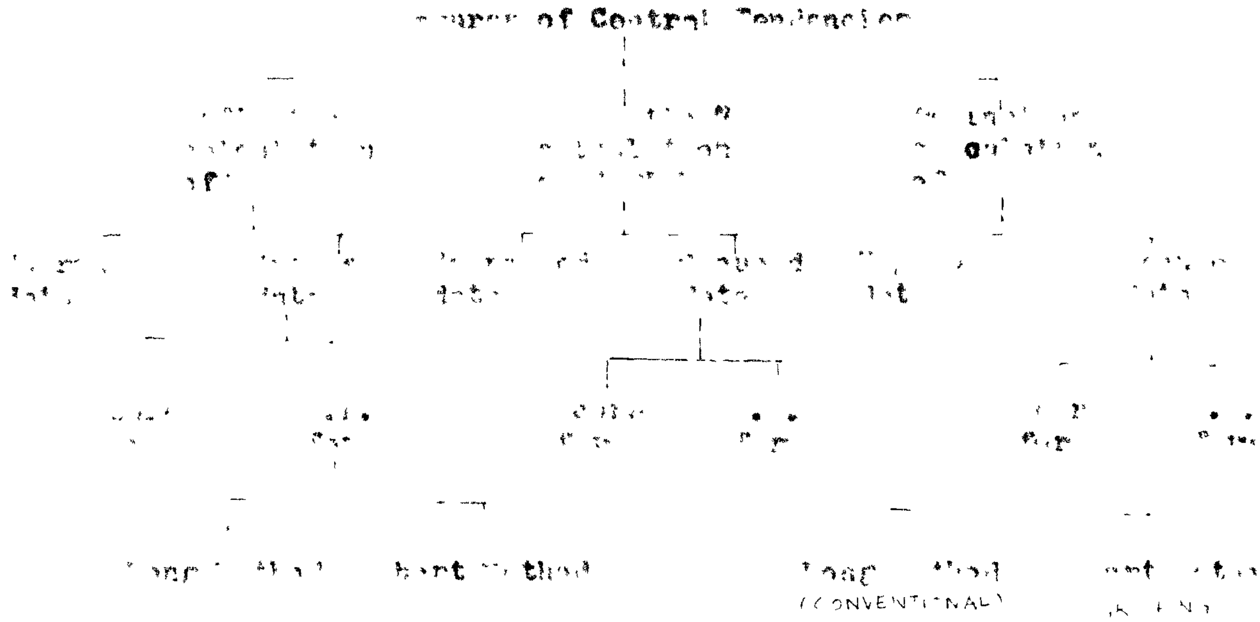
(A) PREPARATION:

(i) Selection of the Topic:

The topic for programming should be selected with great care. The topic should not only be amenable to programming, but the programmer himself should be thoroughly convergent with the subject matter. In view of these criteria it was decided that 'Measures of Central Tendencies', a topic from mathematics course (Statistics) for high school students of the Board of High School and Intermediate Education, Allahabad,was selected. The present investigator is a graduate in mathematics and post-graduate in physics and as such is thoroughly conversant with the topic. The topic itself consists of well designed and fairly sequenced steps which can be easily programmed. Leedhan and Unwin (1971) also points out, "The subject (Mathematics) lends itself readily to a programming approach... At secondary level conventional teaching in mathematical topics already bears a superficial resemblance to programming".

(ii) Content Outline:

The content outline should cover all the material planned to teach (Klaus, 1961). Once the topic has been selected, the next step is to carefully prepare the content outline covering the materials that are to be taught. The outline of the present topic was prepared after a thorough study of a number of text books of high school level and other reference books. Some secondary school teachers with sufficiently long experience of teaching statistics to high school classes were also consulted. The academic background of the present investigator was also of great help in writing the content outline. Since the programme was to be constructed in Hindi, the content outline was also prepared in Hindi. The outline has been presented in Appendix A . This outline was also employed as text for expository method of teaching. The following topics have been covered in the outline.



(iii) Objectives in Behavioural Terms:

The objectives in behavioural terms represent the particular responses that can be expected in the student's repertoire when the programme is completed. Writing specifications of objectives in behavioural terms is the initial step in the development of a programme. The objectives with their behavioural outcomes provides guidance in the planning of instruction. Mager (1975) while emphasising the importance of objectives in developing programmed learning material also points out, "Objectives are useful in providing a sound basis for the selection or designing of instructional content and procedures". He further writes, "... bjectives can serve as a spotlight to illuminate the worth of that existing instruction and they can provide a basis for improving it". The writing of objectives in behavioural terms or task description has been presented in the following paragraphs:

Task Description:

A task description involves the explicit and unambiguous description of terminal behaviours, which are to be achieved through the programme. obert ager (1962) and obert iller (1962) after working independently have proposed two more or less similar procedures of task description. According to Mager's procedure the task description requires identification of the terminal performance which the instruction attempts to produce, specification of the conditions under which the behaviour is expected to occur and the criteria of acceptance of students' performance. Miller's procedure of task description is somewhat more difficult to describe. According to Miller, an unambiguous and clear task description must have indication or the cue which calls for the response, activation or the response to be made and feedback, or the indication of how adequate the response has been. The difference between the two procedures of task description chiefly one of terminology (Deccecco, 1970). Both the procedures of task description involves specifically the description of stimulus conditions under which the student must perform, indication of response he is to make in behavioural terms and provision for feed-back or for assessment of student performance. In the present programme Mager's criteria of task description as briefly described above have been adopted for the present study as they are objective, practicable and have a high degree of precision.

In the present programme development, the following task descriptions were made:

1. The learner must be able to define and descriminate the following measures of central tendencies:
   (a) Mean
   (b) Median
   (c) Mode

2. Given the ungrouped data, the learner must be able to calculate the mean.

3. Given the grouped data in score form, the learner must be able to calculate the mean.

4. Given the grouped data in class internal, the learner must be able to calculate the mean by long method.

5. Given the grouped data in class interval. The learner must be able to calculate the mean by short method.

6. Given the ungrouped data with odd number of cases, the learner must be able to calculate the median.

7. Given the ungrouped data with even number of cases, the learner must be able to calculate median.

8. Given the grouped data in score form, the learner must be able to calculate median.

9. Given the grouped data in class interval, the learner must be able to calculate median.

10. Given the ungrouped data, the learner must be able to calculate the mode.

11. Given the grouped data in score form, the learner must be able to calculate mode.

12. Given the grouped data in score form with more than one score having same highest frequencies, the learner must be able to calculate mode.

13. Given the grouped data in class interval, the learner must be able to calculate mode.

(iv)Task Analysis:

The programmer must not only describe but also analyse the tasks he expects the student to perform. While the task description is the description of terminal behaviours which the programme proposes to achieve, the task analysis examines the component behaviour the learner must acquire in the process of reaching the terminal behaviour. In a proper task analysis, the teacher would correctly classify the behaviour involved in each objective and establish the best learning condition for each. A task analysis may also identify the subtasks which must be learned in the process of learning the task proper. Peter Pipe (1966) suggests that task analysis involves building a pyramid of objectives. On the top of the pyramid is the instructional objectives. This objective is analysed into subobjectives which are needed for achieving the above mentioned instructional objective. These statements of subobjectives will form the next lower level of pyramid. The third level of the pyramid constitutes the component behaviours which are needed for the achievement of the subobjective. This analysis continues downwards till the level of entering behaviours are reached. Fig.6 presents the pyramid of objectives for the general objective of 'Definition and calculation of various measures of central Tendencies'.

In the pyramid presented in Fig.6, programme objectives were further analysed into component behaviours which are involved in the achievement of the objective concerned. Panel-I presents the component behaviours for the programme objective. Given the grouped data in class interval, the learner must be able to calculate the mean by long method. Such component behaviours for every objectives were prepared because they formed the basis for writing the frames.

(v) Test of Entry Level Behaviour:

For every programme of instruction some pre-requisite behaviours are essential. These pre-requisite behaviours form the basis in a programme. They are recalled in the beginning frames for building up the subsequent frames. If the students do not possess these pre-requisites or entry level behaviours, they would not be able to understand the subsequent frames and would also failo on the criterion test. It is, therefore, imparative that the student should be tested for their entry level behaviours before the administration of programme and only those students should be allowed to learn through it who possess the necessary entry level behaviours. A test of entry level behaviour was, therefore, prepared for the present programme. The procedures adopted thereof have been described in the next chapter.

(vi) Test of Terminal Behaviour:

After task analysis, the programmer sets about writing the frames. A criterion frame is constructed for each item of task analysis. A series of teaching frames are also constructed for each of the criterion frames. In this way the whole programme is prepared. Usually two criterion test items are also constructed pertaining to each criterion frame. These criterion test items are also called post-test items or test of terminal behaviours. In the present investigation only one set of criterion test items was prepared. It was considered that, since the criterion test consisted of numerical examples only, there was hardly any need to have duplicate item for every objective. Further, the interval between the pre and post test administration was sufficiently large so that there was hardly any scope for carry over from one situation to another. The test of terminal behaviour was administered both before and after the administration of the programmes. The construction of this test has been described in next chapter.

(B) WRITING THE PROGRAMME:

(i) Frame Writing:

In a linear programme the material is presented through small units called 'frames'. Each frame constitutes a small segment of subject matter which calls forth particular student response (Taber, et al., 1965). The segment of subject matter is based on knowledge already possessed by the student and a very small increment of knowledge, moving the student steadily towards the terminal behaviour. The size of subject matter to be learned in a frame may vary in different parts of the programme. But in the linear programme, the frames are kept as small as possible to provide good learning. Maccopy and Sheffield (1961) concluded that (i) short steps are more effective than large steps for initial learning and (ii) the progressive lengthening of steps lead to best performance on the test of terminal behaviour. While writing the frames for the programme, it should be kept in mind that frames should be as simple as possible, should avoid ambiguous material, should be written keeping in view the mental level of the learner and should try to incorporate the various psychological principles of efficient learning. Leedham and Unwin (1971) suggest a number of guidelines which may assist in frame writing. They point out that while writing the frame, one should be unambiguous, be brief, be simple and be aware of reading ability of students for which programme is intended. Keeping in view all the above criteria of good frame writing, the frames for the present programme were written.

A frame consists of four parts (a) stimulus or stimulus context (b) enrichment material (c) response (d) prompts. Frame number 8 of the programme constructed for the present study has been presented below as an example:

Frame No.8:- The score which appears maximum number of times in a group is called the mode of that group. In the scores 15, 18, 20, 22, 25, 27, 27, 30, 32 of the frame No.4, the score......... appears maximum number of times.

Hence mode = ...

(a) Stimulus or Stimulus Context - Each frame consists of some stimulus or stimulus context. The stimulus or stimulus context acts for evoking the desired response. These are the words which precede and follow the blank. In the frame No.8 presented above, the material 'In the scores 15, 18, 20, 22, 25, 27, 27, 30 and 32 of the frame no.4, the score which appears maximum number of times' and 'mode' are the stimulus and stimulus scontext to evoke the desired response.

(b) Enrichment Material - The fourth ingredient of a frame, the enrichment material, is optional. The enrichment material makes the frame more readable or interesting or recalls previously learned material to facilitate student response. In many instances interest in a programme is heightened by inserting the facts which are relevant to the material being covered but not part of course or programme objectives. Such material is called enrichment material.

In the frame No.8 given above, the material 'The score which appears maximum number of times in a group is cl called the mode of that group', is enrichment material which recalls previously learned material to facilitate student response and makes the frame more readable and interesting.

(c) Response - Each frame requires the students to make response or responses. For this purpose each frame either poses a question or leaves a space blank in which the required response is written. The response may be overt or covert. A covert response is not considered satisfactory because it does not control mathemagenic behaviours of children. An overt response is easily learnt because the student is using most of his senses and his mathemagenic behaviours are also controlled. In the present programme, overt responding has been employed. While writing the programme frames, care was taken to see that the student reads and understands all the parts of the frame before giving the response. Mergulues (1964) has shown that the learner tries to remember only those parts of the frame which he is required to respond to. The frames also required only critical informations to be responded to. Unimportant and trivial informations do not add to the knowledge of the learner. Holland (1960) has shown that frames requiring trivial informations to be given, yield a very low error rate in programme performance but the post test error rates are very high showing thereby that sufficient learning has not taken place through the frames. Contrariwise, the frames requiring critical informations to be responded too yeild comparatively higher programme error rate i.e. they are comparatively more difficoult but the post test scores are fairly high showing a comparativelyb better learning through such frames. Keeping in view this discussion it was considered that the frames in the present programme should require only critical informations as responses. The response blank were kept, as far as possible, near the close end of the frame. Horn (1963) argues that the blank space should appear as close to the end of the frame as possible because this position spares the student the awareness of flipping his eye back and fourth, skidding around inside frame after frame looking for the relevant material. In the above example, the blank space is provided near the end of the frame and requires the 'mode' to be filled in. The correct responses of each frame are given on the back page and as to avoid knowledge of response before writing or filling up the blanks.

(d) Prompts or Cues - Prompts or cues arep provided in the frame to guide the student to the correct response without over controlling his behaviour and to prevent the student from making unnecessary errors. eyer (1966) defines prompt, "any technique employed to make emission of the correct answer more likely". They are supplimentary stimuli in that they are added to a frame to make the frame easier but are not sufficient in themselves to produce the response (Markle, 1964). It has been shown by quite a few researches that prompts or cues help in better learning. Kimble and Jepson Wulff (1953) found that students in the prompted group scores higher on practice items, transfer items and the most difficult items. The students showed the greatest superiority on those items for which prompts had been provided. Oliver Cooke and his associate (1960) also argued that instructional procedures which furnish the student sufficient guidance to avoid mistake is superior to procedures which only correct errors after they are made. A variety of prompts may be employed in the frames. The prompts may be formal or the-matic. Formal prompts are those which give an idea of the form of correct response while the thematic prompt is one which gives an idea of the theme or the subject matter of the correct response.

The formal prompt may be partial response prompt, rhyming prompt, literal prompt and frame structure prompt. These various formal prompts have frequently been used in different parts of the present programme. In frame no.26 of the panel no.1, the method of calculation of midvalue has been explained and that for the first class interval has been calculated, which serves as a prompt for calculating the remaining values. Thus the partial provision of the correct response in this frame has served as a prompt. This prompt is, therefore, called a partial response prompt. Since it gives an idea of the 'form' of the correct response, it is a formal prompt. The frame no.30 of the panel-1 utilises frame structure formal prompt. In this frame the first two items have been partially prompted and for remaining items of the frame, the stimulus material is arranged in such a way as to prompt the correct response.

The thematic prompts may be of picture prompt, context setting prompt, synonyms and antomyms prompt, analogies prompt, rules prompt and inductive prompt. In the present programme, these various thematic prompts have been utilized. In frame no.29 of panel-1 synonyms 'or' is used to prompt the correct response. In frame no.28 of the same panel, rule that 'mean is obtained by dividing the total score of students by the total number of students' serves as a prompt to evoke the correct response.

(ii) Criterion and Teaching Frames:

The criterion or terminal frame is one which tests pupil's knowledge without auxiliary clues or prompts whereas teaching frames is one which is needed inorder to proceed for criterion frame. The teaching frame teaches various interim objectives or objectives of the programme whereas the criterion frame tests the criterion objective.

(C) TRYOUT AND REVISION:

The tryout and revision of programme involves two type of teaching:

(I) Developmental Testing

(II) Validation Testing

(I) Developmental Testing:-

The developmental testing aims at revising the draft programme by the help of students till it is felt that it will be able to achieve the objectives. It is completed in the following two stages:

(i) The individual testing stage

(ii) The group testing stage

(i) The Individual Testing Stage:

This is the most important stage for improving the programme. This stage takes the help of the student themselves in improving the programme. In fact the consumer (here the student) should be the best judge for a teaching programme. Thomas Gilbert (1960) suggests,

> Take your first crude effort to the student. Remember, he is going to teach you. The student can not fail. If he does not get where you want him to go, you have failed. Try something else. In the absence of anything better, let him be your guide. If you come to a dead end, vary your approach until you have gotten him to where you want him to go.

At this stage the original draft of the programme is improved with the help of student one at a time. This individual tryout is repeated till finally the students do not make any mistake and programme seems to achieve the objective for which it has been constructed. The individual tryout works on the principle of 'diminishing return'.

The frames originally prepared were first written on a cards (size approximately 4" X 3") with one frame on one side of the card and its correct response on the reverse side. These cards were employed for individual tryout. Six students cooperated in the individual tryout. In accordance with the suggestions of Leedham and Unwin (1971), these students were selected from the lower end of the intelligence spectrum of the students for whom the programme was intended. The individual tryout required a perfect rapport between the student and the programmer. This was achieved by a very affectionate treatment towards these students and by making his feel that his cooperation, was essential for improving the programme and that the programme does not intend to identify the students mistake. After the selection of students, these cards were presented one by one to individual students and the students were required to write down their responses on a separate response sheet. After writing down the response, the student were required to flip the card over to see the correct response. The programmer also noted the time taken by the student to complete a frame, his response, reaction and editing comments on the following proforma:

TABLE-II

SAMPLE PROFORMA OF INDIVIDUAL TRYOUT FOR LINEAR PROGRAMME

Frame No.	Time Taken	Student's Response			Student's Reaction			Editing Comment
		Right	Wrong	Remark	Easy	Blocked	Tired	
21	2 m			Modification				CRAB
26	3 m			Need for Midvalue				RATIO+ NALE
27	3½m			One more example				PROMPT
28	2 m			-				-
53	2 m			-				OVER VIEW
62	8 m			Requires Clarification				-
83	9 m			- do -				-
84	6 m			- do -				-

As pointed out earlier the time taken to complete a frame and student's reaction to it were noted on the proforma. A perusal of the sample proforma presented above would show that frame nos. 82, 83 and 84 have taken too much time to complete and the student's reaction is also of tiredness or blocking. Such frames were modified. The frame no.82 was originally as follows:

Frame-82:

"Analysis table" of frame-81 will be filled up as follows:

(i) The highest frquencies in the column-1 of grouping table are 18 These are of scores.......and........ Thus these scores hawe been written in front of column-1 below the scores......... and ....

(ii) Similarly, in the column-2 of the grouping table, the highest frquency 29 is of scores.......and....... These scores have been written in the analysis table of frame-81 at appropriate places.

(iii) The highest frequency of column-3 of grouping table is of scores......... and .... Write down these scores in appropriate places in the table.

Analysis Table:

Column	Scores with highest frequencies									
	4	6	7	8	9	10	11	12	13	14
1					9		11			
2					9	10				
3										
4										
5										
6										
frequency										

Apparently the blocking is due to lack of understanding and ambiguous language. It may also be due to the following factors: (a) Dissimilar allignment columns and rows of grouping and analysis table, (b) writing the scores in place of their tally marks and consequent confusion in calculating frequencies, (c) requiring a calculation of large number of frequencies. The frame was therefore, modified as follows:

(i) The allingment of groups and columns was made similar to the grouping table given in frame 80.

Analysis Table:

Score Column	Tally marks of Scores with highest frequency 1	2	3	4	5	6	Freqeuncy 7
4							
6							
7							
8							
9							
10							
11							
12							
13							
14							

This eliminated the confusion regarding the columns and the obtained scores.

(ii) The frame was divided into 3 frames each requiring a small bit of information. The first frame of this series i.e. frame no.81 of modified programme was a prompted frame requiring students to put tally marks against the score/ scores having highest frequency in column 1. Tally marks were preferred to the usual practice of writing the score as such because in the later case there was some confusion in arriving at the total frequency. There was no such confusion when the tallies were used. In each of the following frames, i.e. frame nos.81, 82 and 83 frequencies of one column at a time were required to be filled in by the students. In this way single frame was divided into three frames rendering the sequence comparatively more easy. An individual tryout of this sequence did not show any sign of blockade. The modified frames were:

Frame-81:

For calculating the mode by grouping method, the analysis table was framed after completion of grouping table. In the analysis table tally marks are put in front of scores having the highest frequency of each column. From the grouping table L.H.S., In can be seen that the highest frquency is.... which is of scores..... and ..... Thus in the analysis table the tally marks will be put in front of scores 9 and 11 below column 1.

Frame-82:

In the same way, the highest frequency in the column 2 of frame-80's grouping table is....... ofThis frequency is the sumofof frequencies..... and .... of score 9 and 10. Thus tally marks must be put in front of scores 9 and 10 under column 2. Please put the tally marks in the analysis table in column 2 at an appropriate place.

Frame-83.

The highest frequency in the column-3 of frame-80's grouping table is..... This frequency is the sum of frequencies.......and.....of scores.......and...... Please put the tally marks in the analysis table at an appropriate place.

Analysis Table

Scores	Column	Tally marks of scores with highest frequency						Frequency
	Column	1	2	3	4	5	6	7
4								
6								
7								
8				1				
9		1	1	1				
10			1					
11		1						
12								
13								
14								

Frame no.83 and 84 (of the draft programme) also elucited a feeling of tiredness on the part of the student. These frames were also modifed.

In this way all the frames were scrutinised for the time taken and for evoking a feeling of tiredness or blocking. Some frames were found to possess some minor defects e.g. in some frames the correct response was implied, the frame lacked illustrations or the frames were obviously give away ones. The appropriate editing remarks were noted and the frames were revised accordingly.

Editing and Revision of Frames:

This was done on the basis of editing notes taken during the individual tryout. Some examples would clarify the steps undertaken by the programmer/investigator, original frame no.26 was as follows:

The formula for midvalue

$$= \frac{\ldots\ldots\ldots + \ldots\ldots\ldots}{2}$$

The editing remark for this frame was 'RATIONALE' which indicates that the frame sequence being reinforced fails to provide the student with an adequate picture of the reasons for learning the material covered by the sequence. Thus rationale are needed. The revised frame was as follows:

In this way when scores are given in class intervals then the representative score of that interval is....... ....The formula for calculating midvalue,

$$x = \frac{\ldots\ldots\ldots + \ldots\ldots\ldots}{2}$$

Similarly, frame no.27 was originally as follows:

Find out the midvalues of the class intervals given in the golloving table:

Class Interval	Frequency	Midvalue
0-5	4	-
5-10	5	-
10-15	1	-
15-20	5	-
20-25	3	-

The editing remark for this frame was 'PROMPT' which indicates that the frame does not provide any clue to evoke correct response. So the prompts are needed. The above frame was revised as follows:

Calculate the midvalues of class intervals given in the following table:

Class Interval (C.I.)	Frequency (f)	Midvalues (x)
0-5	4	2.5
5-10	5	...
10-15	1	...
15-20	5	...
20-25	3	...

The midvalue (x) of class interval 0-5 = $\frac{0+5}{2}$ =2.5 similarly calculate the midvalues of other class intervals.

In the frame no.21, the editing remark was C R A B which indicate that the correct response was missing the frame. If the correct response is missing the student is unable to check his response, this creates frustation. The smissing of correct response also results in wastage of students' valuable time because he has to reread the portion of the programme in order to check his response. So the correct response was added to the frame.

The frame no.53 was originally as follows:

The frequency distribution of 12 students of a class was as follows:

x	f
3	2
5	8
6	5
7	2
	N = 17

In the above distribution score 3 is obtained by 2 students, and score 5 by 8 students. Similarly score 6 is obtained by ....students and score 7 is obtained by ....students.

Score 5 or less than this were obtained by 8+2=10 students. This 10 is called the cumulative frequency of score 5. core 6 or less than this were obtained by

5+.....+.....=......students. Thus, the cumulative frequency of score 6 = ...

Similarly, score 7 or less than this were obtained by.......+......+......+.......So cumulative frequency of score 7 =.........

The editing remark for this was 'OVERVIEW', which indicates that the programme has failed to provide the student with an introductory overall picture of the subject being studied. The following overview of the material to be studied is thus provided before the above frame.

"Upto now we have studied the calculation of median in ungrouped data.

The median of any group = $(\frac{N+1}{2})$th students'score

For calculating the median in grouped data first we have to calculate the cumulative frequencies. In the next frames we shall explain you the calculation of cumulative frequency and than median with the help of that cumulative frequencies".

In this way all the frames were revised on the basis of individual tryout.

GROUP TRYOUT:

The individual tryout helps in inproving the frames and making them suitable even for the dullest student of the class. This assures that the student would not commit a mistake while learning through the programme. But the individual tryout does not provide any evidence regarding the efficiency with which the representative group for which the programme is constructed would perform. With this objective in view, the programme was administered to a group of twentyfive students. The student comprising the group consisted of weak, average as well as superior students. The group tryout consists of two steps:

(a) Frame Analysis

(b) Post test Analysis

(a) Frame Analysis - This is conducted to find out the error rate of different frames. The number of students responding to a frame and those responding correctly were entered in the following proforma. This helped in calculating the error rate i.e. % of student attempting the frame incorrectly.

TABLE-III

SAMPLE PROFORMA FOR FRAME ANALYSIS OF LINEAR PROGRAMME

Frame No.	Frame	No.of Student Responded	No.of Student Responded Correctly	Error Rate
8	Criterion	25	23	8%
13	Criterion	25	24	4%
51	Criterion	25	23	8%
53	Teaching	25	23	8%
54	Teaching	25	25	0%

The standards are that the frames which had error rate more than 3% (in case of criterion frames) and more than 10 (in case of teaching frames) indicate a need for revision. It was however, considered that these standards are far too rigorious, therefore, the standard needed to be brought down. It was, therefore, decided that if the error rate of criterion frame exceeds 5% and that of teaching frame exceeds 10 then only the frame needed revision. A frame analysis of all the frames were made and it was found that the frame nos. 8 and 51 did not confirm the aforesaid criterion.

An examination of these frames showed that they contained printers error and possibly the boosted error rate may be due to this reason. It was also considered that in view of the every easy frames, as they were, there was hardly any chance of any other reason for the increased error rate. These printing errors were corrected and frames were attempted easily. The error rate also came down to normal. one of the teaching frames revealed abnormal error rates. The frames were very thoroughly scrutinised at the individual tryout stage, therefore, the group tryout did not reveal any abnormality in the teaching frames.

(b) Post Test Analysis - After the administration of programme, a post test was administered to the group who had completed the programme, to estimate the efficiency of the programme. An analysis of the post test scores was also done to revise and improve the programme. If a student fails at a post test item, it may be due to the defect in the item itself or due to poor frames pertaining to the objective for which the post test item has been constructed. To identify such frames a 'scallogram analysis' was conducted. The post test items were rearranged in the hierarchal order of the objectives and the number of students attempting each items were calculated. The accepted standard in this regard is that in order to accept the item, it should be attempted correctly by 80% of the students. Obviously this would show that post test items as well as the frames pertaining to the objective for which the post test items were written are satisfactory. Again it seems reasonable to argue that this standard is rather too high for situation obtained in India where the educational technology movement is in its infancy. Therefore, even if a post test item was attempted correctly by 60% of the students (standard set up for obtaining the first division) it was deemed to be satisfactory. However, in spite of this watered down standard, some items of the post test were found to be unsatisfactory. The items which were found to be unsatisfactory on the above standard were item nos. 4, 14, 31, 36, 38, 39 and 40. These items were attempted poorly either because of these items themselves or because of defective frame pertaining to them. The frames were examined thoroughly and it was found that the frames did not possess any defect. Therefore, the defect was possibly in the items themselves. An 'option analysis' of the post test items was done to ascertain the possible cause of the student's poor performance. It was found that item no.4 contained a printers mistake, which was corrected. The item no.14 contained a printer's mistake. A score 20 had been printed in addition to the given scores, which rendered the correct alternative to the item incorrect. These mistakes were corrected by removing the last score 20. The option analysis of item no.31 revealed that most of the students write midvalues (x) in place of deviations (d), so this mistake of the post test items were corrected by defining the symbol 'd' i.e. deviation. Furthermore, the option analysis of item nos. 38, 39, 40 showed that the students did not prefer any specific distractors. All the distractors were attempted more or less equally by the students was could not answer the item correctly. Evidently the failure on the part of the students to attempt the item correctly was due to the fact that these items required lengthy calculations. The usual practical in such case is to devide the items into small manageable,parts. But the nature of these items warranted that it is not divided into smaller items. Therefore, the only alternative felt was to increase the weightage of the item, thereby increasing its importance.

(II) Validation Testing:

While the aim of developmental testing is to revise and finalise the draft programme until the programmer is satisfied that the programme will be able to do what it was intended to do, the aim of validation testing is to study how far the draft programme is successful in teaching the specified population in the natural conditions of its use. In specific terms, the purpose of validation testing is to find out:

(1) to what extent the programme has succeeded in achieving the objectives for which it was constructed.
(2) what are the reactions of the participating students and teachers.

In order to find out the extent to which the programme is succeeded in achieving the objectives, the pre and post test scores were compared. The average of the pre test scores (excluding the entering level test item scores) was 0 and that of the post test scores was 125. Thus the programme has succeeded in obtaining an average gain of 125 scores.

The validity of the programme was also ascertained by comparing the efficiency of the programme with that of conventional method.

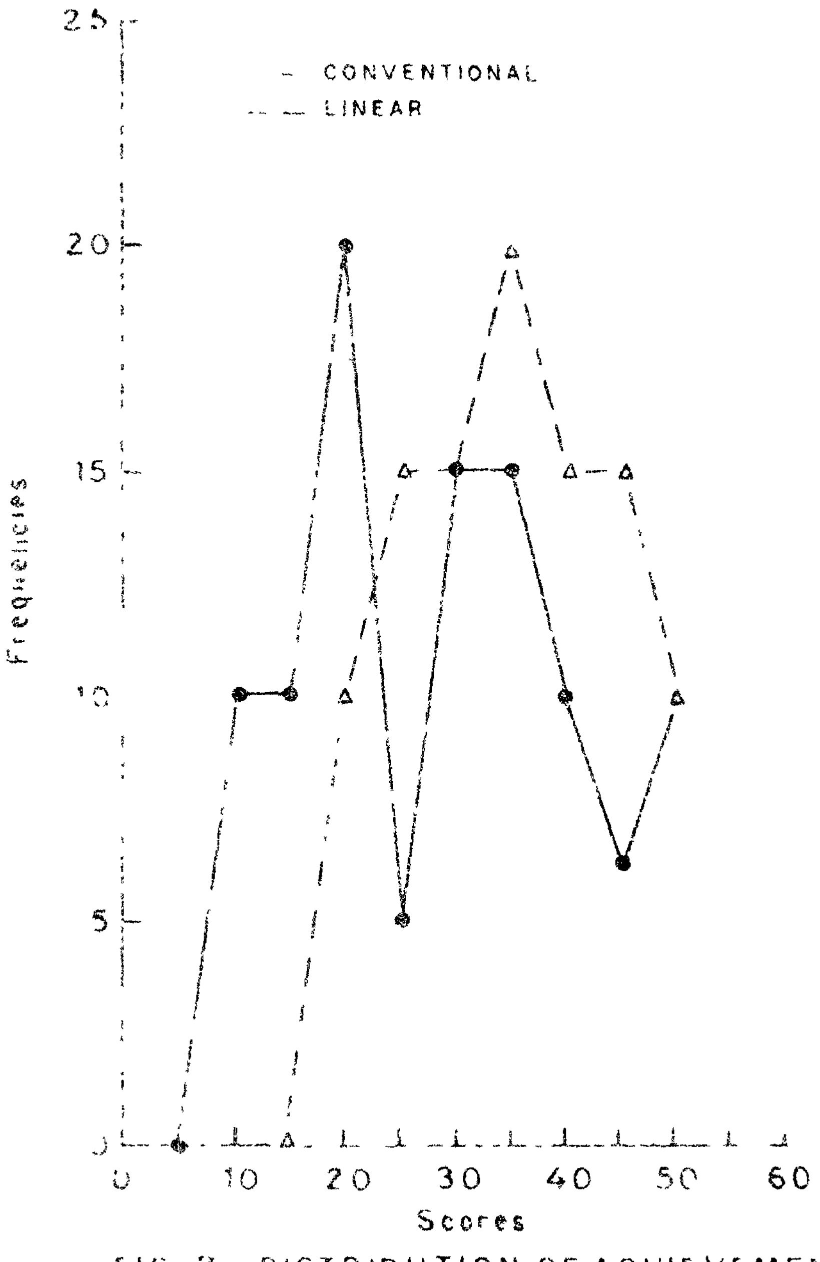


FIG 7 DISTRIBUTION OF ACHIEVEMENT SCORES OF STUDENTS TAUGHT THROUGH CONVENTIONAL METHOD AND LINEAR PROGRAMME

TABLE IV

DISTRIBUTION OF ACHIEVEMENT SCORES OF STUDENTS TAUGHT BY THE CONVENTIONAL AND LINEAR PROGRAMMED METHODS.

C.I.	Percentage of Frequency			
	Conventional		Linear Programme	
0-5	0		0	
5-10	10	20%	0	0%
10-15	10		0	
15-20	20		10	
20-25	5	55%	15	60%
25-30	15		15	
30-35	15		20	
35-40	10		15	
40-45	5	25%	15	40%
45-50	10		10	

It would be seen from the Table IV and figure 7 that the achievement scores obtained through different methods possess distinctly separate distribution. While 20% of the students taught by conventional method occupy the lower end of the distribution of scores i.e. scores between zero and 15 whereas none of the students taught by linear programme occupy this part of distribution of scores. The 55 of the students taught by conventional method occupy the middle part of the distribution of scores i.e. scores between 15 and 35, whereas 60% of the students taught by linear programme occupy this portion of the distribution of scores. The distribution further reveals that the upper end of the distribution of scores (i.e. scores between 35 and 50) is occupied by only 25% of the students taught by conventional method as compared 40% of the students taught by linear programme. This shows that the percentage of students taught by linear programme is higher than that of students taught by conventional method at both, the middle and upper end of the distribution. None of the students taught by linear programme occupy the lower end of the distribution of scores, while 20% students taught by the conventional method fall in this region. This reveals that the present linear programme has resulted in higher achievement as compared to the conventional method.

The efficiency of teaching programme consists not only in the scores obtained by students but also in the time taken to do so. In this respect, the linear programme is also proved to be superior. While the average time taken by the student to learn through conventional method was 4 hours but it was only 3½ hours for learning through the linear programme.

The reactions of the participating students and teachers toward the programme was ascertained by putting questions regarding the frames that are considered either too difficult or too easy and extent to which they like the programme vis a vis the conventional method. None of the frames were considered very difficult or very easy by most of the students and they found the linear programme considerably easier to understand than conventional one.

DEVELOPMENT AND VALIDATION OF BRANCHING PROGRAMME:

Since the branching programme does not utilize any learning theory but depends upon sound techniques of teaching, the strength of the programme lies in its developmental and validation processes. The development of branching programme involves three major stages like that of linear programme:

(a) Preparation

(b) Writing the programme

(c) Tryout and Revision (Developmental and Validation testing) of the programme.

(a) Preparation:

Since the aim of the present study is to develop andv validate linear and branching programme both on the same topic (i.e. 'Measures of Central Tendencies'), the steps of preparation i.e. selection of a unit or a topic, preparation of content outline, defining objectives in behavioural terms, construction of tests of entering and terminal level behaviour would be similar to those for the linear programme. These steps of preparation have already been presented in the development of linear programme.

(b) Writing the Programme:

After the preparation stage, the next stage is actual writing of the programme. In chapter III on design of the study, the structure of a branching programme has already been described. Briefly, however, a branching programme consists of a expository frame which may consist of two or three paragraph (Rowntree, 1963). The expository paragraph is followed by a multiple choice type question and the student is required to select the one which he considers correct. If he selects the correct answer, he proceeds to the next'main line'frame but if he chooses an incorrect answer, the is directed to a 'remedial frame' which points out the causes of his mistake and provides a further elaboration of facts, concepts, given in the original 'main line' frame or expository paragraph and is again directed to the original 'main line' frame. After the remedial excercise the student is expected to respond correctly to the multiple choice question given at the end of expository paragraph or 'main line' frame. Thus the writing of a branching programme is different from that of linear one.

The writing of branching programme, therefore, requires writing of three parts:

(i) Main line frame
(ii) Alternative responses
(iii) Remedial frames

(i) Main line Frame:- The 'main line' frame constitute a primary part of the branching programme. After passing through various frames learner is able to achieve his target. In a 'main line' frame a small unit of subject matter is thoroughly explained with the help of examples. A inductive-deductive approach and egrule-ruleg procedure was followed. The important aspects of the frame are dark printedor or underlined so that greater attention is paid to them.

(ii) Alternative Responses:- Each frame in a branching programme is followed by a multiple choice questions. Growder (1962) points out that multiple choice question may serve:

(1) to detect a procedural errors and lead to a single corrective frame.
(2) to lead to a correctional sequence
(3) to find a mistake in interpretation of a previous point and return to that point in the programme.

Since the alternatives control the pupil's route through the programme, providing feedback to the learner, selection of these alternatives and their arrangements are among the most important tasks. The alternatives were consisted of the correct response and some distractors. The distractors were constructed on the basis of the most probable errors based either on experience of teaching the material or on the basis of an empirical procedures. The empirical procedure in the present programme consisted in examining the responses given to the different multiple choice itemsof the achievement test employed during the developmental testing of the linear programme. An ophon analysis of the post test items was made and the errors that were generally committed by students were noted. These errors were employed for constructing the distractors of the multiple choice questions presented at the end of expository or main line frame. For example, after the option analysis of post test items requiring calculation of median the common mistake committed by the student were (i) mean was calculated in-stead of median (ii) $\frac{N+1}{2}$th term was considered as median in place of scores of $\frac{N+1}{2}$ th term.

There is apparently no restriction on the number of alternatives, In the present branching programme three to four alternatives were generally employed to reduce chances of quessing. As a example, frame no.13 has been presented:

Frame 13:

In a examination, a group have achieved 1,1,4,7 and 9. Find out the mediano of these scores.

Median is that mid point below and above which 50-50 scores of students lie. If N is the number of students then

id score $= \frac{N+1}{2}$ th student's score

$\therefore$ Median $= \frac{N+1}{2}$ th student's score

In the above group

edian $= (\frac{5+1}{2})$th students' score (because N+5)

$=$ 3th student's score

$=$ 4

Example: Ina a examination, few students have achieved following scores: 7, 9, 18, 24, 25, 27, 30, find out the median.

Here N = 7 $\therefore$ Median $= (\frac{N+1}{2})$th students' score

$= \frac{7+1}{2}$ th students' score

$=$ 4 " " "

$=$ 24

Question: Few students of a class have following scores. Calculate the median:

3, 4, 5, 5, 6, 6, 7, 8, 9

If your answer is -

(a) 5	see frame (63) on page 54	
(b) 5.89	" " (64) " " 55	
(c) 6	" " (65) " " 56	

It would be seen in the above example that the concept of median has been explained thoroughly, than an example has been given and solved in the frame itself. The procedure is thus explained to the students. Following the example, a multiple choice question has been given. The alternatives to the multiple choice question have been selected on the basis of errors generally committed by the students on the achievement test item pertaining to the subject matter of this frame as explained in the foregoing paragraph.

(iii) Remedial or branched frames:- The choice of alternatives in the main frame of the branching programmes takes the learner to the remedial or branched frames. These frames either advises the learner to precede further in the main line frames or provide remedial excercises by pointing out the errors committed by the students and giving a more exhaustive exposition of the facts of concepts included in the main line frame. As an example, the remedial or branched frames to the main line frame no.13 presented above have been presented below:

Frame 63 From Frame 13

Your answer is wrong

We know that Median $= \frac{N+1}{2}$ th student's score

Here, $N = 9$ Hence Median $= \frac{9+1}{2}$ th student's score

$= 5$th " "

You have considered 5 as median in place of 5th student's score. Note that 5th student's score is median. Now again study the frame no.13 on page 20 and answer the question:

The student has given value of $\frac{N+1}{2}$ th i.e. $\frac{9+1}{2}=5$ as median. Obviously the students have failed to find out the 5th student's score. The fact has been further eolaborated in the above frame and the correct method of calculation of median of the example presented in the original main line frame has been explained. The student has then been directed to the main line frame.

Frame 64: From frame 13

Your answer is wrong

You have not understood median properly. Median is score below and above which 50-50 cases lie, ile.

Median $= \frac{N+1}{2}$ th student's score

You have calculated mean instead of median because the mean of question of frame 13 is

$$= \frac{3+4+5+5+6+6+7+8+9}{9} = \frac{53}{9} = 5.89$$

<u>Now study the frame 13 on page 20 again and answer the the question again</u>.

The students give mean as the response, therefore, this above frame explains the difference between median and mean and further explains the procedure of calculating the median and then directs to the original main line frame.

Page 65 From frame 13

<u>Your answer is correct</u>.

Your have understood the method of calculating median properly because, Median = $\frac{9+1}{2}$ th student's score (In que, N=9)

= 6

<u>Now study the frame no.14 on page 21</u>.

The students' response was correct and therefore, the above frame direct to the student to next main line frame.

Tryout and Revision:

The tryout and revision of branching programme involves two types of testing.

a. Developmental testing

b. Validation testing

a. evelopmental esting:

he developmental testing aims at revising the draft programme by the help of students till the programmer feels that it will be able to achieve the objectives. It has the following two stages:

(i he ndividual esting tage

(ii The roup esting tage

(i Individual esting:- he original frames were written on cards with one frame on each card. hese cards were presented individually to the students. our students participated in individual tryout. The criteria for selecting students for the individual testing of the branching programme were similar to those employed for the linear programme. he responses and reactions were noted on the following proforma:

TABLE V

S AL PROFORMA OF INDIVIDUAL TESTING FOR BRA HING STYLE OF PROGRAMME

Frame o.	ime aken (minutes)	tudent esponse			tudent eaction		
		Right	rong	Remark	asy	Blocked	ired
17	3			Needs classification in S/2			
18	8						
19	5						
20	1						
25	7						

Editing and Revision of Original Draft:

The frames were revised on the basis of student responses and reaching and consequent remarks. For example, the students were held up in frame no.18. A perusal of this frame revealed that the students are not able to proceed further because the concept of N/2 has not been clarified. The frame was revised accordingly, incorporating more examples to explain this concept. Similarly frame no.25 was also considered very difficult for the student. It was considered that if a blank table of 'analysis table' was provided, the student response would be considerably easy. This frame was revised accordingly and student gave better response. In this way the whole programme was revised.

Group Tryout.

After editing the programme on the basis of individual tryout, the programme was administered to a representative sample of 25 students. The students were asked to said to work carefully and their responses were noted on a separate sheet of paper.

Frame Analysis.- After administering the branching programme booklet, frame by frame analysis was conducted with the help of following proforma in order to find out the error rate of each frame.

TABLE V

[illegible] PROFORMA FOR [illegible] ANALYSIS OF BRANCHING PROGRAMME.

Frame No.	Frame	No.of Student Responded	No.of Student Responded Correctly	Error Rate
3	Main frame	25	23	8%
7	Main frame	25	25	0
52	Remedial frame	25	24	4

The standards of error rate employed in the linear programme was not applicable to a branching programme. Such standards of error rate of branching programme frames were not available in the previous studies. Therefore, the error rate standard for this purpose were set up on theoretical considerations. The main line frame of branching programme assumes that the concepts, facts or principles included in it should be thoroughly explained with the help of suitable examples employing maxim of efficient teaching. Thus in a satisfactory branching programme, main line frame should not yield a high error rate. If an analysis revealed a high error rate for these frames, the frame should be revised a rewritten. However, a low error rate may be expected for such frames. A standard of 10% error rate was set up arbitrarily for the revision of main line frames in the present study. If a frame yields a error rate of more than 10% it was revised. The 10% error rate was considered acceptable as it would be taken care of by the remedial or branched frames. However, the remedial frames could not be expected to yield even a very low error rate. Therefore, if a remedial frames yielded any error rate it was rejected and a new frame was prepared in lieu of that. For example, frame no.52 yielded a error rate of 4 . It was a case of revision and therefore, revised. Similarly other frames were revised.

Validation Testing of Branching Programme:

As in linear programme, while the aim of development testing is to revise and finalise the draft programme until the programmer is satisfied that the programme will be able to achieve its objectives, the aim of validation testing is to study how far the programme is successful in teaching the specified population in natural conditions of its use. In specific terms, the purpose of validation testing is to find out.

i) to what extent the programme has succeeded in achieving the objectives for which it was constructed.

(ii) what are the reactions of participating teachers and students.

In order to find out the extent to which the programme has succeeded in achieving the objectives, the pre and post test scores were compared. The average of pre

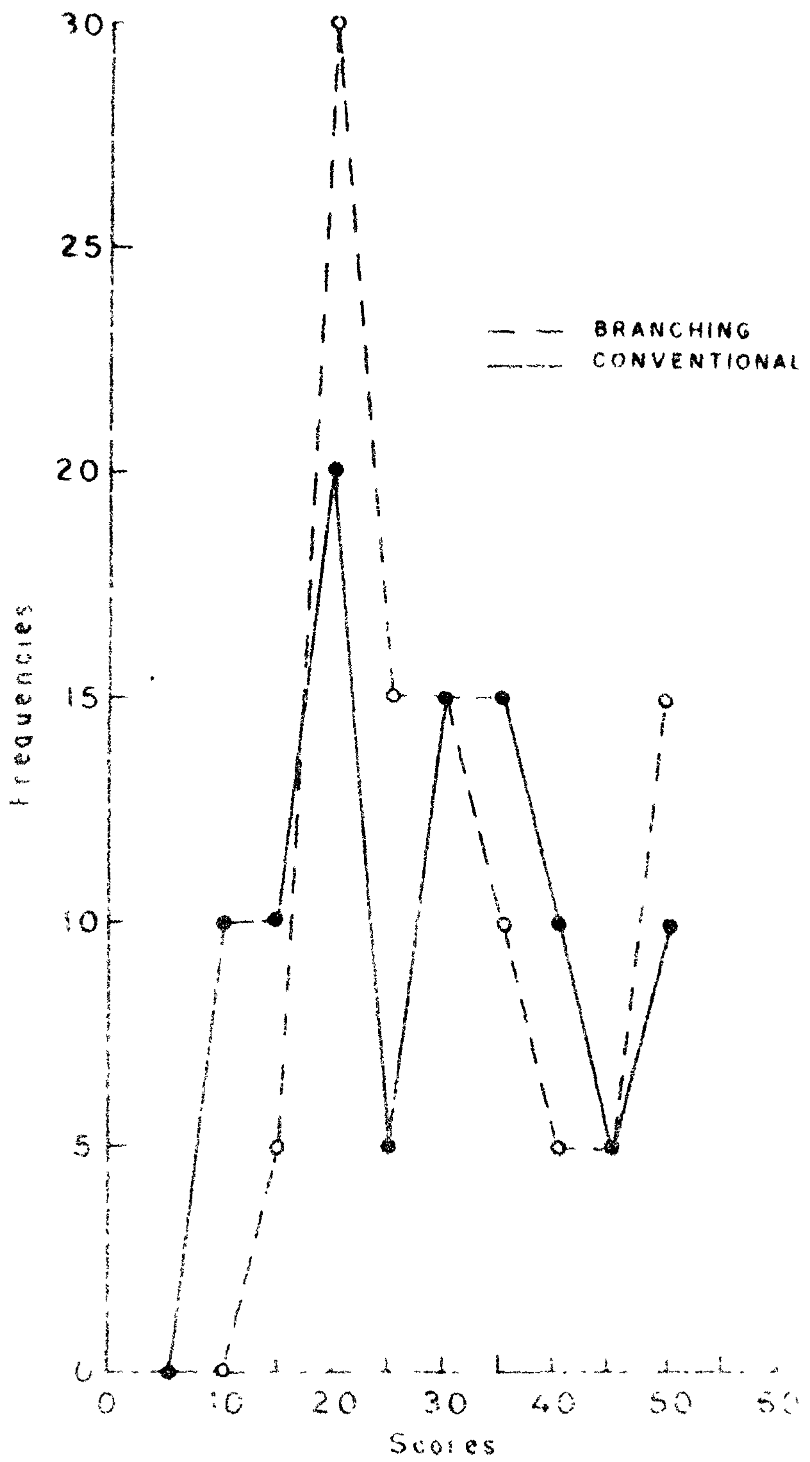


FIG 8 DISTRIBUTION OF ACHIEVEMENT OF STUDENTS TAUGHT THROUGH BRANCHING PROGRAMME AND CONVENTIONAL METHOD

test scores was and that of post test score was 127. Thus the present programme have succeeded in obtaining an average gain of 127 scores.

The validity of the programme was also ascertained by comparing the efficiency of the programme with that of conventional method.

TABLE VII

DISTRIBUTION OF ACHIEVEMENT SCORES OF STUDENTS TAUGHT THROUGH BRANCHING PROGRAMME AND CONVENTIONAL METHOD.

Class Interval	Frequencies: Conventional Method		Frequencies: Branching Programme	
0-5	0	20%	0	5%
5-10	10		0	
10 -15	10		5	
15-20	20	55	30	70
20-25	5		15	
25-30	15		15	
30-35	15		10	
35-40	10	25%	5	25%
40 -45	5		5	
45-50	10		15	

It would be seen from the Table-VII and figure 8 that the achievement scores obtained through two methods possess distinctly separate distributions, while 20% of the students taught by conventional method occupy the lower end of distribution of scores i.e. scores between 0 and 15 whereas 5 of students taught by branching occupy this part of distribution of scores. 55 of the students taught by conventional method occupy the middle part of the distribution of scores i.e. scores between 15 and 35 whereas 70 of the students taught by branching programme occupy this part of the distribution.i.e.the distribution further reveals that the upper end of the distribution of scores (i.e. scores between 35 and 50) is occupied by only 25 of the students taught by conventional method as well as branching programme. This shows that percentage of students taught by branching programme is higher than that of conventional method in middle part of the distribution whereas the percentage of students taught by branching programme is lower than that of percentage of students taught by conventional method out the lower end of the distribution. The percentage of students in the higher part of distribution is equal for both the methods. This shows that the present branching programme has resulted in higher achievement as compared to conventional method.

The efficiency of teaching programme consists not only in the scores obtained by students but also in the time taken to do so. In this respect also, the branching programme is proved to be better. While the average time taken by the students to learn through conventional method was 4 hours, it was only 3 hours and 20 minutes for learning through branching programme.

The reactions of the participating students and teachers toward the programme was ascertained by putting questions regarding the frames that were considered either too difficult or too easy and extent to which they liked the programme, vis a vis the conventional method. None of the frames were considered too difficult or too easy by most of the students and they found the branching programme as considerably easier to understand than conventional one.

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CHAPTER - V

CONSTRUCTION OF PRE AND POST ACHIEVEMENT TESTS

Determination of Objectives

Design

Blue Print or Grid

Composition of Test Items

Composition of Directions for Taking the Test

Item Analysis

Selection of Items

Reliability

Validity

References

The present study seeks to investigate the relationship between different learner characteristics and achievement through different treatment modes. This would require not only a test of terminal behaviours i.e. a measure of achievement of the pupils after a teaching programme, but also a test of entry level behaviours, which would include the measures of behaviours that are prerequisites for the development of terminal behaviours. Therefore, entry level test and a test of terminal behaviours were developed for this study (Appendix D). The procedures adopted for this purpose have been described in the following paragraphs.

Determining Objectives

The most important step in planning a test is to define the objectives of instruction. Separate sets of objectives were formulated for the entry level and terminal behaviours. The learning outcomes of the following objectives are sought to be measured through the achievement test of terminal behaviours.

Objective I : The pupil will acquire the knowledge of terms, symbols, concepts, definitions, formulae and procedures of calculation of mean, median and mode.

Specifications. The pupil will recall and recognise the -

(1) definition of mean, median and mode,

(ii) <u>formulae</u> for calculating mean in ungrouped data, grouped data when scores are given, grouped data when class intervals are given by long as well as short method, midpoint(x), median in ungrouped data when cases are even or odd, median interval, median in class interval, mode in class interval.

(iii) <u>procedures of calculating</u> mean in ungrouped data, mean in grouped data when scores are given, midpoint, mean in grouped data when class intervals are given by long method, assumed mean, mean in grouped data when class intervals are given by short method, median in ungrouped data when cases are even or odd, cumulative frequencies, median in grouped data when scores are given and when class intervals are given, mode in ungrouped data, mode in grouped data when one or more than one score have highest frquency and mode in class interval when class intervals are given.

<u>Objective II</u> : The pupil will develop an understanding of the terms, symbols, concepts, definitions, formulae and procedures of calculation of mean, median and mode.

<u>Specifications</u>: The pupil will

(i) identify the appropriate formula for appropriate purpose e.g. the formula for mean in class interval by short method is $a + \frac{fd}{N} \times i$

(ii) compare related concepts of mean, median and mode, mean in class interval by long and short method, median in even and odd number of cases,

(iii) discriminate between closely related procedures of determining appropriate class interval for assumed mean, formulae for mean, median and mode in ungrouped and grouped data when scores as well as class intervals are given,

(iv) detect errors of various formulae of mean, median and mode in ungrouped and grouped data when scores as well as class intervals are given.

<u>Objective III</u>: The pupil will develop skill in computation of mean, median and mode.

<u>Specifications</u>: The pupil will

(i) orally calculate with speed and accuracy the assumed mean, median in ungrouped data in odd number of cases, mode in ungrouped data and in grouped data when scores are given, model interval in grouped data when class intervals are given;

(ii) calculate with speed and accuracy mean in ungrouped data, grouped data when scores are given, mid-value, grouped data when class intervals are given by long as well as short method, cumulative frequencies, median in grouped data when scores or class intervals are given, mode in grouped data when more than one score have highest frequency and mode in grouped data when class intervals are given.

or the development of the above terminal behaviours, some entry level behaviours are the prerequisites. These entry level behaviours and their specifications are given below:

Objective I : The pupil should possess the knowledge of various arithmetical and statistical symbols and concepts.

Specifications: The pupil should recall and recognise the

(i) symbols - Σ, N, ΣX

(ii) concepts of even-odd numbers, ascending-descending orders, grouped-ungrouped data.

(iii) properties of class interval e.g. upper and lower limits, size of the class interval.

TABLE-VIII

WEIGHTAGE TO DIFFERENT OBJECTIVES IN THE ENTRY LEVEL AND TERMINAL BEHAVIOUR TESTS.

S.No.	Objectives	Percentages		Total
		Part I	Part II	
1.	Knowledge	15	11	13
2.	Understanding	60	22	41
3.	Skill	25	67	46
Total		100	100	100

B. Weightage to Content

One of the important characteristics of an achievement test is that it should be comprehensive i.e. it should have the widest possible coverage. The items to be included in the test should not only cover all the content areas but the coverage should also be in accordance to their relative importance. Therefore, the determination of the relative importance of the different units of the content constituted another important step in the design of the test. A critical analysis of the content matter was attempted. The help of subject experts was also elucit in this regard. Consequently the following weightage was assigned to different content units.

TABLE - IX

WEIGHTAGE ASSIGNED TO CONTENT UNITS.

Part	Units	Percentage
I- Test of entry level behaviours	(A) Arithmetical concepts e.g. x, ascending-descending order, even-odd numbers.	70
	(B) Statistical concepts e.g. grouped-ungrouped data, ", properties of class interval.	30
	Total	-100
II- Test of terminal level behaviours	(A) Measures of central tendencies	10
	(B) Mean	
	(i) Ungrouped	10
	(ii) Grouped scores	10
	(iii) Grouped-class interval	
	(a) Long method	10
	(b) Short method	10
	Total-	40
	( ) Median	
	(i Ungrouped	10
	(ii) Grouped-score	10
	(iii) Grouped-class interval	10
	Total -	30
	( ) Mode	
	(i) Ungrouped	5
	(ii) Grouped-scores	
	(a) General method	5
	(b) Grouping method	5
	(iii) Grouped-class interval	5
	Total -	20

( ) Weightage to form of Test Items:

After assigning the weightage to different objectives and units of contents, the next problem before the investigator was to select the choice of format of test items. The decision as to which type of item is most suitable depends to a large extent upon the content and the skill of the teachers in constructing the different forms. There are a number of different forms of objective type test items: multiple choice type, true-false type, completion type, matching type etc. The multiple choice type is considered to be the best because it reduces the chances of guessing to the minimum. However, the nature of the subject matter under present study warranted the construction of completion type of test items in most cases. For example, for eliciting the cumulative frequency of different step intervals, the whole frequency distribution and some plausible alternatives needed to be presented for each test item. Evidently such a procedure was very uneconomical. Therefore, completion type items were constructed in such cases. Apart from the practicability of such items, these items eliminate the chances of guessing altogether. In some respects these items are even better than the multiple choice type items if they properly constructed. These items were assigned 60% weightage of the whole test. Table-5 presents the weightage assigned to different types of test items included in entry level and terminal behaviour tests.

TABLE - X

WEIGHTAGE TO FORM OF QUESTIONS.

S. No.	Form	Part I	Part II
1.	Multiple choice	100%	40%
2.	Completion type	-	60%

Blue print or Grid -

When the weightage to different objectives, content units and forms of questions have been decided, the next step in developing the test items was to decide the distribution of all the items over different objectives and content units according to the weightage decided. This is generally done by means of a three dimensional chart called blue print or grid. In this blue print or grid, content matter was written in first column and objectives and forms of questions in the rows. The blue print or grid thus, prepared, is presented in Table-XI.

TABLE - XI

[illegible] - WAY [illegible] OF ITEMS

UNIT	OBJECTIVES / Form of Items / Subunits	KNOWLEDGE: Multiple choice	KNOWLEDGE: Completion choice	UNDERSTANDING: Multiple choice	UNDERSTANDING: Completion choice	APPLICATION SKILL: Multiple choice	APPLICATION SKILL: Completion choice	TOTAL NUMBER OF ITEMS		
I	(A) Arithmetical	3(1)	-	7(1)	-	5(1)	-	15	15	21
	(B) Statistical	-	-	6(1)	-	-	-	6	6	
II (A)	MEASURES OF CENTRAL TENDENCIES	3(1)								
	(A) MEAN									
	(i) Ungrouped	-	-	1(1)	-	2(1)		3	3	
	(ii) Grouped-scores	-	-	1(1)	-	-	2(3)	3	3	
	(iii) Grouped-class interval									17
	(1) Long method	-	-	2(1)	-	-	3(4)	5	11	
	2 Short method	-	-	2(1)	-	2(1)	2(4)	6		

....

	(c) MEDIAN									
II	(i) Ungrouped	1(1)	-	-	-	4(1)	-	5	5	
	(ii) Grouped-Score	-	-	1(1)	-	-	2(2)	3	3	11
	(iii) Grouped-class interval	-	-	1(1)	-	-	2(3)	3	3	
	(d) MODE									
	(i) Ungrouped	-	-	-	-	2(1)	-	2	2	
	(ii) Grouped-score									
	(1) Simple	-	-	-	-	2(1)	-	2	4	9
	(2) Grouping	-	-	-	-	-	2(15)	2		
	(iii) Grouped-class interval	-	-	1(1)	-	-	2(2)	3	3	
	TOTAL NUMBER OF	7	-	22	-	17	15			
	ITEMS	7		22		32			61	

NOTE: Numbers outside the parenthesis indicate number of testi items and the numbers inside the parenthesis indicate marks for one question.

The maximum marks to the different test items were allotted according to their relative difficulties. Test items requiring single step computation e.g. calculation of midvalue or calcuation of mean when the total of scores was given, was alloted a maximum score of one. Computation of mean of grouped data required, according to the estimate of the present investigation and some subject matter experts about four times difficult computation than that required for the single step computations. Consequently a maximum score of 4 was alloted to such test item. Some test items were very complex and required a higher maximum score e.g. calculation of mode by grouping method required the calculation of different steps in grouping and analysis tables, which entitled this item a maximum score of 15 according to the judgement of the experts. The table of two way specifications (Table-XI) presents the weightage to different items in parenthesis.

Composing Test Items:

After the preparation of blue print or grid, the different items were constructed. The different precautions suggested by Thorndike and Hagen (1963), Travers (1955), Burt (1972), Lindquest (1951) were observed while constructing these items. The entry level part of the test(part I) consisted of 21 multiple choice items and the terminal behaviour part of the test (part II) consisted of 25 multiple choice and 15 completion test items. Thus, the total test consisted of 46 multiple choice items and 15 completion type items. The item analysis to be described in following pages resulted in the rejection of only one item of the entry level test (part I). Thus the final form consisted of 45 multiple choice and 15 completion type test items.

Composing Directions for Taking the Test:

A good measure of achievement should not only be objective but should also be reliable and valid i.e. the measures should truely represent the achievement of the learner. This requires that the learner should be thoroughly conversant with the method of attempting the question an Extranious factors like quessing etc. should not influence the final score. The present achievement test consisted of the multiple choice and completion type test items. Since the students are not likely to be familiar with these new type test items, efforts were made to enable the students to thoroughly understand the method of taking the two types of test items. The whole test was divided into two parts - one consisting of multiple choice and other that of completion type test items. An example thoroughly explaining the procedure of taking each of the test item and method of recording the response was presented in the beginning of each section. A very easy concept, an understanding of which could be expected of every student constituted the subject matter of this example. The same concept was used for the illustrative examples of two types of items.

A student may be held up in a test item in the beginning of the test which is difficult for him. Thus, he may not be able to attempt comparatively easier item which occur later in the test because of paucity of time. Therefore, the students were required not to waste their time on difficult items, to solve easier items in the beginning and to attempt difficult items if the time permitted (direction no.4). In the multiple choice items the students may be tempted to guess and since their are 25% to 33% chances because there were three or four alternation, that a guess may be correct, some of the responses could be correct without the student knowing anything about them. To obviate this discripancy the students were warned not to guess because marks would be deducted for wrong answers. They should attempt an alternative only when they are sure that it was the correct choice (direction no.5). The test was intended to be a speed test, therefore, the students were required to complete the whole test in 60 minutes which was the time required by the weakest student to complete the test. The usual directions of taking the test carefully and to revise the responses were also included (direction no.8).

The aforementioned test was administered to a representative sample of students for the purpose of tryout. The test was scored with the help of scoring key. The rationale of weight alloted to the different items has already been discussed in the preceding paragraph.

TRYOUT - Item Analysis:

The item of an achievement test should neither be so difficult so that none of the student is able to attempt them nor so easy that everyone of the students would attempt them successfully. The items should, therefore, be neither too easy nor too difficult. To ascertain this, an inded of difficulty (difficulty value) was calculated in the present study. The items which were either too easy or too difficult were rejected and not included in the final form of the test. The items of the test should also discriminate well between a high and a low achievers. This was ascertained by computing the discrimination value of each item. The items with low or negative discrimination value were not included in the final form of the test. The procedure of calculation of these values has been presented in the following paragraphs:

A. Difficulty value: The difficulty value of each item was calculated with the help of the following formula:

$$\text{Difficulty value of any item} = \frac{R}{N}$$

where, R = number of individuals answering the item correctly

N = number of individuals in the group.

B. Discrimination value: For determining the discrimination value of each item of the test, the method suggested by Kelley and Flangar was employed. For this the top 27% and bottom 27% cases i.e. 27% cases who scored the highest marks and 27% cases who scored the lowest marks were selected and number of individuals who answered the item correctly were counted in each group. The discrimination value of each item was, then, calculated with the help of following formula:

$$Dv = \frac{Ru - R_l}{f}$$

here, Dv = Discrimination value of the item

Ru = number of individuals in the top group who answered the items correctly.

R_l = number of individuals in the bottom group who answered the item correctly.

f = Total number of individuals in each group.

The Table-XII presents the difficulty value and discrimination value of each item of the test.

TABLE - XII

DIFFICULTY AND DISCRIMINATION VALUES
OF ACHIEVEMENT TEST ITEMS.

Item No.	Difficulty Value	Discrimination Value
Part I		
1	0.73	0.20
2.	0.80	0.20
3	0.67	0.30
4	0.90	0.40
5	0.85	0.30
6	0.70	0.30
7	0.90	0.20
8	0.90	0.20
9	0.90	0.20
10	0.93	0.10
11	0.75	0.30
12	0.43	0.40
13	0.53	0.40
14	0.58	0.70
15	0.60	0.20
16(i)	0.42	0.30
(ii)	0.52	0.10
17(i)	0.22	0.10
(ii)	0.43	0.20
18(i)	0.48	0.30
(ii)	0.83	0.20
(iii)	0.63	0.50
19(i)	0.53	0.30
(ii)	0.90	0.30
(iii)	0.58	0.10

Cont....

20(i)	0.45	0.30
(ii)	0.82	0.20
(iii)	0.90	0.20
(iv)	0.77	0.20
(v)	0.83	0.20
21(i)	0.67	0.60
(ii)	0.82	0.60
(iii)	0.85	0.30
(iv)	0.77	0.30
(v)	0.83	0.30
Part II		
1	0.82	0.30
2	0.78	0.40
3	0.52	0.60
4	0.50	0.80
5	0.90	0.30
6	0.43	0.60
7	0.47	0.50
8	0.70	0.60
9	0.27	0.10
10	0.68	0.10
11	0.46	0.30
12	0.67	0.20
13	0.68	0.20
14	0.53	0.40
15	0.58	0.10
16	0.47	0.60
17	0.58	0.40
18	0.88	0.10
19	0.17	0.20
20	0.82	0.20
21	0.90	0.30
22	0.90	0.30

Cont....

23	0.55	0.50
24	0.63	0.30
25	0.37	0.10
26 (i)	0.85	0.30
(ii)	0.72	0.60
(iii)	0.56	0.70
27(i)	0.75	0.50
(ii)	0.60	0.40
(iii)	0.45	0.50
28(i)	0.42	0.70
(ii)	0.22	0.30
(iii)	0.10	0.20
29(i)	0.62	0.50
(ii)	0.47	0.50
(iii)	0.30	0.60
30(i)	0.68	0.60
(ii)	0.62	0.80
(iii)	0.30	0.40
31(i)	0.05	0.20
(ii)	0.05	0.20
(iii)	0.02	0.10
32(i)	0.11	0.40
(ii)	0.06	0.10
(iii)	0.06	0.20
(iv)	0.06	0.20
33(i)	0.62	0.60
(ii)	0.12	0.40
34(i)	0.67	0.50
(ii)	0.20	0.50
35(i)	0.63	0.30
(ii)	0.18	0.50
(iii)	0.05	0.10

Cont.....

36(i)	0.72	0.60
(ii)	0.33	0.60
(iii)	0.12	0.30
37(i	0.77	0.50
(ii)	0.03	0.10
38(i)	0.62	0.60
(ii)	0.03	0.10
39(i)	0.23	0.30
(ii)	0.27	0.60
(iii)	0.12	0.20
(iv)	0.15	0.30
40(i)	0.25	0.30
(ii)	0.18	0.70
(iii)	0.12	0.20
(iv)	0.12	0.20

Selection of Items:

It is already been pointed out that a satisfactory item would neither be too difficult nor too easy. ost of the workers in the field (Thorndike and Hagen, 1963) are of the opinionthat an achievement test item with a difficulty value of less than 0.1 i.e. an item which is not attempted correctly even by 10% of the student and an item with a difficulty value of more than 0.9 i.e. an item which more than 90% of the students attempt correctly should not be included in a test. These standards were adopted for rejecting the items in the present study. A satisfactory items should also discriminate well between high and low achievers. Items having discrimination value below 0.2 were considered unsatisfactory and were rejected. Table presents the difficulty value and discimination value of thedifferent items of the test. It would be seen that item no.10 of part I possesses a difficulty value of 0.93 and. discrimination value of 0.1 which reveals that it is very easy and poor in discrimination, An examination of the content of this item revealed that it did not affect the content to be measured and therefore, it was rejected. It will also be seen that item nos. 16,17,19 of part I and items 9,10,15,18,25,31,32,35(iii),37(ii) and 38(ii) of part II did not meet the standard set either for accepting the difficulty value or discrimination value or of both. It would be seen that most of the items with very high or low difficulty value also possess poor discrimination value. This seems to be fairly logical because a very difficult item which is not atte pted correctly by an i.e. neither by low achievers nor by high achievers, would possess poor discrimination value. Similarly, a very easy item which is attempted correctly by all, both low and high achievers, would also not discriminate well. An examination of all the above items revealed that the poor difficulty and discrimination values of these items was due to a variety of defects:

A. Difficult Language: Item nos. 16(ii) and 17 of part I and item nos. 10, 15, 18, 37(ii) and 38(ii) of part II possessed the difficult language. The language was therefore, improved.

For example, the original item nos.16 was as follows:

Q. In an examination, some students have secured following scores: 5,7,7,11,16

(i what is the form of above scores ?

(a) Ungrouped

(b) Grouped

(ii) what is the frequency of score 7

(a) one

(b) two

The above item was modified as follows:

Q. In a examination, the scores secured by some students are presented by two methods:

Method I - 5, 7, 7, 11, 16, 16

Method II -

Scores	Frequency
5	1
7	2
11	1
16	2

(i) what is the form of scores in method I

(a) Ungrouped

(b) Grouped

(ii) what is the form of scores in method II

(a) Ungrouped

(b) Grouped

(iii) what is the frequency of score 7 in method I

(a) One

(b) Two

(iv) what is the frequency of score 11 in method II

(a) One

(b) Two

Similarly other items were modified.

*. Poor format of Items: Item nos.31 and 32 of part II did not have specified space for writing the responses.

For example, item no.31 was originally as follows:

Class Interval (C.I.)	Frequency (f)	Deviation (d)	Frequency X deviat. (fd)
0-10	2		
10-20	4		
20-30	9		
30-40	5		

In the above distribution, class interval 20-30 has highest frequency and lie nearly at the centre of the distribution. Thus assumed mean will be 25.

(i) fill up in the column 3 and 4, the value of d and fd.

(ii) fill up the value of fd in blank space of the above distribution.

(iii) What will be the mean of above distribution ?

Mean =

The above item 31 was modifed as follows:

Class Interval (C.I.)	Frequency (f)	Deviation (d)	Frequency & Deviat. (fd)
1	2	3	4
-10	2	...	...
10-20	4	...	...
20-30	9	...	...
30-40	5	...	...

fd = ...

In the above distribution, class interval 20-30 has highest frequency and lie nearly at the centre of the distribution. Thus the assumed mean will be 25.

(i) Fill up in the column 3 and 4, the value of d & fd ?

(ii) Fill up the value of fd in blank space of the above distribution.

(iii) What will be the mean of the above distribution ?

Mean = ...

Similarly other items were modified.

3. <u>Alternatives were not Plausible</u>: Item nos. 9 and 25 of part II did not have plausible alternatives. Therefore, these alternatives were changed to make them more plausible. For example, the item no.9 was originally as follows:

Q. What is the formula to calculate the mean by short method:

(a) $\frac{fd}{N}$ X i + 1

(b) $\frac{fd}{N}$ X i + A

(c) $\frac{fx}{N}$ X i + A

(d) $\frac{fd}{N}$ X i + A

The above item was changed as follows:

Q. What is the formula for calculating mean by short method:

(a) $A + \frac{fd}{N} X i$

(b) $1 + (\frac{N/2-F}{f}) X i$

(c) $1 + (\frac{f-f_1}{2f-f_1-f_2}) X i$

Similarly item no.25 was also modified.

(4) Printing Mistake:

Item nos. 35(iii) of part II has minor mistake of cyclostyling. The word median was not clearly cyclostyled.

In final form this mistake was remedied. In this way the achievement test items of poor discrimination and difficulty values were modified. These items were not rejected because they measure an important aspects of content under study and therefore, the rejection of these items would affect the validity of the achievement test. These modified test were readministered to different sample. These items yielded satisfactory difficulty and discrimination values at the second administration.

<u>Reliability of the Test</u>

After the item analysis, the evaluation of the instrument was done with respect to reliability. Reliability refers to the consistency and accuracy with which the test measures what it does measure. A instrument is said to be perfectly reliable if it yields the same result on each administration or scoring. But every measurement, even the most exact one, is an approximation and is subject to error. This is much more conspicuously the case with psychometric instruments so reliability is measured in degrees or extent to which instrument measures consistently and accurately.

The reliability coefficient of the present test was estimated by splitting the whole test into two equal parts of even and odd items. The coefficient of reliability thus obtained was corrected by the Spearman-Brown Prophecy formula. The reliability coefficient of the entry level and terminal behaviour test after correction by the Spearman-Brown Prophecy formula were 0.78 and 0.84 respectively. These reliability coefficients are somewhat low according to standard set by Kelley (1927). However, Guilford (1956) recommends that a reliability coefficient between 0.80 and 0.95 is satisfactory for most purposes. The reliability coefficient of the entry level test is nearly 0.80 which is fairly satisfactory in view of the fact that this test was not intended to measure the outcomes of instructional procedures. The terminal behaviours test which measures, the outcomes of instruction and has been used as a criterion measure in the present investigation has yielded a fairly satisfactory reliability index of 0.84.

Validity of the Test

The present achievement test was constructed after very careful analysis of the content and objectives of teaching. Every effort was made to cover all the areas of this analysis. Therefore, this test may be considered to possess sufficient content validity. Attempt was also made to estimate the statistical validity of the test. The test scores were correlated with the examination marks in statistics covering the same subject matter. A validity index of 0.82 was obtained which is quite satisfactory.

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CHAPTER - VI

COLLECTION AND PRESENTATION OF THE DATA

A. Collection of the Data

The Sample

The Method

B. Presentation of the Data

Intelligence Test Score Data

Creativity Test Score Data

Anxiety Test Score Data

Extroversion Test Score Data

Previous Achievement Score Data

Post Test Achievement Test Score Data

The present investigation seeks to study the relationship between intelligence, creativity, anxiety and extroversion and achievement through linear and branching styles of programming and expository method of teaching. Previous achievement and intelligence have been employed as control variables.

COLLECTION OF THE DATA:

The data on the different variables employed in the present study was collected by the means of the following measures:

(1) Intelligence: Mehrotra's 'Mixed Type Group Test of Intelligence'

(2) Creativity: Baquar Mehdi's Verbal Test of Creative Thinking

(3) Anxiety: Sarason's General Anxiety Scale for Children

(4) Extroversion: Eysenck's Maudsley Personality Inventory adopted by Jalota, Kapoor.

(5) Previous Achievement: Marks obtained by the students in mathematics at the junior high school examination conducted by the department of Education, U.P. served as measure of previous achievement.

The instructional treatment modes that have been employed in the present study were as below:

(1) Linear Style of Programming

(2) Branching Style of Programming

(3) Expository Method

The linear as well as branching styles of programmes were developed and validated by the investigator himself. The expository text was also prepared by the investigator.

SAMPLE:

The study was conducted on 287 male students studying in ninth class of three intermediate colleges situated in two towns of Aligarh district - Aligarh and Hathras. In the selection of the three schools, the following criteria were kept in mind for each school:

(1) The school was located in semi-urban locality.
(2) Students from all the three strata-low, medium and high were studying in the school.
(3) Investigator had a good rapport with the teacher and students of school specially with mathematics teachers and students.
(4) Adequate facilities were available in the school to collect the data.

The sample of the present study, thus, consisted of total 287 students as follows:

TABLE-XIII

NUMBER AND AVERAGE AGES OF THE STUDENTS FROM THREE SELECTED SCHOOLS

School	Number	Average Age
1. D.A.V. Inter College, Aligarh	80	14.89
2. B.I. Jain Inter College, Aligarh.	88	14.55
3. S. Inter College, Hathras	119	15.75
Total	287	15.06

METHOD:

Students belonging to each school were divided into three groups as follows:

TABLE-XIV

NUMBER OF STUDENTS IN EACH OF THE GROUP

School	No. of Students			
	Linear I	Branching II	Expository III	Total
D.A.V. Inter College, Aligarh	22	28	30	80
B.I. Jain Inter College, Aligarh	28	29	31	88
S. Inter College, Hathras	41	47	31	119
Total	91	104	92	287

The usual practice in an investigation which seeks to study the relationship between some personological variables and achievement through different treatment modes, is to match the groups on some variables which is employed as a control variable. This was also attempted in the present study, but this needed reorganisation of classes and the time table which was not acceptable to the different heads of institution. The investigator, therefore, had to collect the data on the different variables from the students as they were grouped in different sections in their respective institutions. It was proposed to control the effect of the different variables statistically. For this purpose, analysis of covariance has been employed in the present study.

The measures of intelligence, creativity, anxiety and extroversion were administered to these groups of students meticulously following the instructions given by the author's of the different tests in their manuals. The test were scored by the investigator himself by the help of the scoring keys. An examination of the test scores revealed that the some of the students had not appeared at all the four tests simultaneously. The scores of such students had to be rejected.

After the administration of the tests, the teaching by the three treatment modes was taken up. As mentioned earlier linear and branching styles of programming was developed and validated by the investigator himself (Appendix B C). The expository text was also prepared by the investigator (Appendix A). The success through the linear and branching programme and the expository method depends on the possession of some entry level behaviour. An entry level test was prepared and administered to these students to ensure that they possessed all the previous knowledges which are needed for success through these treatment modes. It was found that almost all the students possessed all the necessary entry level behaviours. Only 3 students failed to achieve well on this test. They were not excluded for the teaching programme because of disciplinary problem however, they were not included in the analysis of the data. The criterion test was also administered in the beginning to ensure the students do not already possess the behaviours that the different treatment modes employed in the present study proposed to teach. It was found that none of the students possessed any of the behaviour proposed to be taught by the different treatment modes. The linear and branching styles of programming are autoinstructional methods of teaching and needed no interference or elaboration by the investigator or any other teacher. It was also attempted to keep the conditions fairly identical at the three administrations of the two programmes. This was done in view of the fact that the achievement scores of the three institutions have to be pooled together in the final analysis. The investigator himself taught the students by the expository method at the three institutions. Every effort was made to make the three teaching programme identical by using the similar expository text and illustrations at the three occasions. This was also done to ensure comparable achievement through expository method at the three institutions.

After the teaching programme a post test (Appendix D) was administered to all the students to measure their achievement through different treatment modes.

Presentation of the Data:

Intelligence, creativity, anxiety and extroversion are the independent variables and achievement through three instructional modes viz. linear, branching programmed style and expository method are the dependent variables in the present study. The previous achievement in the subject under study and intelligence (for those groups in which intelligence was not a independent variable) were employed as control variable.

Intelligence Test Score Data

Mehrotra's'mixed type group test of intelligence'was employed as a measure of verbal and non-verbal intelligence. It yields separate scores on verbal and non-verbal intelligence. Since the range of age of study population was negligible, there was hardly any need to convert these scores by any method. Therefore, raw scores have been employed for further analysis of data.

The Table XV presents the range and other statistics of the scores obtained by students taught through expository methods.

TABLE- XV

RANGE AND OTHER STATISTICS OF SCORES OBTAINED ON THE TEST OF INTELLIGENCE BY STUDENTS TAUGHT THROUGH EXPOSITORY METHOD.

Treatment mode	N	Range	Mean	Median	St. Dev.	Skewness 3(M- Md.)	$\frac{Q}{P_{90} - P_{10}}$ Kurtosis
Expository	88	-51	22.1	21.6	8.56	+0.209	0.275

The above table shows that mean as well as median values of the distribution are quite close to each other. The value of skewness is +0.51, which shows that the distribution is only slightly positively skewed. The value of skewness (0.209) is not too different from the zero. It is also clear that the distribution is only slightly platy kurtic. The obtained value of kurtosis i.e. 0.275 is very much near to its normal value i.e. 0.263. Thus the obtained distribution of intelligence scores of students taught by expository method is found almost normal.

The Table XVI presents range and other statistics of the scores obtained on the intelligence text by students taught through linear programmed instructional style.

TABLE-XVI

RANGE AND OTHER STATISTICS OF SCORES OBTAINED OF THE TEST OF INTELLIGENCE BY STUDENTS TAUGHT THROUGH LINEAR PROGRAMME

Treatment	N	Range	Mean	Median	St. Dev.	Skewness $3(M-Mdn)$	Kurtosis $\frac{Q}{P_{90}-P_{10}}$
Linear	87	8-69	27.53	25.21	13.41	+0.50	0.275

The above table shows that mean of the distribution is 27.53 whereas median is 25.21. These values are quite close to each other. The value of skewness is +0.51, which shows that the distribution is slightly positively skewed. It is also clear that the distribution is only slightly platykurtic. The obtained value of kurtosis i.e. 0.275 is very much equal to its normal value of 0.263. Thus the obtained distribution is also found almost normal.

The Table XVII presents the range and other statistics of the scores obtained on the intelligence test by students taught through branching programmed instructional styles. The mean and median of the above distribution lie not far from each other. The value of skewness (0.54) shows that the distribution is slightly positively skewed. It is also clear that the distribution is slightly leptokurtic i.e. the value of kurtosis (0.12) is less than its normal value (0.263) but not too away with its normal value. Thus the distribution is normal or atleast nearly normal.

TABLE-XVII

RANGE AND OTHER STATISTICS OF SCORES OBTAINED ON THE TEST OF INTELLIGENCE BY STUDENTS TAUGHT THROUGH BRANCHING PROGRAMME.

Treatment mode	N	Range	Mean	Median	St. Dev.	Skewness $3(M-Mdn)$	Kurtosis $\frac{Q}{P_{90}-P_{10}}$
Branching Programme	103	7-84	30.31	28.9	16.07	+0.44	0.12

Creativity Test Score Data:

This test initially yielded three scores one each for fluency, flexibility and originality. Since the rationale for scoring is quite different for each of the dimensions of the creativity and different weightage has been given to a single response for the three dimensions i.e. fluency, flexibility and originality, hence it was not considered reasonable to add the raw scores of the three dimensions in order to obtain the total creativity score. These scores were, therefore, converted into standard scores. These standard scores on the above three aspects of creativity were added to obtain the total creativity score.

(A) Table XVIII presents the range and other statistics of total creativity scores obtained by the students taught through expository method.

TABLE-XVIII

RANGE AND OTHER STATISTICS OF SCORES OBTAINED ON A TEST OF CREATIVITY BY STUDENTS TAUGHT THROUGH EXPOSITORY METHOD

Treatment mode	N	Range	Mean	Median	S.D.	Skewness $3(M-Mdn)$	Kurtosis $\frac{Q}{P_{90}-P_{10}}$
Expository method	87	112.6-189.8	138.82	135.5	22.7	+0.44	0.210

The mean and median of the creativity scores obtained by students taught through expository method is 138.82 and 135.5 respectively, which is not too much different. The value of skewness is +0.44, which shows that the distribution is only very slightly positively skewed. The value of obtained skewness (+0.44) is low. It is also clear that the distribution is very slightly lepto-kurtic. The obtained value of kurtosis (0.210) is also not different from the normal value (i.e. 0.263). Thus the obtained distribution of creativity is found almost normal.

(B) Table XIX presents the range and other statistics of total creativity scores obtained by the students taught through linear method.

TABLE-XIX

RANGE AND OTHER STATISTICS OF SCORES OBTAINED ON A TEST OF CREATIVITY BY STUDENTS TAUGHT THROUGH LINEAR PROGRAMMING.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Linear programme	91	117.2-215.9	157.42	154.25	21.01	+0.45	0.209

The above table reveals that mean and median of the distribution are not different from each other. The value of skewness i.e. +0.45 shows that the distribution is only slightly positively skewed. The table also shows that the distribution is only very slightly lepto-kurtic. The obtained value of kurtosis (i.e.0.209) does not differ significantly from the normal value i.e. 0.263. Thus the obtained distribution is nearly normal.

( ) Table XX presents the range and other statistics of total creativity scores obtained by the students taught through branching method.

TABLE-XX

RANGE AND OTHER STATISTICS OF SCORES OBTAINED ON A TEST OF CREATIVITY BY STUDENTS TAUGHT THROUGH BRANCHING PROGRAMME.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Branching programme	103	112-208	147.31	145.31	20.49	+0.29	0.24

The above table reveals that mean and median lies nearly at the same point. The value of skewness shows that the distribution is very slightly positively skewed. It is also clear that the distribution is only slightly leptokurtic. The obtained value (i.e. 0.24) is not significantly different from the normal value (i.e. 0.263). Thus the obtained distribution is almost normal.

Anxiety Test Score Data:

Sarason's 'General Anxiety Scale for Children' was employed as a measure of anxiety. Total score of a subject is equal to the algebric summation of the number of situations which the person had marked as 'yes'. The maximum score is 45 and minimum zero. Higher scores of anxiety are indicative of high anxiety.

(A) Table XXI presents the range and other statistics of distribution of scores on the anxiety scale obtained by the students taught through expository method.

TABLE-XXI

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY STUDENTS ON A TEST OF ANXIETY TAUGHT THROUGH EXPOSITORY METHOD.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Expository method	88	4-42	25.38	25.33	8.22	+0.018	0.313

The table XXI shows the mean and median of the distribution as 25.38 and 25.33 respectively, which are very much equal to each other. The obtained value of skewness is +0.18 which is not different from zero. The value of kurtosis is 0.313, which is not significantly different from .263. Hence the distribution is nearly normal.

(B) The table XXII presents the range and other statistics of distribution of scores on the anxiety scale obtained by students taught by linear style of programming.

TABLE-XXII

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY THE STUDENTS ON A TEST OF ANXIETY TAUGHT THROUGH LINEAR PROGRAMME.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Linear programme	90	2-38	23.67	24.25	7.19	-0.08	0.20

The above table shows that the mean (23.67) and median (24.25) are nearly equal to each other. There is only a slight difference between the two. The skewness is -0.08 which is very much negligible. The table further reveals that the value of kurtosis is 0.29 which is only a slightly (0.03) different from the normal value (i.e.0.263). Hence the obtained distribution is nearly normal.

(C) The table XXIII presents the range and other statistics of anxiety scores obtained by students taught through branching programme.

TABLE-XXIII

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY STUDENTS ON A TEST OF ANXIETY TAUGHT THROUGH BRANCHING PROGRAMME.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Branching programme	102	0-43	27.84	26.8	7.2	+0.416	0.26

The above table reveals that the mean and median of the distribution are nearly equal to each other. The value of skewness is +0.416, which shows that the distribution is only slightly positively skewed. The obtained value of kurtosis is almost identical with the normal value (0.263). Thus the obtained distribution is normal.

Extroversion Inventory Data:

Eysenck's 'Maudsley Personality Inventory' adopted by Jalota and Kapoor was employed to obtain the scores on the extroversion trait. Total score on the inventory of each student was obtained by summing up their score on each item. The maximum score is 48 whereas the minimum score is zero.

higher score on the trait is an indication of high level of extroversion.

(A) Table XXIV presents the range and other statistics of the distribution of scores on the extroversion inventory obtained by students taught through expository method.

TABLE-XXIV

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY STUDENTS ON A TEST OF EXTROVERSION TAUGHT THROUGH EXPOSITORY METHOD.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Expository method	88	0-36	25.10	24.75	7.15	+0.13	0.23

The above table shows that the mean and median of the distribution (i.e. 25.10 and 24.75) is quite close to each other. The value of skewness is +0.13 which shows that the distribution is very slightly positively skewed. It is also clear from the table that the distribution is slightly lepto-kurtic. The obtained value of 0.23 is slightly lower than its normal value. Thus the obtained distribution of scores on extroversion is nearly normal.

(B) Table XXV presents the range and other statistics of scores on extroversion inventory obtained by students taught through linear style of programming.

TABLE-XXV

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY STUDENTS ON A TEST OF EXTROVERSION TAUGHT THROUGH LINEAR PROGRAMMING.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Linear	90	?-34	23.89	24.4	3.61	-0.42	0.45

It is clear from the table that the mean (23.89) and median (i.e. 24.4) is nearly equal to each other. The obtained value of skewness is -0.42 which shows that the distribution is only slightly negatively skewed. The distribution is slightly platy-kurtic. The obtained value (0.45) is not significantly different from the normal value (0.363). Thus the obtained distribution is nearly normal.

(?) Table XXVI presents the range and other statistics of scores on extroversion inventory obtained by students taught through branching style of programming.

TABLE-XXVI

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Branching	101	?-36	26.36	24.9	9.3	+0.45	.18

The table XXVI clearly shows that mean and median are quite close to each other. The value of skewness (0.45) shows that the distribution is slightly positively skewed. It is also clear that the distribution is slightly lepto-kurtic. The obtained value of kurtosis is slightly less than the normal. Thus the obtained distribution is nearly normal.

Previous Achievement Score Data

Previous achievement in the subject under study i.e. mathematics was employed as control variable in the present study. The scores obtained by the students in mathematics at the Junior High School examination served as measure of previous achievement in the subject.

In the following pages the distribution of previous achievement scores of various groups are presented.

(A) Table XXVII presents the range and other statistics of previous achievement of students taught through expository method.

TABLE-XXVII

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY STUDENTS IN THEIR PREVIOUS EXAMINATION OF MATHEMATICS TAUGHT THROUGH EXPOSITORY METHOD.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Expository	88	2-83	32.84	33.0	15.77	-0.03	0.19

The above table shows that the mean and median of the distribution are very close to each other and hence lie on the more or less same point. The skewness value is -0.03 which is negligible. It is also clear that the distribution is slightly leptokurtic. The obtained value (0.19) is only slightly different from normal value i.e. 0.263. Hence the distribution can be considered as normal.

(B) Table XXVIII presents the range and other statistics of the previous achievement scores obtained by students taught through linear style of programming.

TABLE-XXVIII

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY STUDENTS IN THEIR PREVIOUS EXAMINATION OF MATHEMATICS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Linear	86	6-83	34.09	32.27	16.69	+0.32	0.248

It car be seen from the above table that mean and median of the distribution are 34.09 and 32.02, which are not different from each other. The skewness value of distribution is +0.32 which shows that the distribution is only slightly positively skewed. It is further seen that the distribution is slightly leptokurtic. The kurtosis value (0.248) is only slightly less than the normal value (0.263). Thus the distribution is nearly normal.

(C) Table XXIX presents the range and other statistics of previous achievement scores of student taught through branching style of programming.

TABLE-XXIX

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY THE STUDENTS ON THEIR PREVIOUS EXAMINATION OF MATHEMATICS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Branching	103	2-72	29.86	29.25	18.1	+0.11	0.54

It can be seen from the above table that mean and median of the distribution is 29.86 and 29.25 respectively which are nearly equal. The value of skewness is +0.11 which is only slightly greater than zero. This reveals only a slightly positive skewness which is unimportant for the present purpose. It is further seen that the distribution is slightly platy-kurtic with only a slightly higher value than the normal value. Thus the distribution can be assumed as normal.

Post Achievement Score Data.

An achievement test (hereinafter called the post test) was administered to the students taught through linear and branching programmes and the expository method. The scores on this test constituted the criterion for assessing the efficiency of a treatment mode. The following tables presents the distribution of scores obtained by the students taught through three methods.

(A) Table XXX presents the range and other statistics of scores obtained on the post test by students taught through expository method.

TABLE-XXX

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY THE STUDENTS ON A POST ACHIEVEMENT TEST TAUGHT THROUGH EXPOSITORY METHOD.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Expository	88	9-76	40.22	38.5	14.47	+0.35	0.286

The table presented above, shows that mean and median of the distribution are 40.22 and 38.5 respectively. The difference between the mean and the median is insignificant. The value of skewness is +0.35 which shows that the distribution is only very slightly positively skewed. It is also seen that the distribution is very slightly platykurtic. The obtained value of kurtosis is very slightly different from 0.263. Thus this distribution can also be assumed to be normal.

(F) Table XXXI presents the range and other statistics of scores obtained on the post test by students taught through linear style of programming.

TABLE-XXXI

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY THE STUDENTS ON A POST ACHIEVEMENT TEST TAUGHT THROUGH LINEAR PROGRAMMING.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Linear	90	14-79	41.56	39.6	17.47	+0.470	0.290

It can be seen from the above table that the difference between the mean and median is insignificant. The value of skewness (0.470) shows that the distribution is only slightly positively skewed. The obtained value of kurtosis (0.29) is slightly greater than the normal value i.e. .263. Thus the distribution is only very slightly platy-kurtosis. Thus the distribution is nearly normal.

(c) Table XXXII presents the range and other statistics of scores obtained on the post test by students taught through branching style of programming.

TABLE-XXXII

RANGE AND OTHER STATISTICS OF SCORES OBTAINED BY THE STUDENTS ON A TEST OF POST ACHIEVEMENT TEST TAUGHT THROUGH BRANCHING PROGRAMME.

Treatment mode	N	Range	Mean	Median	S.D.	Skewness	Kurtosis
Branching	103	8-83	34.6	34.1	14.9	+0.04	0.26

It can be seen from the above table that skewness value is approximately equal to zero and the value of kurtosis is also more or less identical with the normal value of 0.263. Thus the distribution can be assumed as normal.

A perusal of the tables from XVI-XXXIII shows that the distribution of the scores obtained by the students on the different predictor and criterion variables are nearly normal. One of the most important assumptions for employing the analysis of covariance and 't' test techniques is that the population from which the samples under study are drawn must be normally distributed. For this purpose, in practice it is usually considered satisfactory that the samples themselves do not depend drastically from normality. Thus the data obtained for the different variables employed in the study is suitable for employing the analysis of covariance and 't' test techniques.

CHAPTER - VII

ANALYSIS OF THE DATA

(A) Basic Assumptions

(B) Relationship between Personality and Mental Ability Variables and Achievement through Treatment Modes

Intelligence and Treatment Modes

Creativity and Treatment Modes

Anxiety and Treatment Modes

Extroversion and Treatment Modes

References

BASIC ASSUMPTIONS

In the present study analysis of covariance and t-tests have been employed to study the interaction between personality and mental ability variables and achievement through three treatment modes viz. linear style of programming, branching style of programming and expository method of teaching. The rationale for employing 'analysis of covariance' for the analysis of the data of the present investigation has already been presented in Chapter III. The proper use of most of the inferential statistical techniques like covariance and 't' test is based on certain assumptions. According to Popham (1967), the assumptions which must be satisfied for the proper interpretation of analysis of covariance results are: (1) The relationship between two variables must be linear, (2) the homoscedasticity between the two variables must exist, (3) the variances in the subgroups must be relatively homogeneous i.e. have equal variances, (4) the measures must be randomly drawn.

The two important assumptions for the interpretation of the results of covariance are that the distributions must be linear and homoscedastic. These assumptions are fairly important in case of coefficient of correlation and linear regression, but in the case of analysis of covariance, a stringent satisfaction of these assumptions is not needed. Popham (1967) writes that for the interpretation of covariance results "stringent satisfaction of these is probably not required but the departure from them should not be too large". It can be argued that if the distribution of the two variables is normal, the relationship between them will also be linear. Guilford (1956) also writes that "normality or near normality of both the distributions correlated is sufficient in itself to promote linearity". Since the stringent satisfaction of the assumption of linearity is not needed in the case of covariance, the normality of the distribution of the variables was considered sufficient for the satisfaction of the assumptions of linearity. Again, homoscedasticity is also satisfied if the distributions are linear. Popham (1967) supports this view and writes "Ordinarily, a data is considered to be distributed in a homoscedastic fashion when the relationship is linear in nature". Since homoscedasticity depends upon linearity and linearity depends upon normality of distribution, therefore, normality of the distribution can be considered sufficient for the satisfaction of the assumptions of linearity and homoscedasticity. This is particularly more so because as pointed out by Popham (1967) that stringent satisfaction of the assumptions of linearity and homoscedasticity are not needed for interpretating results of analysis of covariance.

Therefore, only normality of distribution was considered sufficient to take care of the first two assumptions namely, linearity and homoscedasticity between two variables.

Homogeneity of variances in the subgroups is an another important assumption and can be tested.

The interpretation of 't' test results also requires the satisfaction of following two assumptions (Popham, 1967). (1) the sample data must be drawn randomly from the population and (2) the population from which each sample is drawn must be normally distributed.

Thus in the final analysis, the following three assumptions were needed to be satisfied:

(1) The sample data must be drawn randomly from the population.

(2) The population from which each sample is drawn must be normally distributed.

(3) The variances in the subgroups must be homogeneous.

An attempt has been made to see in the following pages whether these assumptions have been satisfied by the data.

Assumption 1:

Since the assignment to all the three treatment modes was random and observations were independent the assumption that measures must be randomly drawn satisfy in itself.

Assumption 2:

The satisfaction of this assumption requires that the population from which each sample is drawn must be normally distributed. For this purpose, in practice it is usually considered satisfactory that the samples themselves do not depart drastically from normality.

It has already been shown in the preceding pages (presentation of data) that the distribution of scores of various dependent, independent and control variables were normal or near normal. The values of skewness of these distributions ranged from 0.018 to 0.50 which are considered insignificantly different from the normal value which is zero. The values of kurtosis also ranged from 0.128 to 0.509 which are also not significantly different from the normal value of 0.263. Thus the assumption that the samples have been drawn from a population, in which the variables under study are normally distributed, is satisfied.

<u>Assumption 3</u>:

The satisfaction of this assumption requires that the variances in the subgroups must be realatively homogeneous i.e. the subgroups have equal variances. There are several methods by which this hypothesis can be tested. Hartley's (1950) test for homogeneity of variances and Bartlett's (1937) tests for homogeneity of variences are generally employed for this purpose. Hartley's test (1950) is a test of great simplicity and convenience which can be used when all the samples have the same number of cases.

In the present study the top 30% and bottom 30 students constituted the high and low groups. Some cases were dropped inorder to keep the number of cases equal in each of the groups. Since the number of cases in both low and high groups is the same, Hartley's (1950) test was applied to test the homogeneity of variances. This test yields a constant known as Fmax., which can be defined as the ratio of maximum variance to minimum variance of the subgroup under testing. The significance of Fmax. can be checked against the table value of Fmax. for various levels of significance (Walker and Lev).

(A) Test of homogeneity of variances between Low and High Intelligence groups

Table XXXIII presents the Fmax. values of high and low intelligence groups.

TABLE-XXXIII

TEST OF HOMOGENEITY OF VARIANCE OF HIGH AND LOW INTELLIGENCE GROUPS

Intelligence Group	VARIANCES (S^2)		
	Linear	Branching	Expository
LOW	42.250	43.1640	7.2361
HIGH	104.629	114.49	16.3216
$Fmax. = \frac{S^2max.}{S^2min}$	2.47	2.6	2.25
Significance	Insignificant	Insignificant	Insignificant

It can be seen from the table XXXIII that Fmax. values for linear, branching and expository methods are 2.47, 2.6 and 2.25 respectively which are less than the tabled value of Fmax. for n (degree of freedom) = 28 and k (no. of mean squares) = 2 i.e. 2.63 at 0.01 level. Thus the Fmax. are all insignificant, which shows that low and high intelligence groups of each of the methods have equal variances.

(4) <u>Test of homogeneity of Variances between Low and High Creative Groups.</u>

Table XXXIV presents the variances and Fmax. values of high and low creative subgroups.

TABLE-XXXIV

TEST OF HOMOGENEITY OF VARIANCE OF HIGH AND LOW CREATIVE GROUPS

Creative Group	VARIANCES (S^2)		
	Linear	Branching	Expository
LOW	457.96	841.00	686.44
HIGH	424.36	388.09	9[illegible]0.[illegible]
Fmax.	1.07	2.17	1.31
Significance	Insignificant	Insignificant	Insignificant

From the above table, it can be seen that the Fmax. are 1.07, 2.17 and 1.31 for high and low creative groups of linear, branching and expository methods respectively. All the Fmax. values are less than the tables value i.e. 2.63 for n = 28 and k = 2 at 0.01 level. Thus all the values are insignificant at 0.01 level. There are only 1% chances that the difference in high and low creative groups exists. There are variances of all the subgroups of linear, branching and expository methods were homogeneous.

( ) Test of homogeneity of variances between Low and High Anxiety Groups.

Table XXXV presents the variances of Fmax. value of high and low anxiety groups.

TABLE-XXXV

TEST OF HOMOGENEITY OF VARIANCES OF HIGH AND LOW ANXIETY GROUPS

Anxiety Groups	VARIANCES (S^2)		
	Linear	Branching	Expository
LOW	20.4304	30.9136	30.9136
HIGH	9.6721	11.56	10.4829
Fmax.	2.11	2.60	2.98
Significance	Insignificant	Insignificant	Insignificant

The above table reveals the Fmax. values for high and low subgroups of linear, branching and expository methods as 2.11, 2.60 and 2.98 respectively, which are all less than the tables value of Fmax. for k = 2 and n = 28 at 0.01 level. Thus it can be concluded that the groups were possess homogeneous variance at 0.01 level of significance.

( ) Test of homogeneity of Variances between High and Low Extroversion Groups.

The table XXXVI presents the variance and Fmax. values of high and low extroversion groups.

TABLE-XXXVI

TEST OF HOMOGENEITY OF VARIANCES OF HIGH AND LOW EXTROVERSION GROUPS.

Extroversion Group	VARIANCES (S^2)		
	Linear	Branching	Expository
LOW	15.1321	18.1476	19.0096
HIGH	6.1009	6.8129	8.5849
Fmax.	2.48	2.60	2.21
Significance	Insignificant	Insignificant	Insignificant

The above table reveals that all Fmax. values for n = 28 and k = 2 at 0.01 level of significance are insignificant which shows that the high and low groups of extroversion employed for the three modes possess homogeneous variance.

It is clear from the preceeding discussion that groups high and low on the different predictor and criterion measures possess homogeneous variance.

The above discussions shows that the present data satisfy the various assumptions of covariance analysis and 't' test. Therefore, it was fairly reasonable to employ analysis of covariance and 't' test for investigating the relationship between personality and mental ability variables and achievement through the three treatment modes viz. linear, branching and expository methods of teaching.

RELATIONSHIP BETWEEN PERSONALITY AND MENTAL ABILITY VARIABLES AND ACHIEVEMENT THROUGH TREATMENT MODES:

The analysis of the data basically consisted in dividing the subjects of the study into high and low groups on each personality and mental ability variable for each of the three treatment modes viz. linear, branching style of programmed learning and expository method and comparing their post achievement. The upper group consisted of top 33% and the lower group of bottom 33%. The achievement of these two groups after employing either of the three treatment modes was compared. The intelligence and previous achievement were controlled by employing analysis of covariance. In this way the mean achievements of high and low groups adjusted on the basis of the control variables were obtained. A comparison of these adjusted means was made not only between high and low groups of the same treatment modes but also between the high groups taught by different treatment modes and between low groups taught by these treatment modes. Obviously such a comparison necessitated that the high as well as low groups taught by the three treatment modes should be comparable as far as the personality and mental ability variable under investigation was concerned, Unless this was ensured there could be a chance that the range and mean of a high group on any variable for one treatment mode could be similar or nearly so to the range and mean of a low group on that variable for another treatment mode. Such a situation would make the comparison untenable. Therefore, for each variable the high as well as the low groups taught by three treatment modes were tested for comparability. Comparison was also made between high and low groups on that variable for each of the three treatment modes separately.

INTELLIGENCE AND TREATMENT MODES:

The technique of the analysis of covariance employed in the present study requires that the high and low groups should consist of similar number of cases. According to Popham (1967) "Though disproportionate frequencies in the subgroups can be employed, intensive adjustments must be made in the analysis. Usually it is wiser to employ proportionate or equal numbers of subjects in the subgroups"(p.244). The number of students taught by the three treatment modes ranged from 8 to 1[illegible]3. The upper and lower intelligence groups consisted of 2[illegible] students each. These groups were made after including such students as would tend to make the groups comparable.

Table XXVII presents the mean, S.D. and 't' values

TABLE-XXXVII

't' VALUES BETWEEN DIFFERENT PAIRS OF TREATMENT MODES FOR HIGH AS WELL AS LOW INTELLIGENCE GROUPS.

Intelligence subgroups	Statistics	TREATMENT MODES					
		Linear	Branching	Expository	Branching	Expository	Linear
High (N=29)	Mean (X)	41.76	42.56	37.00	42.56	37.00	41.76
	S.D. (σ)	10.23	10.71	4.04	10.71	4.04	10.23
	't'	0.28		2.48		2.34	
LOW (N=29)	Mean (X)	12.69	14.30	12.62	14.30	12.62	12.69
	S.D. (σ)	6.5	6.57	2.69	6.57	2.69	6.5
	't'	0.90		1.28		0.005	

of low as well as high intelligence groups for different pairs of treatment modes.

It would be seen from the table XXXVII that all the 't' values of high as well as low intelligence groups for different pairs of treatment modes are not insignificant even at 0.01 level of significance. The tables value of 't' for 28 degree of freedom at 0.01 level of significance is 2.78. Thus it can safely be concluded that the high as well as the low intelligence subgroups taught through linear, branching and expository treatment modes did not differ significantly whatever differences occured could be due to chance alone.

Comparison was also made between the means of intelligence scores of the high and low groups taught through each of the three treatment modes. The significant difference between the subgroups is important for reliable prediction of the efficiency of treatment modes for high and low intelligence groups. Table XXXVIII presents the mean, S.D. and 't' values of high and low intelligence groups for different treatment modes.

TABLE-XXXVIII

'T' VALUES BETWEEN HIGH AND LOW INTELLIGENCE GROUPS FOR DIFFERENT TREATMENT MODES.

Statistics	Linear		Branching		Expository	
	Low Int.	High Int.	Low Int.	High Int.	Low Int.	High Int.
Mean (M)	12.69	41.76	14.30	42.56	12.61	37.00
S.D. (σ)	6.5	10.23	6.57	11.31	2.69	4.04
't'	12.03*		11.6*		27.35*	

*Significance at 0.05 level.

It would be seen from the table XXXVIII that mean values of high and low intelligence groups for different treatment modes are very significantly different from each other. Thus it can very safely be concluded that the high and low intelligence groups for each treatment modes were far separated from each other.

Analysis of Covariance

After establishing that the high as well as low intelligence groups taught by the three treatment modes were comparable, the next step before the investigator was to find out to what extent the post achievement scores of the high and low intelligence groups differed when taught by the three treatment modes viz. linear, branching and expository method. As pointed out earlier the influence of previous achievement was sought to be controlled. This was achieved by employing the analysis of covariance. This has been explained in the following pages.

(A) Linear Style of Programming:

The sums, sum of squares and mean and cross products of the criterion and control variable are presented in the table XXXIA.

TABLE-XXXIX

SUMS, SUM OF SQUARES, MEAN AND CROSS PRODUCTS OF CRITERION AND CONTROL VARIABLES OF THE HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Intelligence Group	N	CRITERION: Post Achievement			CONTROL: Previous Achievement			Cross Product
		ΣY	ΣY^2	$\bar{Y}$	ΣX_1	ΣX_1^2	$\bar{X}_1$	$\Sigma X_1 Y$
High	29	1478	84840	5097	898	36570	30.97	44638
Low	29	1214	62906	41.86	768	23936	26.48	28869
Total	58	2692	147746	-	1666	60506	-	73507

The values of the above table were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the subgroups. These deviation values are presented in the table XL. The calculation of these deviations values has been presented in appendix E.

TABLE-XL

SUM OF SQUARES AND CROSS PRODUCTS IN DEVIATION FORM FOR THE TOTAL AND WITHIN THE SUBGROUPS OF THE HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Sources of Variation	Post Achievement	Previous Achievement	Cross Product
Total	22800.07	12651.59	-3818.38
Within	21598.42	12360.21	-9047.97

he regression equations for total and for within the subgroups were then calculated with the help of above table. These regression equations have been presented below:

or Total, $Y = -0.3018\ X$ ------------(1)

or ithin $Y = -0.73\ X$ ------------(2)

where X and Y are control and criterion variables respectively.

The above regression equations were employed for calculating the sum of squares of residuals for total and within subgroups. These sum of squares of residuals have been presented in Table XLI. Their calculations appears in appendix .

TABLE-XLI

SUM OF SQUARES OF RESIDUALS F.. TOTAL AN WITHIN SUBGROUPS ... HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

ources of Variation	Residuals		ean Square
	Degree of Freedom	um of Squares	
Total	56	21647.64	
ithin	55	14975.09	272.27
Difference	1	6672.55	6672.55

$$F_{1,55} = \frac{6672.55}{272.27} = 24.51$$

After the calculation of the sum of squares of residuals, 'F' test of significance was made to find whether the difference in the post test achievement scores of the high and low intelligence groups taught through linear programme was significant or not.

It would be seen from the table XII that the difference between the total and within subgroups sum of squares of residuals is 6672.55, which is due to the difference in the achievement of high and low intelligence groups when taught by linear style of programming. The mean of squares for this difference is also 6672.55 because the difference in the degree of freedom between total and within subgroups is 1. The 'F' value is calculated by dividing this mean square difference by the mean square value of the within subgroups which is 272.27. The value of 'F' comes to 24.51, which is significant, beyond the 5% level of confidence with 1 and 55 degrees of freedom. Therefore, when the criterion means of the two subgroups are adjusted for individual differences in previous achievement in the subject, the difference between the post achievement means through linear style of programming of high and low intelligence groups is significant beyond 0.5 level and has not occured due to chance alone.

The means of post-test scores of high and low intelligence groups taught by linear style of programming were adjusted for the influence of the control variable (previous achievement) by the help of the within regression equation. he adjustment terms of criterion means for controlling the effect of previous achievement were 1.63 and 1.65 for the high and low intelligence groups respectively. It would be seen from table XXXIX that the high intelligence group is superior than the low intelligence group on the control variable. herefore, the adjustment term was subtracted from the criterion means of the high group and adjustment term for the low group was added to criterion means of this group. hese adjusted and unadjusted means have been presented in able XIII. he calculation of the adjustment terms and adjusted means have been presented in appendix 4.

TABLE-XLII

UNADJUSTED, ADJUSTED MEANS AND F VALUE BETWEEN THE ADJUSTED MEANS OF THE LOW AND HIGH INTELLIGENCE GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Intelligence Groups	Post Achievement Means		
	Unadjusted	Adjusted	F value
High	50.97	48.34	24.51
Low	41.86	43.51	

(B) Branching tyle of Programming

The sums, sum of squares, cross products and means of the criterion and control variables are shown in table XLIII.

The regression equations for total and for within the subgroups were than calculated with the help of the table XLIV. These equations have been presented below:

$$Y = .037\ X$$

$$Y = 0.05\ X$$

where X and Y are control and criterion variables, respectively. The above regression equations were employed for calculating the sum of squares of residuals for total and within the sub-groups. These sum of squares for residuals have been presented in table XLV.

TABLE-XLV

SUM OF SQUARES OF RESIDUALS FOR TOTAL AND WITHIN THE SUBGROUPS OF THE HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Sources of Variation	Degree of Freedom	RESIDUALS	
		Sum of Squares	Mean Square
Total	56	33341.74	
Within	55	10227.66	185.96
Difference	1	23114.08	23114.08

$$F_{1,55} = \frac{23114.08}{185.96} = 124.30$$

After the calculation of the sum of squares of residuals, 'T' test of significance was made to find whether the difference in the post achievement scores of the high and low intelligence groups taught by the branching style of programming was significant or not.

It would be seen from the table XLV that the difference between the total and within subgroups sum of squares of residuals is 23114.08, which is due to the difference in the post achievement of high and low intelligence groups when taught by branching styles of programming. The mean of square for this difference is also 23114.08 because the difference in the degrees of freedom between total and within subgroups is 1. The 'F' value comes to be 124.30 which is significant, beyond the 5 level of confidence for 1 and 55 degree of freedom. Therefore, when the criterion means of the two subgroups are adjusted for individual differences in previous achievement in the subject, the difference between the post achievement means through branching style of programming of high and low intelligence groups is significant and has not occured due to chance alone.

The means of post test scores of high and low intelligence groups taught by branching style of programming were adjusted for the influence of the control variable by the help of the within regression equation. The adjustment terms of criterion means for controlling the effect of previous achievement were .0115 and 0.0110 for the high and low intelligence groups, respectively. It would be seen from the table XLIII that the high intelligence group is superior than the low intelligence group on the control variable. Therefore, the adjustment term for the high intelligence was subtracted from the criterion mean of the group and adjustment term for the low intelligence group was added to criterion mean of this group. These unadjusted and adjusted means has been presented in table XLVI.

TABLE-XLVI

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE BETWEEN THE ADJUSTED MEANS OF HIGH AND LOW INTELLIGENCE GROUP TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Intelligence Group	Post Achievement Means		
	Unadjusted	Adjusted	'F' Value
High	55.93	55.92	124.3
Low	33.52	33.53	

(C) Expository Method.

The sums, sum of squares, cross products and means of criterion and control variables are shown in table XLVII.

TABLE-XLVII

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF CRITERION AND CONTROL VARIABLE OF THE HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Intelligence Group	N	Post Achievement ΣY	ΣY^2	$\bar{Y}$	Previous Achievement ΣX_1	ΣX_1^2	$\bar{X}_1$	Cross Product $\Sigma X_1 Y$
High	29	1189	60789	41.00	938	35374	32.34	37129
Low	29	1109	50279	38.24	972	41910	33.52	34026
Total	58	2298	111068		1910	77284		71155

With the help of table XLVII, the sum of squares and sum of cross products in deviation form for the total and for the within subgroups are calculated. These deviations are presented in the table XLVIII.

TABLE-XLVIII

SUM OF SQUARES AND CROSS PRODUCTS IN DEVIATION FORM FOR THE TOTAL AND FOR THE WITHIN SUBGROUPS OF THE HIGH AND LOW INTELLIGENCY GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Cross Product $\Sigma X_1 Y$
Total	20019.66	14385.72	-452[illegible].52
Within	19909.31	14368.24	-4473.62

The regression equations for total and for within the subgroups were then calculated with the help of above table. These regression lines are:

For Total, $Y = 0.3141\ X$

For Within $Y = 0.3110\ X$

where X and Y are control and criterion variables respectively.

The above regression lines were employed for calculating the sum of squares of residuals for total and within subgroups. These sum of squares of residuals have been presented in table XLIX.

TABLE-XLIX

SUM OF SQUARES OF RESIDUALS FOR TOTAL AND WITHIN THE SUBGROUPS OF THE HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Sources of Variations	Degree of Freedom	RESIDUALS Sum of Squares	Mean Square
Total	56	18599.15	336.66
Within	55	18516.22	
Difference	1	82.93	82.93

$$F_{1,55} = \frac{82.93}{336.66} = 0.25$$

After calculating the sum of squares of residuals, 'F' test of significance was made to find out whether the difference in the post test achievement of the high and low intelligence groups taught through expository method was significant or not.

It would be seen from table XLIX that the difference between the total and within subgroups sum of squares of residuals is 82.93 which is due to the difference in the post achievement of high and low intelligence groups when taught by expository method. The mean of squares of this difference is also 82.93 because of difference in degree of freedom of total and within subgroups is 1. The 'F' value is calculated by dividing the this mean square difference by the mean square

square value of the within subgroups i.e. 336.66. The 'F' value comes out to 0.25, which is insignificant. Therefore, when the criterion means of the two subgroups are adjusted for individual difference in previous achievement, the difference between the post achievement means through expository method of high and low intelligence groups is insignificant.

The means of post achievement scores for high and low intelligence groups taught by expository method were adjusted for the effect of the control variable (previous achievement) by the help of within regression equations. Though the group means are always adjusted when the analysis of variance 'F' is significant statisticians differ on the question of whether to adjust if the 'F' is not significant. But Popham (1967) recommends that all means should be adjusted to see whether the relative positions of any of the groups change. Therefore, the necessary adjustments were also made in this case. The adjustment terms of criterion means for controlling the effect of previous achievement were 0.18 and 0.18 for the high and low groups respectively. It would be seen from the table XLVII that the high intelligence groups is superior than the low intelligence group on the control variable. Therefore, the adjustment term was subtracted from the criterion mean of high intelligence group and adjustment term for the low intelligence group was added to criterion means of this group. These unadjusted and adjusted means have been presented in table L.

TABLE-L

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE BETWEEN THE ADJUSTED MEANS OF HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Intelligence Group	Post Achievement Means		'F' Value
	Unadjusted	Adjusted	
High	41.00	41.18	.25
Low	38.24	38.06	

CREATIVITY AND TREATMENT MODES

The technique of analysis of covariance employed in the present study requires that the high and low groups should consists of similar number of cases. The number of students taught by the three treatment modes ranged from 88 to 103. The upper and lower creativity groups consisted of 29 students each. These groups were made after including such students as would tend to make the groups comparable.

Table LI presents the mean, S.D. and 't' values of low and high creativity groups for different pairs of treatment modes.

TABLE-LI

't' VALUES BETWEEN DIFFERENT PAIRS OF TREATMENT MODES FOR HIGH AS WELL AS LOW CREATIVITY GROUPS.

Creativity Subgroups	Statistics	TREATMENT MODES					
		Linear	Branching	Expository	Branching	Expository	Linear
High (N=29)	Mean	181.45	172.76	169.86	172.76	169.86	181.45
	S.D.	20.55	19.74	30.01	19.74	30.01	20.55
	't'	1.64		0.43		1.72	
Low (N=29)	Mean	129.00	118.21	122.52	118.21	122.52	129.00
	S.D.	21.39	29.03	26.22	29.03	26.22	21.39
	't'	1.61		0.59		1.05	

It would be seen from the table LI that all the 't' values of high as well as low creativity groups for different pairs of treatment modes are not significant even at 0.1 level of confidence. Thus it can be safely inferred that the high as well as the low creative groups taught by linear, branching and expository treatment modes did not differ significantly. Whatever differences occured could be due to chance alone.

Comparison was also made between the mean creative scores of high and low groups taught by each of the three treatment modes. The significance difference between the subgroups is important for reliable prediction of the efficiency of treatment modes for high and low creative groups. Table LII present the mean, S.D. and 't' values of high and low creative groups for different treatment modes.

TABLE-LII

'T' VALUES BETWEEN HIGH AND LOW CREATIVE GROUPS FOR DIFFERENT TREATMENT MODES.

Statistics	LINEAR		BRANCHING		EXPOSITORY	
	Low Creative	High Creative	Low Creative	High Creative	Low Creative	High Creative
Mean	129.00	181.45	118.21	172.76	122.52	169.86
S.D.	21.39	20.55	29.03	19.74	26.22	30.[illegible]1
't'	9.54*		6.59*		7.44*	

*Significant at 0.05 level.

It would be seen from the table LII that the mean values of high and low creative groups for different treatment modes are very significantly different from each other. Thus it can very safely be concluded that the high and low creative groups for each treatment mode were far separated from each other.

Analysis of Covariance:

After establishing that the high as well as low creative groups taught through the three treatment modes were comparable, the next task before the investigator was to find out to what extent the post achievement of the high and low creative groups differed when taught through the three treatment modes viz. linear, branching and expository treatment modes. As pointed out earlier the influence of previous achievement and intelligence were sought to be controlled. This was achieved by employing the analysis of covariance. This has been explained in the following pages.

(A) Linear Style of Programming:

The sums, sum of squares, cross products and means of the criterion and control variables are shown in table LIII.

The values of the table LIII were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the subgroups.

TABLE-LIII

SUM, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF THE CRITERION AND CONTROL VARIABLES FOR THE HIGH AND LOW CREATIVITY GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Creativity Group	N	Post Achievement			Previous Achievement			Intelligence			Cross Products		
		ΣY	ΣY^2	$\bar{Y}$	ΣX_1	ΣX_1^2	$\bar{X}_1$	ΣX_2	ΣX_2^2	$\bar{X}_2$	$\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
High	29	1329	74125	45.83	1091	51847	37.62	849	41448	29.28	97760	40[illegible]95	44632
Low	29	1222	63722	42.14	675	93934	23.28	730	23250	25.17	54578	19408[illegible]	159583
Total	58	2551	137847		1766	145841		1579	64698		152338	234184	204215

TABLE-LIV

SUM OF SQUARES AND CROSS PRODUCTS IN DEVIATION FORM FOR THE TOTAL AND WITHIN SUBGROUPS OF THE HIGH AND LOW CREATIVITY GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Sources of Variance	Post Achievement	Previous Achievement	Intelligence	Cross Products		
	ΣY^2	ΣX_1^2	ΣX_2^2	$\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
Total	25647.00	92063.34	21711.09	74564.45	164735.22	156137.17
Within	25449.65	89085.62	21466.[illegible]3	73997.00	164515.69	155783.65

These deviation values are presented in the table LIV.

The regression equations for total and within the subgroups were then calculated. These regression equations have been presented below:

For Total, $Y = 1.08\ X_1 - 0.16\ X_2$

For Within $Y = 1.08\ X_1 - 0.14\ X_2$

where X_1 and X_2 are control variables and Y is criterion variable.

With the help of these regression equations, the sum of squares of residuals for total and within subgroups were worked out. These sums of squares of residuals have been presented in table LV.

TABLE-LV

SUM OF SQUARES OF RESIDUALS FOR TOTAL AND WITHIN THE SUB-GROUPS OF THE HIGH AND LOW CREATIVITY GROUP TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Sources of Variation	Degree of Freedom	RESIDUALS	
		Sum of Squares	Mean Squares
Total	55	22659.81	
Within	54	25415.78	470.66
Difference	1	2755.97	2755.97

$$F = \frac{2755.97}{470.66} = 5.8$$

After the calculation of sum of squares of residuals, 'F' test of significance was made to find whether the difference in post achievement scores of the high and low creativity groups taught by linear style of programming was significant or not. It would be seen from the table LV that the difference between the total and within subgroups of sum of squares of residuals is 2755.97, which is due to the difference in the achievement of high and low creative groups when taught by the linear style of programming. The mean of square for this difference is also 2755597 because the difference in degree of freedom between total and within subgroups is 1.

The F value is calculated by dividing this difference mean square by the mean square value of the within subgroups which is 47 .66. The F value comes to 5.8, which is significant beyond 0.05 level with 1 and 54 degree of confidence. Therefore, when the criterion means of two subgroups are adjusted for individual difference in previous achievement and intelligence, the difference between the post achievement through linear style of programming of high and low creative groups is significant beyond 0.05 level and has not occured due to chance alone.

The means of post achievement scores of high and low creative groups taught through the linear programme were adjusted for the influence of the control variables such as previous achievement in the subject and intelligence by the help of within regression equation. The adjustment terms of criterion means for controlling the effect of previous achievement and intelligence were 7.74 and 7.74 and for controlling the effect of intelligence it were 0.29 and 0.29 for high and low creativity groups respectively. It would be seen from the table LIII that the high creativity groups is superior than the low creativity group on both the control variables. Therefore, the adjustment term for both the control variables were subtracted from the criterion mean of the high creativity group and added in criterion means of the low creativity group. The unadjusted and adjusted means have been shown in table LVI.

TABLE-LVI

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE BETWEEN THE ADJUSTED MEANS OF THE LOW AND HIGH CREATIVE GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Creative Group	Post Achievement Means		
	Unadjusted	Adjusted	'F' Value
High	45.83	37.80	5.8
Low	42.14	50.17	

(B) Branching Style of Programming:

The sums, sum of squares, cross products and means of the criterion and control variables are shown in table LVII.

TABLE-LVII

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF THE CRITERION AND CONTROL VARIABLES OF THE HIGH AND LOW CREATIVE GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Creative Group	N	Post Achievement			Previous Achievement			Intelligence			Cross Product		
		ΣY	ΣY^2	$\bar{Y}$	ΣX_1	ΣX_1^2	$\bar{X}_1$	ΣX_2	ΣX_2^2	$\bar{X}_2$	ΣX_1Y	ΣX_2Y	ΣX_1X_2
High	29	1382	90868	47.66	828	30388	26.55	1125	55091	38.79	83329	107117	139084
Low	29	1039	71240	35.83	573	15309	19.76	745	84246	25.68	38981	27074	43289
Total	58	2421	161908		1401	45697		1870	139337		122310	134191	182373

TABLE-LVIII

SUM OF SQUARES AND CROSS PRODUCTS IN RESIDUAL FORM FOR THE TOTAL AND WITHIN SUBGROUPS OF THE HIGH AND LOW CREATIVE GROUPS TAUGHT THROUGH BRANCHING STYLE.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Intelligence ΣX_2^2	Cross Products ΣX_1Y	ΣX_2Y	ΣX_1X_2
Total	60852.12	11855.60	79045.62	63830.33	56134.62	137202.83
Within	58823.09	10734.48	76555.97	62322.31	53897.38	135532.14

The value of table LVII were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the subgroups. These deviation values are presented in the table LVIII.

The regression equations for total and within the subgroups were then calculated. These regression equations have been presented below:

$$Y = 0.15\ X_1 + 0.45\ X_2$$

$$Y = 0.14\ X_1 + 0.45\ X_2$$

where X_1 and X_2 are control variables and Y is a criterion variable.

With the help of these regression equations, the sum of squares of residuals for total and within the subgroups were worked out. These sum of squares of residuals have been presented in table LIX.

TABLE-LIX

SUM OF SQUARES OF RESIDUALS FOR TOTAL AND WITHIN THE SUBGROUPS OF THE HIGH AND LOW CREATIVE GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Sources of Variation	Degree of Freedom	RESIDUAL	
		Sum of Squares	Mean Square
Total	55	26016.99	
Within	54	25849.25	478.69
Difference	1	167.74	167.74

$$F_{1,54} = \frac{167.74}{478.69} = 0.35$$

After the calculation of sum of squares of residuals, 'F' test of significance was made to find whether the difference in post achievement scores of high and low creative groups taught through branching style of programming was significant or not. It would be seen from the table LIXthat the difference between the total and within subgroups of sum of square of residual is 167.74, which is due to the difference in the achievement of high and low creative groups when taught through the branching style of programming. The mean of square for this difference is also 167.74 because the difference in degree of freedom between total and within subgroups is 1. The F value is calculated by dividing this difference mean square by the mean square value of the within subgroup which is 478.69. The 'F' value appears to be 0.35, which is insignificant. Therefore, when the criterion means of the two subgroups are adjusted for individual difference in previous achievement and intelligence, the difference between the post achievement through branching style of programming of high and low creative groups is insignificant.

he means of post achievement scores of high and low creative groups taught through the branching programme were adjusted for the effect of control variables of previous achievement in the subject and intelligence by the help of within regression equation. The adjustment terms of criterion means for controlling the effect of previous achievement were 0.62 and 0.62 and for controlling the effect of intelligence, the adjustment terms were 2.93 and 2.93 for high as well as low creativity groups, respectively. It would be seen from the table LVII that the high creativity group is superior than the low creativity group on both the control variables. Therefore, the adjustment terms for both the control variables were subtracted from the criterion mean of the high creativity group and added in the criterion mean of the low creativity group. The unadjusted and adjusted means have been presented in table LX.

TABLE-LX

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE BETWEEN THE ADJUSTED MEANS OF HIGH AND LOW CREATIVE GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Creative Group	Post Achievement Mean		
	Unadjusted	Adjusted	'F' Value
High	47.66	44.11	0.35
Low	35.83	39.38	

(c) Expository Method:

The sums, sum of squares, cross products and means of the criterion and control variables are shown in table LXI.

TABLE-LXI

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF CRITERION AND CONTROL VARIABLES OF THE HIGH AND LOW CREATIVE GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Creative Group	N	Post Achievement			Previous Achievement			Intelligence			Cross Products		
		ΣY	ΣY^2	$\bar{Y}$	ΣX_1	ΣX_1^2	$\bar{X}_1$	ΣX_2	ΣX_2^2	$\bar{X}$	$\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
High	29	1191	61013	41.07	1070	46282	36.9	857	758335	29.55	89408	46052	50191
Low	29	1054	75975	36.34	795	28061	23.41	515	10865	17.76	58535	28533	19284
Total	58	2245	136988		1865	74343		1372	769200		147943	74685	69475

TABLE-LXII

SUM OF SQUARES AND CROSS PRODUCTS IN DEVIATION FORM FOR THE TOTAL AND WITHIN THE SUBGROUPS OF THE HIGH AND LOW CREATIVE GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Intelligence ΣX_2^2	Cross Products $\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
Total	50091.02	14373.60	736745.10	75754.64	21579.14	25358.10
Within	49767.41	13069.72	734728.48	75105.07	20771.31	23736.56

The values of the abtable LXI were then used to compute the sum of squares and sum of cross producte in deviation form for the total and for within the subgroups. These deviation values are presented in the table LXII.

The regression equation for total and within the subgroups were then calculated. These regression equations have been presented below:

$$Y = 5.56\,X_1 - 0.16\,X_2$$

$$Y = 6.05\,X_1 - .17\,X_2$$

where X_1 and X_2 are two control variables and Y is criterion variable.

With the help of these regression equations, the sum of squares of residual for total and within the subgroups were worked out. These sums of squares of residuals have been presented in table LXIII.

TABLE-LXIII

SUM OF SQUARES OF RESIDUALS FOR TOTAL AND WITHIN THE SUB-GROUPS FOR THE HIGH AND LOW CREATIVE GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Sources of Variation	Degree of Freedom	RESIDUALS	
		Sum of Squares	Mean Square
Total	55	367652.12	
Within	54	397331.47	7356.13
Difference	1	29679.35	29679.35

$$F_{1,54} = \frac{29679.35}{7356.13} = 4.34$$

After the calculation of sum of squares of residuals, test of significance, 'F' test, was made to find whether the difference in post achievement scores of high and low creative groups taught through expository method was significant or not. It would be seen from table LXIII that the difference between the total and within the subgroups of sum of squares of residuals is 29679.35, which is due to the difference in the achievement of high and low creative groups when taught through expository method. The mean of square of this difference is also 29679.35 because of degree of freedom for difference is 1. The 'F' value is calculated by dividing this difference mean square by the mean square value of within the subgroups, which is 7356.13. The 'F' value comes to be 4.34 which is significant beyond 0.05 level. Therefore when the criterion means of the two subgroups are adjusted for previous achievement and intelligence, the difference between the post achievement through expository method of high and low creative groups is significant. Whatever the difference is not due to chance alone.

The means of post achievement of high and low creative groups taught through the expository method were adjusted for the effect of control variables of previous achievement and intelligence by the help of within regression equation. The adjustment terms of criterion means for controlling the effect of previous achievement were 28.74 and 28.74 and for controlling the effect of intelligence were 1 and 1 for high as well as low creativity groups respectively. It would be seen from the table LXI that the high creativity groups is superior than low creativity group on both the control variables. Therefore, the adjustment term for both the control variables were subtracted from the criterion mean of the high creativity group and added to the criterion mean of the low creativity group. The unadjusted and adjusted means have been presented in table LXIV.

TABLE-LXIV

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE BETWEEN ADJUSTED MEANS OF HIGH AND LOW CREATIVE GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Creative Group	Post Achievement Means		
	Unadjusted	Adjusted	'F' Value
High	41.07	17.33	4.34
Low	36.34	66.08	

ANXIETY AND TREATMENT MODE

The technique of analysis of covariance employed in the present study requires that the high and low groups should consist of similar number of cases. The numbers of students taught by the three treatment modes ranged from 88 to 103. The upper and lower anxiety groups consisted of 29 students each. These groups were made after including such students as would tend to make the groups comparable.

TABLE-LXV

't' VALUES BETWEEN DIFFERENT PAIRS OF TREATMENT MODES FOR HIGH AS WELL AS LOW ANXIETY GROUPS

Anxiety Groups	Statistics	TREATMENT MODES					
		Linear	Branching	Expository	Branching	Expository	Linear
High (N=29)	Mean	31.90	32.52	33.31	32.52	33.31	31.90
	S.D.	3.11	3.40	3.41	3.40	3.41	3.11
	't'	0.73		0.907		1.67	
Low (N=29)	Mean	15.52	15.14	16.69	15.14	16.69	15.52
	S.D.	4.52	5.56	5.56	5.56	5.56	4.52
	't'	0.29		1.06		0.88	

Table LXV presents the mean, S.D. and 't' values of low as well as high anxiety groups for different pairs of the treatment modes.

It would be seen from the table LXV that all the 't' values of high as well as low anxiety groups for different pairs of treatment modes are highly insignificant at 0.01 level of confidence. The tables value for 28 degree of freedom at 0.01 level is 2.78. Thus it can safely be concluded that the high as well as low anxiety groups taught through linear, branching and expository treatment modes did not differ significantly. Whatever differences occured could be due to chance alone.

Comparison was also made between the mean anxiety scores of high and low groups taught through each of the three treatment modes. The significant difference between the high and low groups is important for reliable prediction of the efficiency of treatment modes for high and low anxiety groups. The table LXVI presents the mean, S.D. and 't' values of high and low anxiety groups for different treatment modes.

TABLE-LXVI

't' VALUES BETWEEN HIGH AND LOW ANXIETY GROUPS FOR DIFFERENT TREATMENT MODES.

tatistics	Linear		Branching		Expository	
	Low Anxiety	High Anxiety	Low Anxiety	High Anxiety	Low Anxiety	High Anxiety
ean	30.90	15.52	32.52	15.14	32.31	16.69
S.D.	3.11	4.52	2.33	5.56	3.23	5.56
't'	15.3*		15.51*		13.20*	

*Significant at 0.05 level.

It would be seen from the table LXVI that the mean values of high and low anxiety groups for different treatment modes are highly significant. Thus it can be very safely concluded that the high and low anxiety groups for each treatment mode were far seperated from each other.

<u>Analysis of Covariance</u>:

After establishing that the high as well as the low anxiety groups taught through three treatment modes were comparable, the next task before the investigator was to find out to what extent the post achievement scores of the high and low anxiety groups differed when taught through the three treatment modes viz. linear, branching and expository. As pointed out earlier the influence of previous achievement and intelligence was sought to be controlled. This was done by employing the analysis of covariance. This have been explained in the following pages:

(A) Linear Style of Programming:

The sums, sum of squares, cross products and means of the criterion and control variables are shown in table LXVII.

The values of the table LXVII were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the groups. These deviation values are presented in the table LXVIII.

TABLE-LXVII

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF THE CRITERION AND CONTROL VARIABLES OF THE HIGH AND LOW GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Anxiety Group	N	Post Achievement			Previous Achievement			Intelligence			Cross Products		
		ΣY	ΣY^2	$\bar{Y}$	ΣX_1	ΣX_1^2	$\bar{X}_1$	ΣX_2	ΣX_2^2	$\bar{X}_2$	$\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
High	29	1556	97976	53.66	882	39855	30.41	787	41185	27.14	99832	55247	77420
Low	29	1155	54965	39.83	872	32606	30.07	823	74338	28.38	66922	111400	138831
Total	58	2711	152941		1754	72461	30.24	1610	115521		166754	166647	216251

TABLE-LXVIII

SUM OF SQUARES AND CROSS PRODUCTS IN DEVIATION FORM OF HIGH AND LOW ANXIETY GROUP TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Intelligence ΣX_2^2	Cross Products $\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
Total	26225.12	19417.62	70829.62	84769.62	91393.38	167562.38
Within	23452.69	19415.89	70807.28	84700.48	91642.28	167568.58

The regression equations for total and within the subgroups were then calculated. These regression equations have been presented below:

$$Y = 0.352\ X_1 + 0.47\ X_2$$

$$Y = 0.35\ X_1 + 0.46\ X_2$$

where X_1 and X_2 are control variables and Y is criterion variables.

With the help of these regression equations, the sum of squares of residuals for total and for within the subgroups were worked out. These sum of residuals have been presented in table LXIX.

TABLE-LXIX

SUM OF SQUARES OF RESIDUALS FOR THE TOTAL AND WITHIN THE SUBGROUPS OF HIGH AND LOW ANXIETY GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Sources of Variation	Degree of Freedom	RESIDUALS	
		Sum of Squares	Mean Squares
Total	55	46398.92	
Within	54	48347.93	895.33
Difference	1	1949.01	1949.01

$$F_{1,54} = \frac{1949.01}{895.33} = 2.18$$

After the calculation of sum of square of residuals, F test of significance was made to find whether the difference in post achievement score of the high and low anxiety groups taught through linear, style of programming, was significant or not. It would be seen from the table LXIX that the difference between the total and within subgroups of sum of squares of residuals is 1949.01, which is due to the difference in the achievement of high and low creative groups when taught through the linear style of programming. The mean of squares for this difference is also 1949.01 because the difference in degree of freedom between total and within subgroup is one. The F value is calculated by dividing this difference mean square by the mean square value of the within subgroup which is 895.7[illegible]. The F value, 278 is significant. Therefore when the criterion means of two subgroups are adjusted for individual difference in previous achievement and intelligence, the difference between the post achievement through linear style of programming of high and low creative groups is insignificant.

The means of post achievement scores of high and low anxiety groups taught through the linear programme were adjusted for the effect of the control variables of previous achievement and intelligence by the help of within regression equation. The adjustment terms of criterion means for controlling the effect of previous achievement were [illegible].06 and 0.06 and for controlling the effect of intelligence were 0.29 and

.29 for high and low anxiety groups respectively. It would be seen from the table LXVII that the high anxiety group is superior than the low anxiety group on previous achievement and inferior on intelligence. Therefore, the adjustment terms for previous achievement was subtracted and the adjustment term for intelligence was added to the criterion mean of high anxiety group. The adjustment term for previous achievement was added and the adjustment term for intelligence was subtracted from the criterion mean of low anxiety groups. The unadjusted and adjusted means have been shown in table LXX.

TABLE-LXX

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE BETWEEN ADJUSTED MEANS OF HIGH AND LOW ANXIETY GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Anxiety Group	Post Achievement Means		
	Unadjusted	Adjusted	'F' Value
High	55.66	53.31	2.18
Low	39.83	39.60	

(B) Branching Style of Programming:

The sums, sum of squares, cross products and means of the criterion and control variables are shown in table LXXI.

TABLE-LXXI

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF CRITERION AND CONTROL VARIABLES OF LOW AND HIGH ANXIETY GROUP TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Anxiety Group	N	Post Achievement			Previous Achievement			Intelligence			Cross Products		
		ΣY	ΣY^2	$\overline{Y}$	ΣX_1	ΣX_1^2	$\overline{X}_1$	ΣX_2	ΣX_2^2	$\overline{X}_2$	$\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
High	29	1202	83863	41.45	842	95313	29.03	1055	86837	36.38	22404	124125	134275
Low	29	1047	71402	36.10	798	90564	27.52	789	26535	22.07	60040	52280	76383
Total	58	2249	155265		1640	185877		1844	113372		82444	176405	210658

TABLE-LXXII

SUM OF SQUARES AND CROSS PRODUCTS IN DEVIATION FOR FOR THE TOTAL AND WITHIN SUBGROUPS OF THE LOW AND HIGH ANXIETY GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Intelligence ΣX_2^2	Cross Products $\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
Total	68058,09	139504.59	54745.52	18851.59	104902.31	158517.31
Within	67643.86	1394[illegible]1.21	53525.59	18734.00	104191.45	158315.52

The value of the table LXXI were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the subgroups. These deviation values are presented in the table LXXII.

The regression equations for total and within the subgroups were then calculated. These have been presented below:

$$Y = 0.89\ X_1 - 0.66\ X_2$$

$$Y = 0.88\ X_1 - 0.66\ X_2$$

where X_1 and X_2 are control variables and Y is criterion variable.

With the help of these regression equations the sum of squares of residuals for total and within the subgroups were worked out. These sum of squares of residuals have been presented in table LXXIII.

TABLE-LXXIII

SUM OF SQUARES OF RESIDUAL FOR TOTAL AND WITHIN THE SUB-GROUPS OF LOW AND HIGH ANXIETY GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Sources of Variation	Degree of Freedom	RESIDUALS Sum of Squares	Mean Square
Total	55	15600.49	
Within	54	15363.42	284.51
Difference	1	237.07	237.07

$$F_{1,54} = \frac{237.07}{284.51} = 0.83$$

After the calculation of sum of squares of residuals 'F' test of significance was made to find whether the difference in post achievement scores of high and low anxiety groups taught through branching style of programming was significant or not. It would be seen from the table LXXIII that the difference between the total and within the subgroups of sum of square of residual is 237.07, which is due to the difference in the achievement of high and low anxiety group when taught through the branching style of programming. The mean of square for this difference is also 237.07 because the difference in degree of freedom between total and within subgroups is 1. The 'F' value comes to be 0.83, which is insignificant. Therefore when the criterion means of the two subgroups are adjusted for previous achievement in the subject an intelligence, the difference between the post achievement through branching style of programming of high and low anxiety group is insignificant.

The means of post achievement scores of high and low anxiety groups taught through the branching programme were adjusted for the effect of control variables of previous achievement and intelligence by the help of within regression equation. The adjustment term of criterion mean for controlling the effect of previous achievement were 0.67 and 0.67 and for controlling the effect of intelligence were 4.76 and 4.69 for the high and low anxiety groups respectively. It would be seen from the table LXXI that the high anxiety group is superior than the low anxiety group on both the control variables. Therefore, the adjustment terms for both the variables were subtracted from the criterion means of the high anxiety group and added to the criterion means of low anxiety group. The unadjusted and adjusted means have been presented in the table LXXIV.

TABLE-LXXIV

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUES BETWEEN HIGH AND LOW ANXIETY GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Anxiety Group	Post Achievement Mean		
	Unadjusted	Adjusted	'F' Value
High	41.45	36.09	
Low	36.10	41.53	0.83

(C) Expository Method.

The sums, sum of squares, cross products and means of the criterion and control variables are shown in the table LXXV.

The values of the table LXXV were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the group. These deviation values are presented in table LXXVI.

TABLE-LXXV

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF THE CRITERION AND CONTROL VARIABLES OF HIGH AND LOW ANXIETY GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Anxiety Group	N	Post Achievement			Previous Achievement			Intelligence			Cross Products		
		ΣY	ΣY^2	$\overline{Y}$	ΣX_1	ΣX_1^2	$\overline{X}_1$	ΣX_2	ΣX_2^2	$\overline{X}_2$	$\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
High	29	1262	64768	43.52	849	29981	29.28	624	15074	21.52	68186	170552	18554
Low	29	1106	47952	38.14	934	33990	32.21	686	19682	23.66	34068	32216	23294
Total	58	2368	112720		1783	63971	23.68	1310	34756	22.56	102254	202768	41848

TABLE-LXXVI

SUM OF SQUARES AND SUM OF CROSS PRODUCTS IN DEVIATION FORM FOR TOTAL AND WITHIN SUBGROUPS OF THE HIGH AND LOW ANXIETY GROUP TAUGHT THROUGH EXPOSITORY METHOD.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Intelligence ΣX_2^2	Cross Products $\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
Total	16040.3	9159.12	5168.07	29458.42	149283.87	1486.5
Within	15620.7	9034.56	5101.85	29590.50	149450.70	1576.8

The regression equations for total and within the subgroups are then calculated from the table LXXVI. These regression equations have been presented below:

For Total, $Y = -1.85\ X_1 + 29.43\ X_2$

For Within $Y = -2.60\ X_1 + 35.70\ X_2$

where X_1 and X_2 are control variables and Y is criterion variable.

With the help of these regression equations, the sum of squares of residuals for total and for within the subgroups were worked out. These sum of squares of residuals have been presented in table LXXVII.

TABLE-LXXVII

RESIDUALS OF MEANS FROM TOTAL AND WITHIN SUBGROUPS OF THE HIGH AND LOW ANXIETY GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Sources of Variation	Degree of Freedom	RESIDUALS Sum of Squares	Mean Square
Total	55	-4318405.4	
Within	54	-5262414.3	-97452.116
Difference	1	-944008.9	944008.9

$$F_{1,54} = \frac{944008.9}{97452.116} = 9.686$$

After the calculation of sum of squares of residuals, test of significance 'F' test was made to find whether the difference in post achievement scores of high and low anxiety groups taught through expository method was significantly or not. It would be seen from the table LXXVII that the difference between the total and within the subgroups of sum of squares of residual is 944008.9 which is due to the difference in the achievement of high and low anxiety groups when taught through expository method. The mean square of this difference is also 944008.9 because the degree of freedom for difference is one. The 'F' value thus turn to be 9.68 which is highly significant beyond 0.05 level for 1 and 54 degrees of freedom. Therefore when the criterion means of the two subgroups are adjusted for previous achievement and intelligence the difference between the post achievement through expository method of high and low anxiety groups is highly significant. Whatever the difference is not due to chance alone.

The means of post achievement scores of high and low anxiety groups taught through the expository method were adjusted for the effect of control variables of previous achievement and intelligence with the help of regression equation for within the group. The adjustment terms of criterion means for controlling the effect of previous achievement were 2.13 and 2.13 and for controlling the effect of intelligence were 31.58 and 33.71 for the high and low anxiety groups respectively. It would be seen from the table LXXV that the high group is inferior than the low anxiety group on both the control variables. Therefore, the adjustment terms for both the control variables were added to the criterion mean of the high anxiety group and subtracted from the criterion mean of the low anxiety group. The unadjusted and adjusted means have been presented in table LXXVIII.

TABLE-LXXVIII

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE BETWEEN ADJUSTED MEANS OF HIGH AND LOW ANXIETY GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Anxiety Group	Post Achievement Means		'F' Value
	Unadjusted	Adjusted	
High	43.52	84.47	9.68
Low	38.14	-4.97	

EXTROVERSION AND TREATMENT MODE.

The technique of analysis of covariance employed in the present study requires that the high and low groups should consists of similar number of cases. The number of students taught through the three treatment modes ranged from 88 to 103. The upper and lower extroversion groups consisted of 29 students each. These groups were made after including such students as would tend to made the groups comparable.

Table LXXIX presents the mean, S.D. and 't' values of low as well as high extroversion groups for different pairs of treatment modes.

TABLE-LXXIX

't' VALUES BETWEEN LOW AND HIGH EXTROVERSION GROUPS FOR DIFFERENT PAIRS OF TREATMENT MODES

Extroversion Group	Statistics	Linear	Branching	Expository	Branching	Expository	Linear
		TREATMENT MODES					
High	Mean	29.69	29.14	29.86	29.14	29.86	29.69
	S.D.	2.47	2.61	2.93	2.61	2.93	2.47
	't'	0.833		0.98		.239	
Low	Mean	16.28	15.38	14.34	15.38	14.34	16.28
	S.D.	3.89	4.76	4.36	4.76	4.36	3.89
	't'	0.84		0.91		1.78	

It would be seen from the table LXXIX that all the 't' values of high as well as low extroversion groups for different pairs of treatment modes are highly insignificant at 0.01 level. The tables value for 28 degree of freedom at 0.01 level is 2.78. Thus it can safely be concluded that the high as well as low extroversion groups taught through linear, branching and expository treatment modes did not differ significantly.

Comparison was also made between the means extroversion scores of high and low groups taught through each of the three treatment modes. The significant difference between the high and low groups is important for reliable prediction of the efficiency of treatment modes for high and low extroversion groups. Table LXXX presents the mean, S.D. and 't' values of high and low extroversion groups for different treatment modes.

TABLE-LXXX

't' VALUES OF THE HIGH AND LOW EXTROVERSION GROUPS FOR DIFFERENT TREATMENT MODES.

Statistics	Linear		Branching		Expository	
	Low Extro-version	High Extro-version	Low Extro-version	High Extro-version	Low Extro-version	High Extro-version
Mean	16.28	29.69	15.38	29.14	14.34	29.86
S.D.	3.89	2.47	4.76	2.61	4.36	2.93
't'	15.776*		14.843*		16.00*	

*Significant at 0.05 level.

It would be seen from the table LXXX that the mean values of high and low extroversion groups for different treatment modes are highly significant. Thus it can be very safely be concluded that the high and low extroversion groups for each treatment mode were far separated from each other.

Analysis of Covariance.

After establishing that the high as well as the low extroversion groups taught through three treatment modes were comparable, the next task before the investigator was to find out to what extent the post achievement scores of the high and low extroversion groups differed when taught through the three treatment modes viz. linear, branching and expository. As pointed out earlier the influence of previous achievement and intelligence was sought to be controlled. This was done by employing the analysis of covariance. This have been explained in the following pages.

(A) Linear Style of Programming.

The sums, sum of squares, cross products and means of the criterion and control variables are shown in the table LXXXI.

The values of the above table were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the groups. The deviation values are presented in table LXXXII.

TABLE-LXXXI

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF CRITERION AND CONTROL VARIABLES OF THE HIGH AND LOW EXTROVERSION GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Extroversion Group	N	Post Achievement ΣY	ΣY^2	$\overline{Y}$	Previous Achievement ΣX_1	ΣX_1^2	$\overline{X}_1$	Intelligence ΣX_2	ΣX_2^2	$\overline{X}_2$	Cross Products ΣX_1Y	ΣX_2Y	ΣX_1X_2
High	29	1317	68689	45.41	954	34920	32.90	764	25546	26.34	85154	87929	113890
Low	29	1239	68303	42.72	894	37042	30.83	702	21316	24.21	135850	31153	53684
Total	58	2556	136992		1848	71962		1466	46862		221004	119082	167574

TABLE-LXXXII

SUM OF SQUARES AND SUM OF CROSS PRODUCTS OF CRITERION AND CONTROL VARIABLES IN DEVIATION FORM OF LOW AND HIGH EXTROVERSION GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Intelligence ΣX_2^2	Cross Products ΣX_1Y	ΣX_2Y	ΣX_1X_2
Total	24351.72	13080.90	9807.59	139564.55	54476.90	120864.21
Within	24246.82	13018.83	9741.31	139483.86	14393.52	120800.06

The regression equations for total and within the subgroups were then calculated. These regression equations have been presented below:

For Total, $Y = 0.36 X_1 + 1.12 X_2$

For Within $Y = 0.35 X_1 + 1.19 X_2$

where X_1 and X_2 are control variables and Y is criterion variable.

With the help of these regression equations, the sum of squares of residuals for total and for within the subgroups were worked out. The sum of residuals have been presented in table LXXXIII.

TABLE-LXXXIII

RESIDUALS OF MEANS OF TOTAL AND WITHIN SUBGROUPS OF THE HIGH AND LOW EXTROVERSION GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Sources of Variation	Degree of Freedom	RESIDUAL Sum of Squares	Mean Square
Total	55	87241.65	
Within	54	89300.20	1653.7
Difference	1	2058.55	2058.55

$$F_{1,54} = \frac{2058.55}{1653.7} = 1.24$$

After the calculation of sum of squares of residuals, 'F' test of significance was made to find whether the difference in post achievement score of the high and low extroversion groups taught through linear style of programming was significant or not. It would be seen from the table LXXXIII that the difference between the total and within the subgroups of sum of squares of residuals is 2058.55 which is due to the difference in achievement of high and low extroversion groups when taught through linear style of programming. The mean of squares for this difference is also 2058.55 because the difference in degree of freedom between total and within the subgroups is one. The 'F' value is calculated dividing this difference mean square by the mean square value of the within the subgroup which is 1653.7. The 'F' value, 1.24 is significant. Therefore, when the criterion means of two subgroups are adjusted for individual difference in previous achievement and intelligence, the difference between the post achievement through linear style of programming of high and low extroversion group is insignificant.

The means of post achievement scores of high and low extroversion groups taught through the linear style of programming were adjusted for the effect of the control variables of previous achievement and intelligence by the help of within regression equations. The adjustment terms of criterion means for controlling the effect of previous achievement were .36 and 0.36 and that of for controlling the effect of intelligence were 1.26 and 1.26 for the high and low extroversion group respectively. It would be seen from the table LXXXI that the high extroversion group is superior than the low one on both the control variables. Therefore, the adjustment terms for both the control variables were subtracted from the criterion means of the high extroversion and added to the criterion mean of the low extroversion group. The unadjusted and adjusted means have been shown in table LXXXIV.

TABLE-LXXXIV

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE OF LOW AND HIGH EXTROVERSION GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING.

Extroversion Group	Post Achievement Means		'F' Value
	Unadjusted	Adjusted	
High	45.41	43.79	1.24
Low	42.72	44.34	

(B) Branching Style of Programming:

The sums, sum of squares, cross products and means of the criterion and control variables are shown in table LXXXV.

The values of table LXXXV were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the subgroups. These deviation values are presented in the table LXXXVI.

TABLE-LXXXV

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF THE CRITERION AND CONTROL VARIABLES OF THE LOW AND HIGH EXTROVERSION GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Extroversion Group	N	Post Achievement ΣY	ΣY^2	$\bar{Y}$	Previous Achievement ΣX_1	ΣX_1^2	$\bar{X}_1$	Intelligence ΣX_2	ΣX_2^2	$\bar{X}_2$	Cross Products $\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
High	29	1062	57074	36.62	805	28159	27.76	884	32228	30.48	59494	179070	199105
Low	29	1336	103873	46.07	603	18691	20.79	941	41191	32.45	42492	250307	20634
Total	58	2398	160947		1408	46850		1992	73419		101986	429377	219739

TABLE-LXXXVI

SUM OF SQUARES AND CROSS PRODUCTS OF CRITERION AND CONTROL VARIABLES IN DEVIATION FORM OF THE LOW AND HIGH EXTROVERSION GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Intelligence ΣX_2^2	Cross Products $\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
Total	61802.10	12669.59	5004.10	43772.48	347018.10	171381.48
Within	60507.69	11966.07	15938.41	44726.75	353653.41	175634.07

The regression equations for total and within the subgroups were then calculated. These have been presented below:

For Total, $Y = 2.02\, X_1 + 0.17\, X_2$

For Within $Y = 2.0\, X_1 + 0.12\, X_2$

where X_1 and X_2 are control variables and Y is criterion variable.

With the help of these regression equations the sum of squares of residuals for total and within the subgroups were worked out. These sum of residuals have been presented in table LXXXVII.

TABLE-LXXXVII

RESIDUALS OF [illegible] OF TOTAL AND WITHIN SUBGROUPS OF THE HIGH AND LOW EXTROVERSION GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Sources of Variation	Degree of Freedom	RESIDUALS Sum of Squares	Mean Square
Total	55	85611.37	
Within	54	71384.22	1321.93
Difference	1	14227.15	14227.15

$$F_{1,54} = \frac{14227.15}{1321.93} = 10.76$$

After the calculation of sum of squares of residuals, 'F' test of significance was made to find whether the difference in post achievement scores of high and low extroversion groups taught through branching style of programming was significant or not. It would be seen from the table LXXXVII that the difference between the total and within the subgroups of sum of square of residual is 14227.15, which is due to the difference in the achievement of high and low extroversion group when taught through the branching style of programming. The means of square for this difference is also 14227.15 because the difference in degree of freedom between total and within the subgroup is 1. The F value is calculated by dividing this difference mean square by the mean square value of the within subgroup which is 1321.93. The F value comes to be 10.76, which is significant. Therefore, when the criterion achievement means of the two groups are adjusted for previous achievement and intelligence, the difference between post achievement through branching style of programming of high and low groups is significant and the difference is only due to chance alone.

The means of post test achievement scores of high and low extroversion groups taught through the branching programme were adjusted for the effect of control variables of previous achievement and intelligence with the help of within regression equation. The adjustment terms of criterion means for controlling the effect of previous achievement were 6.96 and 6.96

and for controlling the effect of intelligence were 0.12 and 0.12 for the high and low extroversion groups respectively. It would be seen from the table LXXXV that the high extroversion group is superior than the low extroversion group on previous achievement and inferior on the intelligence. Therefore, the adjustment term for the previous achievement was subtracted and adjustment term for intelligence was added to the criterion mean of the high extroversion group and adjustment term for the previous achievement was added and adjustment term for intelligence was subtracted to the criterion mean of the low extroversion group. The unadjusted and adjusted means have been presented in table LXXXVIII.

TABLE-LXXXVIII.

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE OF LOW AND HIGH EXTROVERSION GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING.

Extroversion Group	Post Achievement Mean		'F' Value
	Unadjusted	Adjusted	
High	36.62	29.79	10.76
Low	46.07	52.91	

(C) Expository Method:

The sums, sum of squares, cross products and means of the criterion and control variables are shown in table LXXXIX.

TABLE-LXXXIX

SUMS, SUM OF SQUARES, CROSS PRODUCTS AND MEANS OF THE CRITERION AND CONTROL VARIABLES OF THE LOW AND HIGH EXTROVERSION GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Extroversion Group	N	Post Achievement ΣY	ΣY^2	$\overline{Y}$	Previous Achievement ΣX_1	ΣX_1^2	$\overline{X}_1$	Intelligence ΣX_2	ΣX_2^2	$\overline{X}_2$	Cross Products $\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
High	29	1179	56525	40.66	864	33812	29.79	654	16978	20.25	69864	27546	18739
Low	29	1070	47364	36.90	803	31243	27.69	525	11173	18.10	58428	19862	14636
Total	58	2249	103889		1667	65055		1179	28151		128392	47408	33375

TABLE-XC

SUM OF SQUARES AND CROSS PRODUCTS OF CRITERION AND CONTROL VARIABLES IN DEVIATION FORM OF THE LOW AND HIGH EXTROVERSION GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Sources of Variation	Post Achievement ΣY^2	Previous Achievement ΣX_1^2	Intelligence ΣX_2^2	Cross Products $\Sigma X_1 Y$	$\Sigma X_2 Y$	$\Sigma X_1 X_2$
Total	16682.09	17143.12	4184.78	63752.64	1691.26	511.09
Within	16477.24	17078.97	3897.86	63638.00	1448.83	646.76

The values of the table LXXXIX were then used to compute the sum of squares and sum of cross products in deviation form for the total and for within the subgroups. These deviation value are presented in the table XC.

The regression equations for total and within the subgroups are then calculated from the table XC. These regression equations have been presented below:

For Total, $Y = 3.73\ X_1 - 0.05\ X_2$

For Within $Y = 3.72\ X_1 - 0.25\ X_2$

where X_1 and X_2 are control variables and Y is criterion variable.

With the help of these regression equations, the sum of square of residuals for total and within the subgroups were worked out. These sum of squares of residuals have been presented in table XCI.

TABLE-XCI

RESIDUALS OF MEANS OF TOTAL AND WITHIN SUBGROUPS OF THE HIGH AND LOW EXTROVERSION GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Sources of Variation	Degree of Freedom	RESIDUALS Sum of Squares	Mean Square
Total	55	221030.69	
Within	54	219893.92	4072.11
Difference	1	1136.77	1136.77

$$F_{1,54} = \frac{1136.77}{4072.11} = 0.281$$

After the calculation of sum of squares of residuals a test of significance, i.e. 'F' test was made to find whether the difference in post achievement scores of high and low extroversion groups taught through expository method was significant or not. It would be seen from the table XCI that the difference between the total and within the subgroups of sum of squares of residual is 1136.77 which is due to the difference in achievement of high and low extroversion groups when taught through expository method. The mean square of this difference is also 1136.77 because the degree of freedom for difference is 1. he F value is calculated by dividing this difference mean square by the mean square value of the within subgroup which is 4072.11. The F value comes out to be 0.281 which is insignificant at 0.05 level for 1 and 54 degree of freedom. herefore, when the criterion means of the two subgroups are adjusted for previous achievement through expository method of high and low extroversion groups is insignificant.

The means of post achievement scores of high and low extroversion groups taught through expository method were adjusted for the effect of control variables of previous achievements and intelligence with the help of regression equation for within the group. The adjustment terms of criterion means for controlling the effect of previous achievement were 3.91 and 3.91 and for controlling the effect of individual intelligence were 0.518 and 0.518 for the high and low extroversion groups respectively. It would be seen from the table LXXXIX that the high extroversion group is superior than the low one on both of the control variables. Therefore, the adjustment terms for both the control variables were subtracted from the criterion mean of the high extroversion group and added to the criterion mean of low extroversion group. The adjusted and unadjusted means have been presented in table XCII.

TABLE-XCII

UNADJUSTED, ADJUSTED MEANS AND 'F' VALUE OF HIGH AND LOW EXTROVERSION GROUPS TAUGHT THROUGH EXPOSITORY METHOD.

Extroversion Group	Degree of Freedom	Post Achievement Means		F Value
		Unadjusted	Adjusted	
High	55	40.66	36.23	0.281
Low	54	36.90	41.33	

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CHAPTER - VIII

INTERPRETATION AND DISCUSSION OF THE DATA

Intelligence-Treatment Interaction

Creativity-Treatment Interaction

Anxiety-Treatment Interaction

Extroversion-Treatment Interaction

References

Three groups of students were separately taught by the three treatment modes namely, linear and branching style of programming and expository method. Students of each of these three groups were then divided into high and low ability groups on the basis of the different personality and mental ability variables under investigation. Only such students were included in these groups as would make the high as well as low groups on any personological variable comparable for the three treatment modes. Means of post test achievement scores for the high and low groups taught by different treatment modes were computed. These means were then adjusted for the two control variables namely previous achievement and intelligence* by employing analysis of covariance. This analysis has been presented in the preceding chapter.

The interaction between a personological variable and different treatment modes was studied by comparing the post test achievement scores obtained by students high and low on each personological variable for different treatment modes. The 'F' value obtained in the analysis of covariance were employed for comparing these high and low groups. It was considered that if these values are significant even for one of the treatment modes under investigation, the inter-

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*The post test means for high and low intelligence groups were adjusted for previous achievement only. Other variables were adjusted for intelligence as well as previous achievement.

action could be disordinal and significant. If the difference between the high and low groups on any personological variable for two treatment modes was insignificant, the treatment lines could cross but the angle between them would be so small as not to warrant any significant interaction.

Adjusted post achievement scores obtained by students high as well as low on any personological variable for different pairs of treatment modes were then compared to secure evidence for assigning treatment modes for optimal achievement. The interaction between different personological variables and treatment modes has been presented in the following pages.

INTELLIGENCE-TREATMENT INTERACTION:

Table XCIII presents the adjusted, post test means and 'F' values for difference between post test scores of high and low intelligence groups taught through the three treatment modes.

TABLE-XCIII

ADJUSTED POST ACHIEVEMENT MEANS OF HIGH AND LOW INTELLIGENCE GROUPS AND THEIR 'F' VALUES FOR DIFFERENT TREATMENT MODES.

Intelligence	TREATMENT MODES					
	Linear Style		Branching Style		Expository Method	
	Mean	F Value	Mean	F Value	Mean	F Value
High	48.34	24.51*	55.92	124.3*	41.18	0.25
Low	43.51		33.52		38.06	

* Significant at .01 level

The 'F' values for the linear style of programming is 24.51 which shows that there is significant interaction between intelligence and achievement through linear style of programming. The high intelligence group achieved higher as compared to the low intelligence group. This finds support in the studies by Bhushan (1973), Gangwal (1978), O'Reilly (1969), Kapadia (1972), Agarwal (1978), Davis and Leith (1967), Evans (1965), Hatch and Flint (1962), Limbert and others (1962) and Shay (1961), but contradicts with those of Stolurow (1964), Glaser and Reynolds (1962) and Porter (1961) who did not obtained any significant interaction.

The 'F' value for branching style of programming is 124.3, which is also highly significant. The high intelligence group achieved significantly higher through branching style than the low intelligence group, which shows significant interaction between intelligence and achievement through branching style of programming. This finding is supported by the studies of Agarwal (1978), Kapadia (1972), Hartley (1965) and Silberman (1961).

The expository method has yielded a 'F' value of 0.25 only which is highly insignificant showing that the achievement of high and low intelligence groups when taught through expository method is not statistically different. This finding seems to be a little unbecoming, for it has long been considered rather axiomatic that high intelligence students

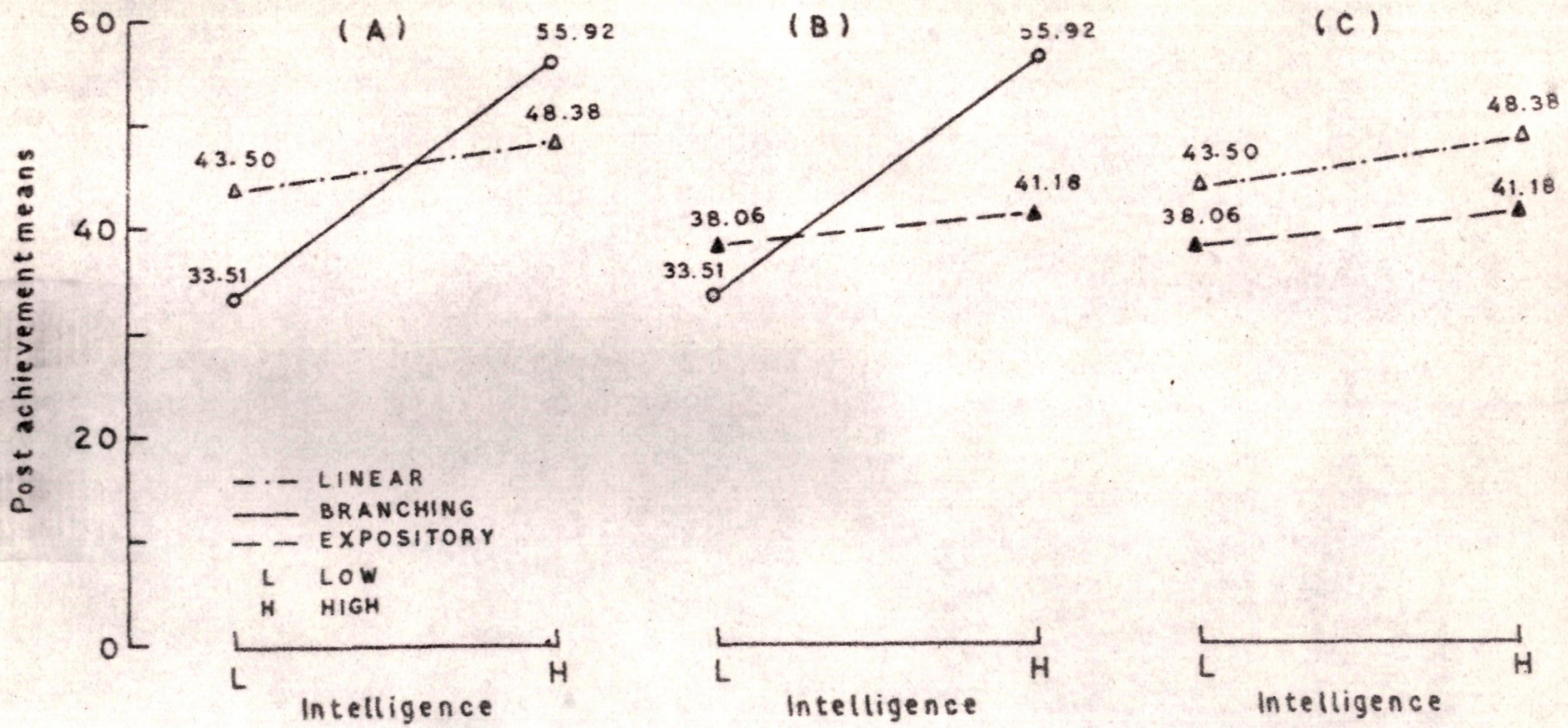

FIG. 9 INTELLIGENCE - TREATMENT INTERACTION FOR DIFFERENT PAIRS OF TREATMENT MODES

should achieve higher than their lesser endowed counterparts when taught by any method let alone by conventional method of teaching. The expository method employed in the present study can well be considered as conventional one. This could have been due to the nature of the subject matter and subjects under study. However, high intelligence group achieved slightly higher by this method in comparison to low intelligence ones.

The above analyses do not provide any evidence for assignment of treatment modes to the two groups of the students. Evidence for this purpose was obtained by comparing the achievement through any of the two treatment modes for either of the intelligence groups. For this purpose, graph of adjusted post test means of students high and low on intelligence for different pairs of treatment modes were drawn and 't' values for difference in post test means were computed for each pair. The interaction between these different pairs were studied according to suggestions of Lubin (1967), Bracht and Glass (1968) with some modifications by the present investigator. The rationale of these modifications have been presented in Chapter III.

A. Comparison between Linear and Branching Styles of Programming

Table XCIV and Fig.3(a) presents the comparison of post test means for high and low intelligence groups of students taught through linear and branching style of programming.

TABLE-XCIV

't' VALUES FOR DIFFERENCE BETWEEN POST TEST MEANS OF HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH LINEAR AND BRANCHING STYLES OF PROGRAMMING.

Treatment modes	Low Intelligence		High Intelligence	
	Adjusted mean	't' Value	Adjusted mean	't' Value
Linear	43.51	1.91*	48.38	1.50
Branching	33.53		55.92	

*Significant at 0.1 level.

It will be seen from the Fig.9(a) that the two treatment lines cross which shows that while the linear style of programming is better for the low intelligence students the branching style has proved to be more effective for the high intelligence group. The interaction is disordinal according to the standard suggested by Lubin (1967). It will also be seen from the table that the 't' values between the post test means of high intelligence students taught by the two modes is 1.50, which is insignificant at 0.05 level for 28 degree of freedom.

The 't' values between the post test means for the low intelligence group taught through the two treatment modes is 1.91, which is also insignificant at 0.05 level. Thus the interaction though disordinal according to Lubin (1967) is not so according to Bracht and Glass (1968) and the present investigator. The low intelligence students may be taught more profitably by the linear style of programming and the high intelligence students by the branching style of programming but the evidence is not very trustworthy as the 't' values for both the groups are insignificant at 0.05 level. However, since 't' value for the low intelligence students is significant at 0.1 level, such students may be taught more profitably by the linear style of programming. These results find sufficient support in previous researches. The review of previous researches presented in Chapter IV shows that more than forty five research workers attempted to study the interaction between intelligence and different treatment modes. Of these only two (Singh, 1977; and Hartley, 1965) attempted to study the interaction between linear and branching style of programming and intelligence of the linear learner. The results of these investigations corroborate the findings of the present study to a fair extent.

B. <u>Comparison between Branching Style of Programming and Expository Method</u>:

Table XCV and Fig.9(b) presents the comparison of post test means for high and low intelligence groups of students taught through the branching style of programming and expository method of teaching.

It will be seen from the Fig.9(b) that the two treatment lines cross each other. The interaction is disordinal according to the standard suggested by Lubin (1967). The expository method has come out to be only slightly better for

TABLE-XCV

't' VALUES FOR THE DIFFERENCE BETWEEN POST TEST MEANS OF HIGH AND LOW INTELLIGENCE STUDENTS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING AND EXPOSITORY METHOD.

Treatment odes	Low Intelligence Adjusted means	't' Value	High Intelligence Adjusted mean	't' Value
Branching	33.53	1.08	55.92	2.77*
Expository	38.06		41.18	

*Significant at 0.01 level

low intelligence students. The 't' value being 1.08, which is significant at 0.01 level for 28 degree of freedom. Thus the interaction which is disordinal according to Lubin (1967) is not so according to Bracht and Glass (1968) because the difference at both the levels of intelligence are not significant. However, according to the present investigator, the interaction is disordinal since the 't' values at one of the levels (high intelligence level) is significant beyond 0.05 level. The high group may be assigned the branching style of programming for better achievement but the evidence is not equally sound for low intelligence students because the 't' value of this group is not significant, each at 0.1 level. Singh (1977) also found that branching programme is superior than conventional for high intelligence students.

C. Comparison between Linear Style of Programming and Expository Method:

Table XCVI and Fig.9(c) presents the comparison of post test means of high and low intelligence students taught through linear and expository treatment modes.

TABLE-XCVI

't' VALUES FOR THE DIFFERENCE BETWEEN POST TEST MEANS OF HIGH AND LOW INTELLIGENCE STUDENTS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING AND EXPOSITORY METHOD.

Treatment modes	Low Intelligence Adjustment means	't' Value	High Intelligence Adjustment means	't' Value
Linear	43.51	-	48.38	-
Expository	38.06		41.18	

It will be seen from the Fig.9(c) that the two treatment lines do not cross which shows that the interaction is ordinal according to Lubin (1967), Bracht and Glass (1968) and the present investigator. Both high as well as low intelligence students have secured better through linear style of programming as compared to expository method. Thus, there is no evidence of interaction i.e. alternative treatments can not be assigned on the basis of this evidence. Thus there was hardly any need to calculate 't' values for the difference between post test means. Sheehan and Hambleton

(1977), Sodhi (1977), Govinda (1976), Bhushan and Sharma (1975), Patel (1975), Pandaya (1974), Nagar (1971), Ripple and others (1969), Jamieson and others (1969), Desai (1966), and Shah (1964) also could not obtain any significant interaction between intelligence and linear style of programming and expository method. Reed and Hayman (1962) however, obtain significant interaction between these variables which revealed that high intelligence students did better on linear style of programming while the low intelligence students did better by conventional or expository method. The subject matter of their study was 'English-2600' text. Tallmadge and Shearer (1969) have shown that subject matter also interacts with treatment modes and personological variables. Therefore, the disparity between the findings of Reed and Hayman (1962) and the present investigator could possible be due to the difference in the subject matter employed in these investigations.

CREATIVITY-TREATMENT INTERACTION:

Table XCVII presents the adjusted, means and 'F' values for the difference between post test means of high and low creativity groups taught through linear and branching styles of programming and expository method.

The table XCVII shows that the 'F' value for linear style of programming is 5.8 which shows the significant interaction between creativity and achievement through linear style of programming. The low creative students achieved

TABLE-XCVII

ADJUSTED POST ACHIEVEMENT MEANS OF HIGH AND LOW CREATIVITY GROUPS AND THEIR 'F' VALUES FOR DIFFERENT TREATMENT MODES.

Creativity	TREATMENT MODES					
	Linear Style		Branching Style		Expository Method	
	Mean	'F'Value	Mean	'F'Value	Mean	'F'Value
High	37.80	5.80*	44.11	0.35	17.33	4.34*
Low	50.17		39.38		66.08	

*Significant at 0.05 level.

significantly higher in comparison to high creative ones. This finds support in the studies of Doty and Doty (1964), Gotkin and Massa (1963) and Stolurow (1962). Welcome (1973) also obtained a significant interaction between creativity and achievement through linear style of programming, but high creative students achieved higher as compared to the low creativity students in his study. O'Reilly (1969), Ripple and others (1969) and Agarwal (1978) did not find any significant difference between low and high creativity groups.

The 'F' value for branching style of programming is 0.35, which is insignificant showing that the post achievement scores of high and low creatives taught through branching style do not differ significantly. Agarwal (1978)corroborates the findings of the present study.

The expository method has yielded a 'F' value of 4.34 which is significant at 0.05 level for 28 degree of freedom. This shows that a significant interaction exists between creativity and achievement through expository method.

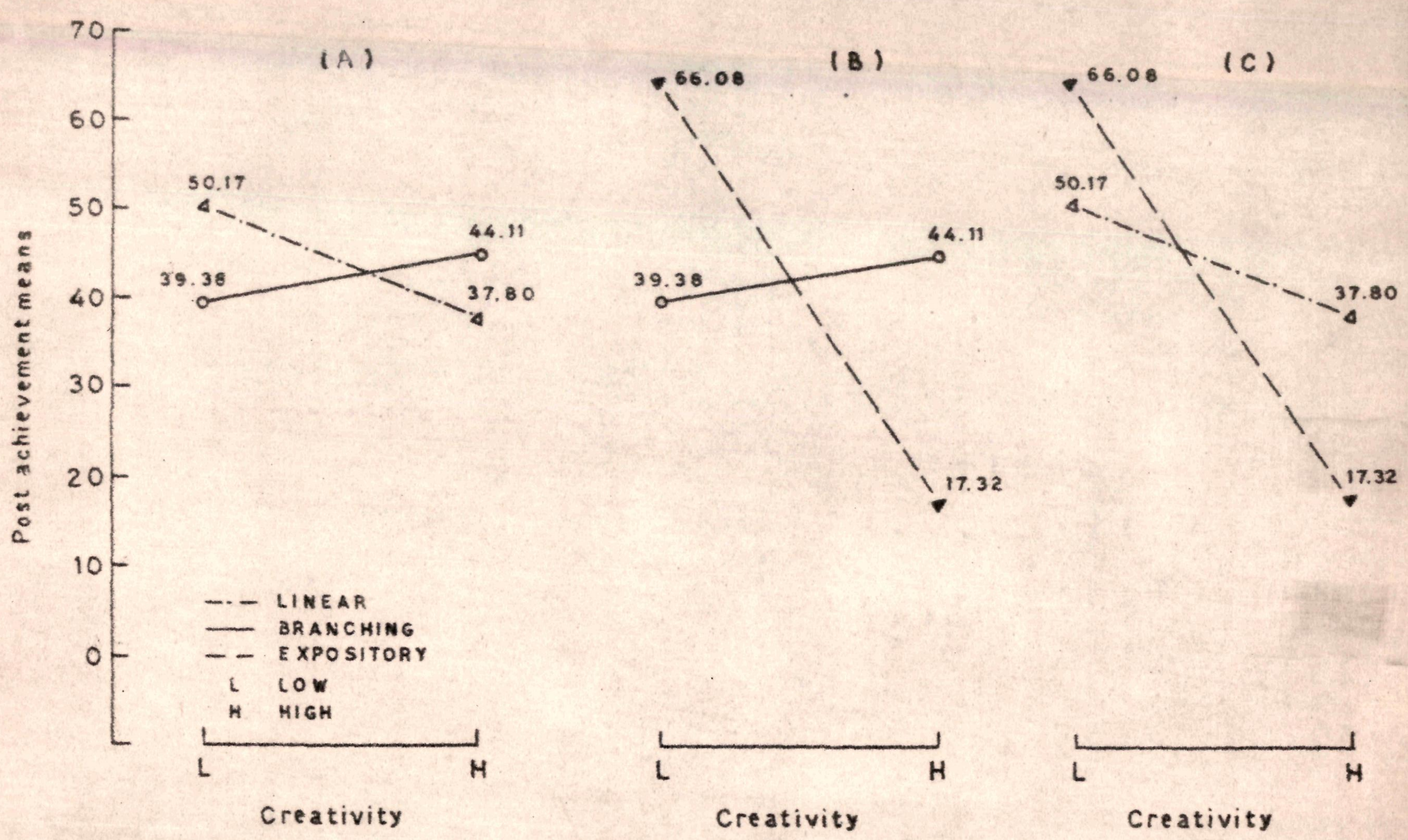

FIG. 10 CREATIVITY - TREATMENT INTERACTION FOR DIFFERENT PAIRS OF TREATMENT MODES

The achievement of low creatives is also higher than that of high creatives when taught through this treatment mode. This is probably due to the fact that expository method has a authoritiar approach, which is known to dampen creativity.

As in the case of intelligence, the above analyses do not provide any evidence for assignment of alternative treatment modes to student of differing levels of creativity. Evidence for this purpose was obtained by comparing the achievement through any of two treatment modes for either of the creativity groups. For this purpose, the graph of adjusted post test means of student high and low on creativity for different pairs of treatment modes were drawn and 't' values for difference in post test means were computed for each pair. The interaction between the different pairs were studied according to the established criteria.

A. Comparison between Linear and Branching Styles of Programming:

Table XVIII and Fig.10(A) presents the comparison of post achievement test means for high and low creative groups of students taught through linear and branching styles of programming.

It will be seen from the Fig.10(A) that the two treatment lines cross each other which shows that while the linear styles is better for low creative students, the branching style has proved to be better for high creative

TABLE-XXVIII

't' VALUES FOR DIFFERENCE IN POST TEST MEANS OF HIGH AND LOW CREATIVITY GROUPS TAUGHT THROUGH LINEAR AND BRANCHING STYLE OF PROGRAMMING.

Treatment modes	Low Creative		High Creative	
	Adjusted mean	't' Value	Adjusted mean	't' Value
Linear	50.17	1.64*	37.80	0.94
Branching	39.38		44.11	

*Significant at 0.1 level

students. The interaction is, thus, disordinal according to the standard of Lubin (1967). It will also be seen from the above table that the branching style is superior for high creative students and linear style is superior for low creative students. The 't' value between the post test means of high creative students taught through the two modes i.e. linear and branching style is 0.94, which is insignificant. The 't' value between the post test means of low creative students taught through the two modes is 1.64 which is also insignificant at 0.05 level for 28 degree of freedom. Thus the interaction through disordinal according to standard suggested by Lubin (1967) is not so according to that suggested by Bracht and Glass (1968) and the present investigator. The low creatives may be taught by linear style of programming. But the evidence is not very sound as the 't' value is not significant at 0.05 level in both the cases. However, since the 't' value of low creative taught by two modes is significant at 0.1 level, such students may more profitably be taught by linear style of programming. This is in agreement with Agarwal (1978), who obtained a very low coefficient of correlation between creativity and linear and branching styles of programming i.e. +0.116 and +0.24 respectively.

(B) <u>Comparison between Branching Style of Programming and Expository Method:</u>

Table IC and Fig.10(b) presents the comparison of post achievement means for high and low creatives taught through branching expository method.

TABLE-IC

't' VALUES FOR DIFFERENCE IN POST TEST MEANS OF HIGH AND LOW CREATIVITY GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING AND EXPOSITORY METHOD.

Treatment modes	Low Creative		High Creative	
	Adjusted mean	't' Value	Adjusted mean	't' Value
Branching	39.38	3.63*	44.11	4.05*
Expository	66.08		17.33	

*Significant at 0.01 level.

It will be seen from the Fig.10(b) that the two treatment lines cross each other which shows that while the expository method is better for the low creatives, the branching style has proved to be better for the high creatives. The interaction is, thus, disordinal according to the criteria of Lubin (1967). Table I also shows that the expository method is better in term of post achievement for lo low creatives and the branching style is better for high creatives. The 't' value between the post achievement means of high and low creatives taught through the two modes are 40.5 and 3.63 respectively, which are very significant beyond 0.05 level for 28 degree of freedom. Thus the interaction which is disordinal according to the criteria of Lubin (1967) is also disordinal according to Bracht and Glass (1968) and the present investigator. The low creatives may be assigned expository method and the high creatives may be assigned branching style for maximal achievement.

(C) Comparison between Linear Style of Programming and Expository Method:

Table C and Fig.10(c) presents the comparison of post achievement means for high and low creative students taught through linear style of programming and expository method.

It will be seen from the Fig.10(c) that the two treatment lines cross each other which reveals that while the expository method is better than the linear style of programming for low creatives, the linear style has proved

TABLE-C

'T' VALUES FOR DIFFERENCE IN POST TEST MEANS OF HIGH AND LOW CREATIVITY GROUPS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING AND EXPOSITORY METHOD.

Treatment Modes	Low Creatives Adjusted mean	Low Creatives 't' Value	High Creatives Adjusted mean	High Creatives 't' Value
linear	50.17	1.95*	37.80	3.74**
Expository	66.08		17.33	

*Significant at 0.1 level
**Significant at 0.01 level.

to be more effective in comparison to expository method for the high creative students. The interaction is thus disordinal according to standard put forth by Lubin (1967). It will also be seen from the table that the 't' value between the post test means of low creative students taught by the two modes is 1.95 which is insignificant at 0.05 level for 28 degree of freedom. The 't' value for high creative students is 3.74 which is significant at 0.01 level for 28 degree of freedom. Thus the interaction which is disordinal as per Lubin (1967) is not so according to that suggested by Bracht and Glass (1968). According to present investigator hthe interaction is disordinal since the 't' value is significant beyond 0.05 level at least at one of the end i.e. at high creativity end.

he low creatives may be taught more profitably by the expository method and high creatives may be assigned linear style of programming be better achievement. But the evidence is not very sound for low creatives as the 't' value is not significant at 0.05 level for 28 degree of freedom. However, since the 't' value for low creatives is significant at 0.1 level, such students may be taught through expository method for better achievement. he above findings are supported by elcome (1973) who reported that high creative students achieved well under individualized learning situation which the linear style of programming provides. But the study contradicts with that of ipple et al. (1969) and Crocker et al.(1976) who did not obtained any significa t interaction between creativity and the two treatment modes employed.

ANXIETY-TREAT ENT INTERACTIO .

able I presents an adjusted means and 'F' values for the difference between post test means of high and low anxiety groups taught through the three treatment modes.

TABLE-I

ADJUSTED POST ACHIEVEMENT MEANS OF HIGH AND LOW ANXIETY GROUPS AND THEIR 'F' VALUES FOR DIFFERENT TREATMENT MODES

Anxiety	TREATMENT MODES					
	Linear tyle		ranching -tyle		xpository ethod	
	Mean	'F' Value	Mean	'F' Value	Mean	'F' Value
High	53.31	5.90*	36.09	0.83	84.87	).68*
Low	39.60		41.53		-4.97	

*Significant at 0.05 level.

The table CI shows that the 'F' value for the linear style of programming is 5.90 which shows that there is significant interaction between anxiety and achievement through linear style of programming. The high anxiety group achieved higher in comparison to low anxiety group. This finds support in the study by Light and Jasserenth (1966),, Gangopadhyay (1971) and Davis and Leith (1967), however, did not obtained any significant interaction. Kapadia (1972) and O'Reilly and Ripple (1966) also obtained a significant interaction between anxiety and achievement through linear style of programming, but low anxiety group achieved higher as compared to the high anxiety group in their studies. However, the conclusions of the present study i.e. the high anxiety students achieve better as compared to low anxiety students through linear style of programming find support in Tobias (1973) who writes:

> The reduced difficulty of programme, their tighter organisation, and reduction of students uncertainity while studying a programme would suggest that these instructional modes ought to be especially beneficial for the performance of high anxiety individuals.

The 'F' value for branching style of programming is 0.83 which is insignificant showing that the achievement of high and low anxiety students when taught through branching style of programming is not statistically different. The finding of this study is supported by the studies of Flynn and Morgan (1966) and Kapadia (1972).

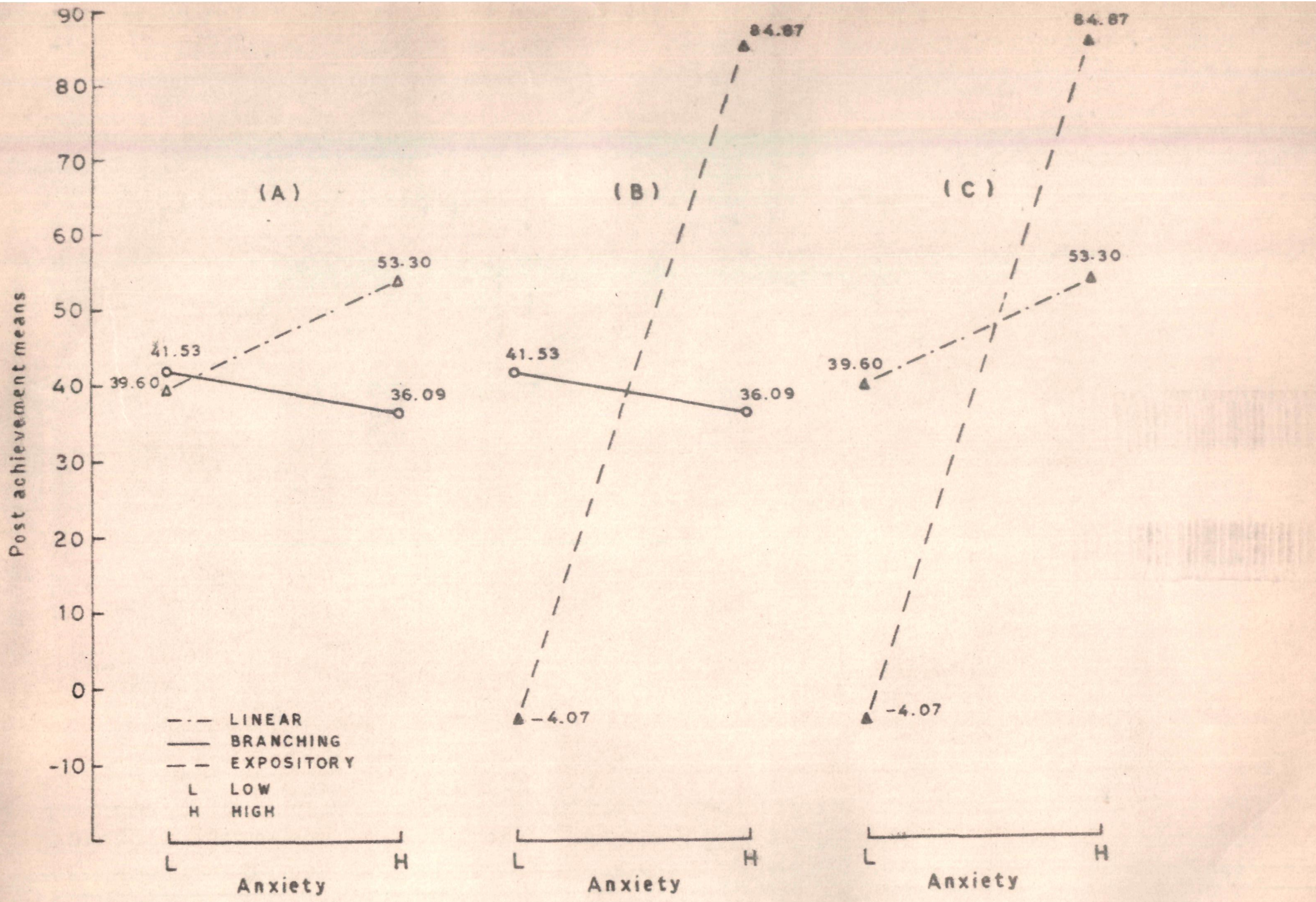

FIG. 11 ANXIETY-TREATMENT INTERACTION FOR DIFFERENT PAIRS OF TREATMENT MODES

The expository method has yielded a 'F' value of 9.68, which is highly significant beyond 0.05 level for 28 degree of freedom. The high anxiety students achieved higher than the low anxiety students when taught through expository method. This finding is in agreement with most of the related studies (Sarason, 1957; Spielberger, 1962; Malnig, 1964).

As in the case of mental ability variables the above analyses do not provide any evidence for assignment of treatment modes to the two groups of the students. Evidence for this purpose was obtained by comparing the achievement through any of the two treatment modes for either of the anxiety groups. For this purpose, graph of adjusted post test means of students high and low on anxiety, for different pairs of treatment modes were drawn and 't' values for difference in post test means were computed for each pair. The interaction between these different pairs were studied according to suggestions of Lubin (1967), Brecht and Glass (1968) with some modifications by the present investigator. The rationale of these modifications have been presented in Chapter III.

(A) <u>Comparison between Linear and Branching Styles of Programming</u>:

Table VII and Fig.11(a) presents the comparison of post test means for high and low anxiety groups of students taught through linear and branching styles of programming.

TABLE-CII

't' VALUES FOR DIFFERENCE IN POST TEST MEANS OF HIGH AND LOW ANXIETY GROUPS TAUGHT THROUGH LINEAR AND BRANCHING STYLES OF PROGRAMMING.

Treatment modes	Low Anxiety Adjusted mean	Low Anxiety 't' Value	High Anxiety Adjusted mean	High Anxiety 't' Value
Linear	39.60	0.27	53.3	2.532*
Branching	41.53		36.09	

*Significant at 0.05 level

It will be seen from the Fig.11(a) that the two treatment lines cross which shows that while the linear style of programming is better for high anxiety group, the branching style has proved to be more effective for the low anxiety group. The interaction is disordinal according to the standard suggested by Lubin (1967). It will be seen from the table that the 't' value between the post test means of high anxiety students taught by the two modes is 2.53, which is significant at 0. 5 level for 28 degree of freedom. The 't' value between the post test means for the low anxiety students taught by the two modes is 0.27, which is insignificant. Thus the interaction though disordinal according to Lubin (1967) is not so according to Bracht and Glass (1968) as the difference at both the levels of anxiety are not significant. However, according to the present investigator, the interaction is disordinal since the 't' value at one of the anxiety levels is significant at 0.05 level. The high anxiety group may be assigned the linear style for better achievement but the evidence is not equally sound for low anxiety group, because the 't' value of this group is not significant even at 0.1 level. This find support from Leith (1966) who writes, "...anxious pupil will do well with linear programmes... the less persistent (less anxious) might fair better with branching programme".

(B) <u>Comparison between Branching Style of Programming and Expository Method</u>.

Table CIII and Fig.11(b) presents the comparison of post achievement means for high and low anxiety students taught through branching style of programming and expository method.

<u>TABLE-CIII</u>

't' VALUES FOR DIFFERENCE IN POST TEST MEANS OF HIGH AND LOW ANXIETY GROUPS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING AND EXPOSITORY METHOD.

Treatment mode	Low Anxiety		High Anxiety	
	Adjusted mean	't' Value	Adjusted mean	't' Value
Branching	41.53	6.3*	36.09	6.5*
Expository	-4.97		84.87	

*Significant at 0.01 level.

It will be seen from the Fig.11(b) that the two treatment lines cross each other. The interaction is disordinal according to the standard suggested by Lubin (1967). The expository method is better in terms of post achievement for high anxiety students and the branching styles of programming is better or low anxiety students. The 't' values between the post achievement means of high and low anxiety groups taught by the two modes are 6.5 and 6.3 respectively, which are both very significant beyond 0.05 level for 28 degree of freedom. Thus the interaction which is disordinal according to Lubin (1967) is also disordinal according to Bracht and Glass (1968) and the present investigator. The low anxiety group may be assigned branching style of programming while the high anxiety group may be assigned expository method for maximal achievement. Leith (1966) also concluded that low anxiety students achieve high with branching style of programming.

(c) <u>Comparison between Linear Style of Programming and Expository Method</u>:

Table IV and Fig.11(c) presents the comparison of post achievement means for high and low anxiety students taught through linear style of programming and expository method.

It will be seen from the Fig.11(c) that the two treatment lines cross each other. The figure reveals that the linear style of programming is better for low anxiety students,

TABLE-XIV

'T' VALUES FOR THE DIFFERENCE BETWEEN POST TEST MEAN OF HIGH AND LOW ANXIETY STUDENTS TAUGHT THROUGH LINEAR STYLE OF PROGRAMMING AND EXPOSITORY METHOD.

Treatment modes	Low Anxiety Adjusted mean	Low Anxiety 't' Value	High Anxiety Adjusted mean	High Anxiety 't' Value
Linear	39.6	8.2*	53.3	7.3*
Expository	-4.97		84.89	

*Significant at 0.01 level.

It will be seen from the Fig.11(c) that the two treatment mlines cross each other. The figure reveals that the linear style of programming is better for low anxiety students, the expository method is superior for high anxiety students. The interaction is disordinal according to the standard suggested by Lubin (1967). The 't' values between the post achievement means of high and low anxiety students taught by two modes are 7.3 and 8.2 respectively, which are both very significant beyond 0.05 level for 28 degree of freedom. Thus the interaction is also disordinal according to Bracht and Glass (1968) and the present investigator. The low anxiety students may be assigned linear style of programming while the high anxiety students may be assigned expository method for optimal achievement. This finds support in the studies of Sheehan and Hambleton (1977), Papey and others

(1975), Brown and Leith (1975) and Moweliby (1977), Patel (1978). Sipple and others (1969) and Leche (1967) however, did not obtain any interaction between anxiety and these treatment modes. Grime and Allinsmith (1961) and Traweek (1964) also obtained a disordinal interaction but in his study high anxiety student profit more from linear style of programming and low anxiety students by expository method. The subject matters of the programmes employed by Grime and Allinsmith (1961) and Traweek (1964) are quite different from that employed in the present study. Tobias (1969) has shown that interaction is not only a function of the treatment modes but also of the subject matter employed. He studied interaction between response modes and programme constructed on different types of subject matter (i.e. technical and familiar) and concluded that constructed response led to superior achievement for technical subject matter with which students had little prior familiarity, whereas on subject matter with which students were previously acquinted, there was little difference between constructed responses mode or reading. Sarason and Palola (1960) also indicated that anxiety had differential effects depending upon the difficulty of the subject matter. Therefore, the difference in achievement by high and low anxiety students may be attributed to the difference in the subject matter employed by them.

EXTROVERSION-TREATMENT INTERACTION:

Table CV presents the adjusted means and 'F' values for the difference between post test scores of high and low extroversion groups taught through the three treatment modes.

TABLE-CV

ADJUSTED POST ACHIEVEMENT MEANS OF HIGH AND LOW EXTROVERSION GROUPS AND THEIR 'F' VALUES FOR DIFFERENT TREATMENT MODES.

Extroversion	TREATMENT MODES					
	Linear Style		Branching Style		Expository Method	
	Mean	'F'Value	Mean	'F'Value	Mean	'F'Value
High	43.79	1.24	29.79	10.76*	36.23	0.28
Low	44.34		52.91		41.33	

*Significant at 0.05 level.

The table CV shows that the 'F' value for the linear style of programming is 1.24, which is insignificant showing that the achievement of high and low extroversion groups when taught through linear style of programming is not statistically different. This finds support in the studied by Greer (1978), Trawn and Leith (1975) and Kapadia (1972) but contradicts with Davis and Leith (1967) who obtained negative and significant interaction between extroversion and achievement through linear style of programming.

The 'F' value for branching style of programming is

10.76 which is highly significant. The low extroversion students achievement significantly higher through branching style than the high extroversion students which shows significant interaction between extroversion and achievement through branching style of programming. This contradicts with the study of Kapadia (1972) who did not obtained any significant interaction between extroversion and achievement through branching style of programming. Kapadia (1972) employed branching programmes in Geography while in the present study a branching programme on mathematics (statistics) has been employed. The contradictory findings of the above two studies may be due to this difference of subject matter employed by them.

The expository method has yielded a 'F' value of 0.28 only which is highly insignificant showing that the achievement of high and low extroversion groups when taught through expository method is not statistically different.

As in the case of anxiety, the above analyses do not provide any evidence for assignment of treatment modes to the two groups of the students. Evidence for this purpose was obtained by comparing the achievement through any of the two treatment modes for either of the extroversion groups. For this purpose, graph of adjusted means of students high and low extroversion for different pairs of treatment modes were drawn and 't' values for difference in post test means were

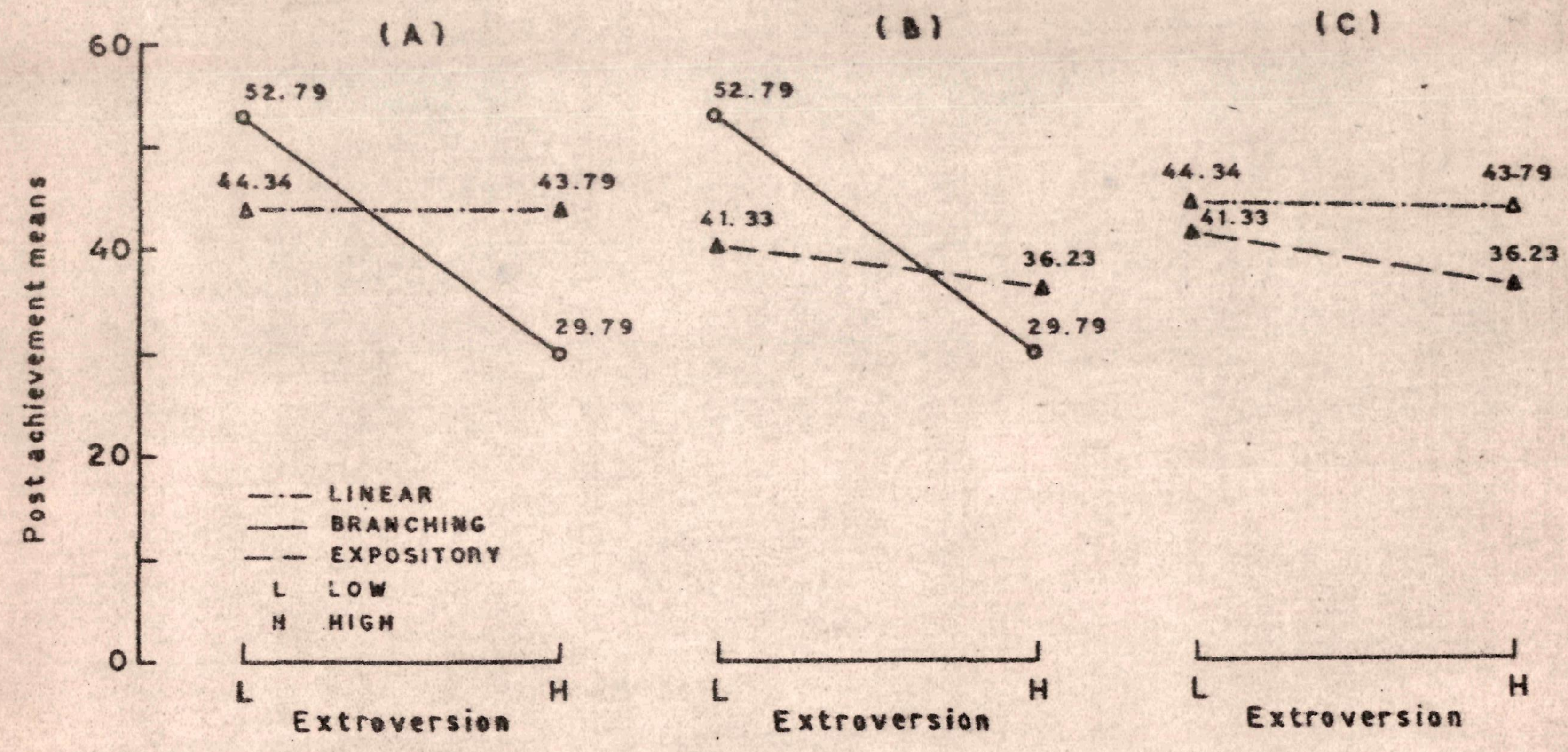

FIG. 12 EXTROVERSION-TREATMENT INTERACTION FOR DIFFERENT PAIRS OF TREATMENT MODES

computed for each pair. The interaction between these different pairs were studied according to the established criteria.

(A) Comparison between Linear and Branching Styles of Programming:

Table CVI and Fig.12(a) presents the comparison of post achievement means for high and low extroversion students taught through linear and branching style of programming.

TABLE-CVI

't' VALUES FOR DIFFERENCE BETWEEN THE POST TEST MEANS OF HIGH AND LOW EXTROVERSION STUDENTS TAUGHT THROUGH LINEAR AND BRANCHING STYLES OF PROGRAMMING.

Treatment modes	Low Extroversion		High Extroversion	
	Adjusted mean	't' Value	Adjusted mean	't' Value
Linear	44.34	1.04	43.79	2.48*
Branching	52.79		29.79	

*Significant at 0.01 level.

It will be seen from the Fig.12(a) that the achievement lines cross which shows that while the branching style of programming is better for low extroversion students, the linear style of programming is proved to be more effective for the high extroversion students. The interaction is disordinal according to the standard suggested by Lubin (1967). It will also be seen from the table CVII that 't' value between the post test means of high extroversion students taught by the two modes is 2.48, which is significant at 0.05 level for 28 degree of freedom. The 't' value between the post test means for the low extroversion students taught through the two modes is 1.04, which is insignificant. Thus the interaction through disordinal according to Lubin (1967), is not so according to Bracht and Glass (1968) as the difference at both the levels of extroversion are not significant. However, according to the present investigator, the interaction is disordinal since the 't' value at one end of the extroversion levels is significant at 0.05 level. The high extroversion students may be assigned the style of programming and extroversion students may be assigned branching style of programming but the evidence is not equally sound for low extroversion students because the 't' value of this group is not significant even at 0.1 level. This conclusion does not fall in line with Leith (1966) hypothesis, who writes "... introverted.... pupil will do well with linear programmes. The... extrovert pupil might fare better with branching programmes".

(a) <u>Comparison between Branching Style of Programming and Expository Method</u>:

Table VII and Fig.12(b) presents the comparison of post achievement means of low and high extroversion students taught through branching style of programming and expository method.

TABLE-CVII

't' VALUES FOR THE DIFFERENCE IN POST TEST MEANS OF HIGH AND LOW EXTROVERSION STUDENTS TAUGHT THROUGH BRANCHING STYLE OF PROGRAMMING AND EXPOSITORY METHOD.

Treatment Modes	Low Extroversion		High Extroversion	
	Adjusted mean	't' Value	Adjusted mean	't' Value
Branching	52.79	1.77*	29.79	1.14
Expository	41.33		36.23	

*Significant at 0.1 level.

It will be seen from the Fig.12(b) that the two treatment lines cross each other which shows that while the branching style of programming is better for the low extroversion students, the expository method has proved to be more effective for the high extroversion students. The interaction is disordinal according to the standard suggested by Lubin (1967). It will be seen from the above table that the 't' values between the post test means for high and low extroversion students taught by the two treatment modes are 1.14 and 1.77 respectively, which are not significant at 0.05 level. Thus the interaction which is disordinal according to the standard of Lubin (1967) is not so according to the Bracht and

Glass (1968) and the present investigator. The low extroversion students may be taught more profitably by the branching style of programming and the high extroversion students by the expository method but the evidence is not very trustworthy as the 't' value for both the groups are insignificant at 0.05 level. However, since 't' value for the low extroversion students taught through branching style and expository method is significant at 0.1 level for 28 degree of freedom, such students may be taught more profitably by the branching style of programming.

( ) <u>Comparison between Linear style of Programming and Expository Method</u>.

Table CVIII and Fig.12(c) presents the comparison of post test means for high and low extroversion students taught through linear style of programming and expository method.

TABLE-CVIII

't' VALUES FOR THE DIFFERENCE IN POST TEST MEANS FOR HIGH AND LOW EXTROVERSION STUDENTS TAUGHT BY LINEAR STYLE OF PROGRAMMING AND EXPOSITORY METHOD.

Treatment modes	Low Extroversion		High Extroversion	
	Adjusted mean	't' Value	Adjusted mean	't' Value
Linear	44.34	-	43.79	-
Expository	41.33		36.32	

It will be seen from the Fig.12(c) that the two treatment lines do not cross, which shows that the interaction is ordinal according to Lubin (1967), Bracht and Glass (1968) and the present investigator. Both high as well as low extroversion students have secured better through linear style of programming as compared to expository method. Thus there is no evidence of interaction i.e. alternative treatments can not be assigned on the basis of this evidence. Thus, there was hardly any need to calculate 't' values for the difference between post test means. Greer (1978) and Irawn and Leith (1975) also could not obtain any significant disordinal interaction between extroversion and linear style of programming and expository method. The ordinal interaction in the studies may be due to the interaction of extroversion and some other personality characteristics within the individual. Davis and Leith (1967) suggested, "it is not one aspect of individuals personality that leads to success, but rather the interaction of personality characteristics within the individual". They found that it was the neuro-introverts who succeeded best on programmed learning task.

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CHAPTER - IX

SUMMARY, CONCLUSIONS, SUGGESTIONS AND SUGGESTIONS FOR FURTHER RESEARCHES

SUMMARY.

Learning effectiveness is a function of personality and mental ability variables of the learner and treatment modes. Therefore, for the optimal learning, treatment modes should be individualized, which implires that the treatment modes should be in accordance with personality and mental ability variables of the learner. Researches concerned with the interaction between treatment modes and personality and mental ability variables of the learner have been terms as ATI (Aptitude-Treatment Interaction) or TTI (Trait-Treatment Interaction) studies. The present study is a modest attempt in this direction. In specific terms, the present study seeks to investigate the relationship between the following personality and mental ability variables as measured by the tools shown against each and achievement through linear and branching styles of programming and expository method of teaching.

(1) Anxiety - Measured at Sarason's General Anxiety Scale for Children (GASC).

(2) Extroversion - Indian Adaptation to Eysneck's Modsley Personality Inventory (MPI) by Kapoor and Jalota.

(3) Intelligence - Mixed Type Group Test of Intelligence by Mehrotra.

(4) Creativity - Non-Verbal Test of Creative Thinking by Baquer Mehdi.

Previous achievement in mathematics measured by junior high school marks in mathematics and intelligence, as measured above, have been employed as control variables. The present study is likely to help in assigning treatment modes to students of differing personality and mental ability variable/characteristics for optimal achievement.

Two programmes - linear and branching on 'Measure of Central Tendencies' (Statistics) suitable for high school students of Board of High School and Intermediate Education, U.P., were developed and validated for the present study. This constituted an important part of the present study. A pre-test consisting of items for measuring entry level behaviours and terminal behaviours was constructed to ensure that the students possess the required prerequisite behaviours and do not possess the terminal behaviour before starting instructions. A post-test consisting of only items measuring terminal behaviours served as a criterion measure.

PROCEDURE:

287 students studying in class IX of three Intermediate Colleges in Urban areas of U.P. constituted the subjects of the present study. The students were administered the two measures of personality (anxiety and extroversion) and the two measures of mental ability (intelligence and creativity). The pre-test was then administered to ensure that the subject possessed the required entry level behaviours

and do not possess any of the behaviours which are proposed to be taught. Three groups of students from each of the above three schools were taught separately by linear and branching styles of programming and expository method of teaching. The post-test was administered after the instructional programme. The post-test scores served as criterion measure.

The personality and mental ability scores and criterion scores of students from three institutions taught through each of the treatment modes were pooled separately. Thus, three groups were constituted each taught by one of the treatment modes. The three groups taught by different treatment modes were divided into high and low groups on the basis of the personality and mental ability scores. Comparison were made between the high groups on any personality or mental ability variables taught by the three treatment modes to ensure that the groups were matched. Similar comparisons were also made for the low groups. In doing so scores of some students had to be rejected. All the 't' values were insignificant showing thereby that the differences between these groups were negligible. Comparisons were also made between the high and low groups on each variable. All the 't' values were significant beyond 0.01 level which indicated that very significant differences existed between high and low groups.

Means of post-test scores obtained by students high and low on each personality and mental ability variables taught through different treatment modes were adjusted for previous achievement in mathematics and intelligence by means of analysis of covariance. Comparisons of these adjusted means were made for different pairs of treatment modes to study the interaction between different personality and mental ability variables and achievement through different treatment modes for assigning treatment modes for optimal achievement. For this purpose, graphs of adjusted post-test means of students high and low on any personality or mental ability variable for different pairs of treatment modes were constructed. If the two treatment lines crossed and the post-test means for students either high or low on any personality and mental ability variable differed significantly, the interaction was considered significant (disordinal). This was regarded as sufficient evidence for assigning treatment mode for optimal achievement.

CONCLUSIONS:

INTELLIGENCE-TREATMENT INTERACTION:

Table CIX presents the adjusted means and 'F' values for the difference between adjusted means of high and low intelligence students for different modes viz. linear and branching styles of programming and expository method.

TABLE - IX

ADJUSTED MEANS AND 'F' VALUES FOR THE DIFFERENCE BETWEEN ADJUSTED MEANS OF HIGH AND LOW INTELLIGENCE GROUPS TAUGHT THROUGH THREE MODES.

Intelligence	POST ACHIEVEMENT MEANS					
	Linear Style Adjusted mean	'F' value	Branching Style Adjusted mean	'F' value	Expository Method Adjusted mean	'F' value
High	48.34	24.51*	55.92	124.3*	41.18	0.25
Low	43.51		33.52		38.06	

*Significant at 0.05 level.

It will be seen from the above table that 'F' values for linear as well as branching styles of programming is significant at 0.05 level for 28 degree of freedom which shows that there is significant interaction between intelligence and achievement through linear as well as branching style of programming. The high intelligence students achieved higher as compared to low intelligence students when taught through any of the above two treatment modes or styles of programming. But the 'F' values for expository method is not significant at 0.05 level for 28 degree of freedom, which shows that the expository method is not significant. However, high intelligence students achieved slightly higher through expository method in comparison to low intelligence ones.

The above analyses do not provide any evidence for assignment of treatment modes to the high and low intelligence groups. Comparisons were therefore, made of the achievements through any of the two treatment modes for either of the intelligence groups for assignment of treatment modes to the high and low groups of intelligence.

It will be seen from the above table CIX that while the linear style is better for low intelligence students in comparison to branching style, the branching style of programming has proved to be more effective for the high intelligence students incomparison to linear style of programming. The interaction is disordinal according to Lubin (1967), but is not so according to Bracht and Glass (1968) and the present investigator as the 't' values for the difference between achievement scores of linear and branching styles of programming for low and high two intelligence groups were 1.91 and 1.50 respectively, which are not significant at 0.05 level for 28 degree of freedom. The low intelligence student may be taught more profitable by the linear style of program ing in comparison to branching style and the high intelligence students by the branching style of programming in comparison to linear style, but the evidence is not very sound. However, since the 't' value for thel low intelligence students is significant at 0.1 level for 28 degree of freedom, such students may be taught more profitably by the linear style in comparison to branching style of programming.

It will also be seen from the table CIX that expository method is better in comparison to branching style of programming for low intelligence students, the branching style is better in comparison to expository method for high intelligence students. The interaction is disordinal according to Lubin (1967) but is not so according to the Bracht and Glass 1968), as the 't' values for the difference between achievement of branching style and expository method for high and low intelligence groups were 2.77 and 1.08 respectively, of which the 't' value of 1.08 is not significant at 0.05 level for 28 degree of freedom. However, according to the present investigator, the interaction is disordinal as the 't' value (i.e. 2.77) at one of the end (i.e. high intelligence) is significant at 0.05 level for 28 degree of freedom. The high intelligence students may be taught through branching style of programming in comparison to expository method for better achievement but the evidence is not equally sound for low intelligence students as 't' value for low intelligence students is not significant even at 0.1 level for 28 degree of freedom.

Furthermore, it will be seen from the table CIX that both high as well as low intelligence students have secured better through linear style of programming as compared to expository method. he interaction is ordinal according to Lubin (1967), Bracht and Glass (1968) and the present investigator. Alternative treatments can not be assigned on the basis of this evidence.

CREATIVITY-TREATMENT INTERACTION

Table CX presents the adjusted means and 'F' values for the differenée between adjusted means of low and high creative students for different treatment modes viz. linear and branching style of programming and expository method.

TABLE-CX

ADJUSTED MEANS AN 'F' VALUES FOR THE DIFFERENCE BETWEEN ADJUSTED MEANS OF HIGH AND LOW CREATIVE STUDENTS TAUGHT THROUGH THREE MODES.

Creative	POST ACHIEVEMENT MEANS					
	Linear Style Adjusted mean	'F' Value	Branching Style Adjusted mean	'F' Value	Expository Method Adjusted mean	'F' value
High	37.80	5.8*	44.11	0.35	17.33	4.34*
Low	50.17		39.38		66.08	

*Significant at 0.05 level.

It will be seen from the table CX that 'F' values for linear style of programming and expository method are significant at 0.05 level for 28 degree of freedom which shows that there is a significant interaction between creativity and achievement through linear style of programming and expository method. The low creative students achieved higher as compared to high creative students when taught through any of the two treatment modes viz. linear style or expository method. But the 'F' value for branching style of programming is insignificant which shows that there is no significant interaction between creativity and achievement through branching style of programming.

The above analyses do not provide any evidence for assignment of treatment modes to the high and low creative students. Comparisons were, therefore, made of the achievements through any of the two treatment modes for either of the creativity groups for assignment of treatment modes.

It will be seen from the table X that while the linear style of programming is better for low creative students in comparison to branching style of programming the branching style has proved to be better for high creative students in comparison to linear style of programming. The interaction is disordinal according to Lubin (1967) but is not so according to Bracht and Glass (1968) and the present investigator as the 't' values for the difference of achievement scores between linear and branching style of programming for low and high creative groups were 1.64 and 0.94 respectively which are not significant at 0.05 level for 28 degree of freedom. The low creative students may be taught more profitably by the linear style in comparison to branching style of programming and high creative students may be taught more profitably by the branching style in comparison to linear style, but the evidence is not sound. However, since the 't' value for low creative group is significant at 0.1 level for 28 degree of freedom such students may be assigned linear style of programming in comparison to branching style of programming for maximal achievement.

It will also be seen from the table X that expository method is better in comparison to branching style for low creative students, the branching style is better in comparison to expository method for high creative students. The interaction is disordinal according to Lubin (1967). The 't' values for the difference between achievements of branching and expository modes for high and low creative groups were 4.05 and 3.63 respectively, both of which are the significant at 0.05 level for 28 degree of freedom. Thus the interaction is also disordinal according to Bracht and Glass (1968) and the present investigator. The low creative students may be assigned expository method in comparison to branching style and high creative students may be assigned branching style of programming in comparison to expository method for maximal achievement.

Furthermore, it will be seen from the above table CX that while the expository method is better than the linear style for low creatives, the linear style has proved to be more effective in comparison to expository method for the high creative students. The interaction is disordinal according to Lubin (1967) but is not so according to Bracht and Glass (1968) as the 't' values for high and low creative groups taught by the two modes were 3.74 and 1.95 respectively, of which 1.95 is not significant at 0.05 level for 28 degree of freedom. According to the present investigator, the interaction is disordinal as the 't' value (i.e. 3.74 at one of the end (i.e. at high creative) is significant at 0.05 level for 28 degree of freedom. The high creative students may be assigned linear style in comparison to expository method for maximal achievement. But the evidence is not very sound for low creative students as the 't' value is not significant at 0.05 level. However, since 't' values (1.95) for low creative students is significant at 0.1 level, such students may be taught a little more profitably by expository method in comparison to linear style for better achievement.

ANXIETY-TREATMENT INTERACTION:

Table CXI presents the adjusted means and 'F' values for the difference between adjusted means of high and low anxiety students for three treatment modes i.e. linear and branching styles of programming and expository method.

It will be seen from the table CXI that 'F' values for linear style and expository method is significant, which shows that there is a significant interaction between anxiety

TABLE-CXI

ADJUSTED MEANS AND 'F' VALUES FOR THE DIFFERENCE BETWEEN ADJUSTED MEANS AT HIGH AND LOW ANXIETY STUDENTS TAUGHT THROUGH THREE MODES.

	POST ACHIEVEMENT MEANS					
Anxiety	Linear Style		Branching Style		Expository Method	
	Adjusted mean	'F' value	Adjusted mean	'F' value	Adjusted mean	'F' value
High	53.31	5.9*	36.09	0.83	84.87	9.68*
Low	39.60		41.53		-4.97	

*Significant at 0.05 level.

and achievement through linear style of programming as well as expository method. The high anxiety students achieved higher as compared to low anxiety students when taught through any of the above two treatment modes. But the 'F' value for branching style is insignificant which shows that there is no significant interaction between anxiety and achievement through branching style of programming.

The above analyses do not provide any evidence for assignment of the treatment modes to the high and low anxiety students. Comparisons were, therefore, made of the achievements through any of the two treatment modes for either of the anxiety groups of assignment of modes to high and low anxiety groups.

It will be seen from the table CXI that while the linear style of programming is better in comparison to branching style for high anxiety students, the branching style has proved to be better in comparison to linear style for low anxiety students. The interaction is disordinal according to the standard of Lubin (1967) but is not as according to Brach and Glass (1968) as the 't' values for the difference between achievement means of linear and branching style of programming at low as well as high anxiety students were 0.27 and 2.532 respectively of which 0.27 is not significant at 0.05 level for 28 degree of freedom. According to the present investigator the interaction is disordinal as the 't' value (2.532) at high anxiety level is significant at 0.05 level for 28 degree of freedom. The high anxiety students may be assigned the linear style in comparison to branching style for better achievement but the evidence is not satisfactory for low anxiety students as the 't' value of this group is not significant even at 0.1 level.

It will also be seen from the table CXI that expository method is better is comparison to branching style for high anxiety students and the branching style has proved to be better in comparison to expository method for low anxiety students for maximal achievement. The interaction is disordinal according to Lubin (1967). The 't' values for the difference between achievement of expository and branching style for high and low anxiety students were 6.5 and 6.3 respectively, which are both significant at 0.05 level for 28 degree of freedom. Thus the interaction is also disordinal according to Bracht and Glass (1968) and the present investigator. The low anxiety students may be assigned branching style in comparison to expository method and high anxiety students may be assigned expository method in comparison to branching style of programming for better achievement.

Further, it will be seen from the table XXI that while the linear style of programming is better in comparison to expository method for low anxiety students, the expository method is superior in comparison to linear style for high anxiety students. The interaction is disordinal according to Lubin (1967). The 't' value for the difference between achievement means of linear style and expository method for high and low anxiety students were 7.3 and 8.2 respectively, which are both significant at 0.05 level for 28 degree of freedom. Thus the interaction, which is disordinal as per Lubin (1967), is also disordinal as per Bracht and Glass (1968) and the present investigator. The linear style may be assigned in comparison to expository method for low anxiety students and expository method may be assigned in comparison to linear style of programming for high anxiety students for maximal achievement.

EXTROVERSION-TREATMENT INTERACTION:

Table XII presents the adjusted means and 'F' values for the difference between adjusted means of high and low extroversion students for linear and branching styles of programming and expository method.

TABLE-CXII

ADJUSTED MEANS AND 'F' VALUES FOR THE DIFFERENCE BETWEEN ADJUSTED MEANS OF LOW AND HIGH EXTROVERSION STUDENTS TAUGHT THROUGH THREE MODES

Extro-version	POST ACHIEVEMENT MEANS					
	Linear Style		Branching Style		Expository Method	
	Adjusted mean	'F' value	Adjusted mean	'F' value	Adjusted mean	'F' value
High	43.79	1.24	29.79	10.76*	36.23	0.28
Low	44.34		52.91		41.33	

*Significant at 0.05 level.

It will be seen from the table CXII that the 'F' values is significant only for branching styles of programming whic shows that there is a significant interaction between extroversion and achievement only through branching style of programming. The low extroversion students achieved higher as compared to high extroversion students when taught through branching style of programming. The 'F' values for linear style of programming and expository method are both insignificant which shows that there is no significant interaction between extroversion and achievement either through linear style of programming or through expository method.

The above analyses do not provide any evidence for assignment of treatment modes to the high and low extroversion students. Comparisons were, therefore, made of the achievements through any of the two treatment modes for either of the extroversion groups for assignment of treatment modes to the high and low extroversion groups.

It will be seen from the above table CXII that while the branching style is better in comparison to linear style for low extroversion students, the linear style of programming has proved to be more effective in comparison to branching style for high extroversion students. The interaction is thus disordinal according to the criteria of Lubin (1967) but is not as according to Bracht and Glass (1968) as the 't' values for the difference between achievement score of linear and branching styles of programming for low and high extroversion students were 1.04 and 2.48 respectively which 1.04 is not significant at 0.05 level for 28 degree of freedom. According to the present investigator, the interaction is disordinal as the 't' value (i.e. 2.48) for one end (i.e. at high extroversion is significant at 0.05 level for 28 degree of freedom. The high extroversion students may be assigned the linear style of programming in comparison to branching style of programming but the evidence is not equally sound for low extroversion students.

It will also be seen from the table CXII that while the branching programming is better in comparison to expository method for low extroversion students the expository method has proved to be more effective in comparison to branching style for high extroversion students. The interaction is disordinal according to Lubin (1967), but is not so according to Bracht and Glass (1968) and the present investigator as the 't' values for the difference between achievement scores of branching style and expository method for low and high extroversion students were 1.77 and 1.14 respectively, both of which are not significant at 0.05 level for 28 degree of freedom. The low extroversion students may be taught more profitably by branching style in comparison to expository method and the high extroversion students by the expository method in comparison to branching styles,but the evidence is not very trustworthy. However, since 't' value for low extroversion students is significant at 0.1 level for 28 degree of freedom, such students may be taught more profitably by branching style of programming in comparison to expository method.

Furthermore, it will be seen from the table CXII that both high as well as low extroversion students have secure better through linear style of programming as compared to expository method. he interaction is ordinal according to the standards of Lubin (1967), Bracht and Glass (1968) and the present investigator. Alternative treatment can not be assigned on the basis of this evidence.

SUGGESTIONS:

tudents with different personality and mental ability variables should be taught through instructional modes suited to them. he conclusions of the present study may be employed by the classroom teacher for this purpose. he following suggestions are being submitted in this regard.

1. Intelligence has been found to interact very significantly with linear and branching styles of programming which means that high intelligence students are likely to achieve higher as compared to low intelligence students when taught through either of these odes. But the interaction between intelligence and expository method of teaching is not significant. A comparison of students achievement through different pairs of treatment modes revealed that high intelligence students may be taught through branching style of programming an' the low intelligence students through expository method of teaching for maximal achievement.
2. he interaction between creativity and achievement through linear style of programming and expository method is significant. ow creatives 'ave been found to achieve higher as compared to high creatives when taught through these two

modes. Creativity has yielded insignificant interaction with branching style of programming. A comparison of student's achievement through different pairs of treatment modes revealed that low creatives may be taught through expository method of teaching and the high creatives through either of the programming styles employed in the study preferably the branching programme for maximal achievement.

3. General anxiety has been found to interact significantly with linear style of programming and expository method. High anxiety students have achieved better through these modes. General anxiety has not yielded significant interaction with branching style of programming. A comparison of student's achievement through different pairs of treatment modes revealed that high anxiety students may be taught through expository method and low anxiety students through either of the programming styles preferably the branching style. If the choice is between the two styles of programming, the high anxiety students may be taught through the linear style of programming and the low anxiety students through branching styles of programming.

4. The personality dimension extroversion/introversion has not yielded any interaction with either linear style of programming or expository method. However, the interaction between extroversion and branching style of programming is significant. High extroversion students achieve lower than the low extroversion students when taught through branching style of

programming. A comparison of student's achievement through different pairs of treatment modes revealed that the high extroversion students may be taught through linear style of programming and low extroversion students may be taught through branching style of programming. The evidence for high extroversion students is more dependable as compared to low extroversion students in this regard.

5. The present study has employed 'Measures of Central Tendencies' a topic of High School statistics as the subject matter of the different instructional treatment modes employed. This subject matter is highly structured and follows a regular sequence. The result of the present study may be applied with caution to other subject matter areas. The applicability of the conclusions of the present study to other areas will depend on the similarity of the structure of the subject matter of these areas and the area investigated.

6. The conclusions of the present study may be generalized to other instructional treatment modes. The treatment modes employed in the present study range from a highly structured teaching-learning situation provided by the linear programme to an unstructured teaching-learning situation provided by the expository method. The branching style seems to be intermediate between these two situations. The generalization to other treatment modes would perhaps, depend upon the degree of structured situation which a treatment mode provides.

SUGGESTIONS FOR FURTHER RESEARCH

1. The present study has employed only four (two personality and two mental ability) variables for assigning treatment modes. Variables like neuroticism, study habits, achievement motivation, cognitive styles etc. have also been found to interact significantly with some treatment modes but the studies for assignment of treatment modes to students high or low on these variables do not seem to have been attempted. Studies should, therefore, be undertaken to investigate the interaction between these variables and different treatment modes with a view to assign treatment modes for maximal achievement.

2. The present investigation has employed only three treatment modes. There are many other treatment modes which seem to require different learner characteristics for success through them. Treatment modes which are known to require different personality and mental ability variables for success through them should, therefore, be studied for interaction effects.

3. Patterns of class room behaviours assessed through technique like 'Flander's Interaction Analysis have been found to have different associations with achievement at different levels when different learning activities are employed. Attempts do not seem to have been made to study the interaction of these patterns and achievement of

student's having different personality and mental ability characteristics. Attempts should, therefore, be made to study the interaction of different patterns of classroom behaviours and achievement of student possessing different personality and mental ability characteristics.

4. It is not one aspect of individuals personality that leads to success, but rather the interaction of different personality characteristics within the individual. For example, it may not be the introverts who achieve high on programmed learning task but the neurotic-introverts who may succeed best on it. Therefore, future ATI researches must take into consideration the interaction of different personality variables within the individual.

5. The present study has employed only 'Measures of Central Tendencies' a topic of high school statistics as subject matter of different treatment modes. A review of previous researches has shown that ATI may appear in one type of subject mat'er and not with the other. Researches should therefore, be undertaken to investigate the interaction between different treatment modes employing a variety of subject matter areas and different learner characteristics.

6. A review of previous researches has shown that variables like age, grade and sex play an important role in interaction between learner attributes and treatment modes.

Studies should, therefore, be undertaken to investigate the interaction between these variables and different treatment modes controlling important learner characteristics.

7. The present study has employed extreme group design with analysis of covariance for studying the interaction between learner attributes and treatment modes. This technique tend to create artificial levels of a rather continuous personological variables. Regression analysis can help in the study of interaction without creating such artificial levels. However, the regression analysis can not help in the assignment of treatment modes to students high and low on the personological variables investigated. Therefore, this technique can not be employed in investigations like the present one, which aim at the assignment of treatment modes. Johnson-Neyman Technique which helps to define regions of personological variables in which the treatment are significantly different on the criterion can be employed for this purpose when hypothesis of homogeneous regression lines in the treatment groups rejected. Studies should, therefore, be undertaken employing this technique for the assginment of treatment modes for maximal achievement.

Study of interaction between learner characteristics and treatment modes with a view to assigning treatment modes for maximal achievement, is a complex problem requiring well designed studies employing sophisticated statistical techniques. The present investigation is an humble attempt in this direction and has merely touched the fringe of the problem. The investigator is conscious of the many limitations of the study, but it is submitted in the hope that it might stimulate better and more detailed research.

APPENDICES

A. Expository Text

B. Linear Programmed Instructional Style Booklet

C. Branching Programmed Instructional Style Booklet

D. Statistics Achievement Test

E. Procedure for Calculation of Analysis of Covariance and Adjusted Means

APPENDIX-A

MEASURES OF CENTRAL TENDENCIES

प्रस्तावना

सांख्यिकी में सामग्री संग्रहण का उद्देश्य किसी विषय के बारे में जानकारी प्राप्त करना होता है। पर संग्रहीत सामग्री की राशि अधिक होने के कारण उसे समझ सकना बहुत कठिन हो जाता है। अगर कोई ऐसी संख्या या प्राप्तांक जो इस समूह का स्तर प्रदर्शित करे, मालूम हो तो इस कठिनाई से बचा जा सकता है। सांख्यिकी में ऐसी संख्याओं को दिये गये समूह का 'केन्द्रीय प्रवृत्ति के मान' कहते हैं।

अध्याय १ में हम केन्द्रीय प्रवृत्ति के विभिन्न मानों- मध्यमान, मध्यांक व बहुलांक की परिभाषा के बारे में जानेंगे।

अध्याय १ केन्द्रीय प्रवृत्ति के विभिन्न मान

मध्यमान

किसी कक्षा के ५ छात्रों ने विज्ञान में क्रमशः १५, २०, ३०, ३५ व ३५ अंक प्राप्त किये।

उपरोक्त छात्रों द्वारा प्राप्त अंकों का

$$\text{औसत} = \frac{१५ + २० + ३० + ३५ + ३५}{५} = \frac{१३५}{५} = २७$$

उपरोक्त विधि से ज्ञात किया हुआ औसत

ही सांख्यिकी में मध्यमान कहलाता है। मध्यमान को $\bar{x}$ से प्रदर्शित करते हैं। औसत ज्ञात करने के लिए समूह या कक्षा के छात्रों द्वारा प्राप्त अंकों के कुल योग को उनकी संख्या से भाग देते हैं। क्योंकि सांख्यिकी में औसत ही मध्यमान कहलाता है। अतः मध्यमान

$$= \frac{\text{प्राप्त अंकों का कुल योग}}{\text{छात्रों की संख्या}}$$

उदाहरण - माना कुछ छात्रों के प्राप्तांक दिये हैं -

१५, १८, २०, २२, २५, २७, २७, ३० व ३२

उपरोक्त प्राप्तांकों का मध्यमान-

$$= \frac{१५ + १८ + २० + २२ + २५ + २७ + २७ + ३० + ३२}{९}$$

$$= \frac{२१६}{९} = २४$$

<u>मध्यांक</u>

पिछले उदाहरण में दिये निम्न प्राप्तांकों का पुनः अध्ययन कीजिये-

१५, १८, २०, २२, २५, २७, २७, ३०, ३२

इन प्राप्तांकों में बीच का अंक अर्थात् वह अंक जिसके ऊपर ५० प्रतिशत तथा नीचे ५० प्रतिशत छात्रों के प्राप्तांक हैं, २५ है।

आँकड़ों में वह अंक जिसके ऊपर ५० प्रतिशत तथा नीचे ५० प्रतिशत छात्रों के अंक होते हैं, मध्यांक कहलाता है। उपरोक्त प्राप्तांकों में अंक २५ के ऊपर ५० प्रतिशत अर्थात् ४ छात्रों के प्राप्तांक हैं। अतः

मध्यांक = २५

उदाहरण - एक कक्षा के कुछ छात्रों के प्राप्तांक निम्नलिखित हैं-

५, ७, १०, १८, २१, २५, २८

उपरोक्त प्राप्तांकों का मध्यांक = १८

<u>बहुलांक</u>

प्राप्तांकों १५, १८, २०, २२, २५, २७, ३० व ३२ में प्राप्तांक २७ दो बार तथा शेष सभी प्राप्तांक एक-एक बार ही आये हैं।

जो प्राप्तांक समूह में सबसे अधिक बार आता है। बहुलांक कहलाता है।

उपरोक्त समूह का बहुलांक = २७

उदाहरण- एक कक्षा के कुछ छात्रों ने विज्ञान में निम्नलिखित अंक प्राप्त किए-

२१, २५, २५, २७, २६, ३०, ३७

उपरोक्त प्राप्तांकों का बहुलांक २५ है, क्योंकि यह समूह में सबसे अधिक बार आया है।

इस प्रकार

(१) मध्यमान वह मान है जो अंकों के कुल योग को उनकी संख्या से भाग देकर प्राप्त होता है।

(२) मध्यांक वह अंक है जिसके ऊपर ५० प्रतिशत तथा नीचे ५० प्रतिशत छात्रों के अंक होते हैं।

(३) बहुलांक वह अंक है जो समूह में सबसे अधिक बार आता है।

उपरोक्त तीनों मान मध्यमान, मध्यांक तथा बहुलांक सांख्यिकी में केन्द्रीय प्रवृत्ति के मान कहलाते हैं।

अभ्यास प्रश्न

(१) एक परीक्षा में ७ छात्रों ने क्रमशः निम्नलिखित अंक प्राप्त किये-

०, १, १, २, ३, ५, ६

इनका मध्यमान, मध्यांक व बहुलांक ज्ञात कीजिए ?

(२) केन्द्रीय प्रवृत्ति के मान कौन-कौन से हैं। प्रत्येक की परिभाषा लिखिए ?

अध्याय- २ मध्यमान की गणना

अव्यवस्थित आंकड़ों में

एक परीक्षा में ५ छात्रों ने क्रमशः १६, १८, १९, २१ व २२ अंक प्राप्त किये। इनका मध्यमान ज्ञात कीजिये ?

हम जानते हैं कि मध्यमान = $\dfrac{\text{प्राप्त अंकों का योग}}{\text{छात्रों की संख्या}}$

प्राप्त अंकों के योग को X तथा छात्रों की संख्या को N से प्रदर्शित करते हैं अतः

$$\text{मध्यमान } (\bar{X}) = \frac{\Sigma X}{N}$$

प्रश्न में, ΣX = १६ + १८ + २१ + २२ = ९६

N = ५

मध्यमान ($\bar{X}$) $\dfrac{९६}{५}$ = १९.२

अभ्यास प्रश्न - कुछ छात्रों ने एक परीक्षा में क्रमशः १, ७, ८, ८, ९, ११, १६, १८ व २० अंक प्राप्त किये।

इनका मध्यमान ज्ञात कीजिये ?

व्यवस्थित आंकड़ों में -

(अ) बारम्बारता बंटन में : एक कक्षा के कुछ छात्रों द्वारा प्राप्त

अध्याय - २ मध्यमान की गणना

अव्यवस्थित आंकड़ों में मध्यमान

एक परीक्षा में ५ छात्रों ने क्रमशः १६, १८, १९, २१, २२ अंक प्राप्त किये। इनका मध्यमान ज्ञात कीजिए ?

हम जानते हैं कि किसी समूह या कक्षा का मध्यमान प्राप्त अंकों के कुल योग को उनकी संख्या से भाग देकर प्राप्त होता है।

उपरोक्त प्राप्तांकों में अंकों का कुल योग =

१६ + १८ + १९ + २१ + २२ = ९६

अंकों के कुल योग को ΣX से प्रदर्शित करते हैं। अतः ΣX = ९६

छात्रों की संख्या को N से प्रदर्शित करते हैं। उपरोक्त प्राप्तांकों में N = ५

$$\text{अतः मध्यमान} = \frac{\text{प्राप्त अंकों का कुल योग}}{\text{छात्रों की संख्या}}$$

$$= \frac{९६}{५} = १९.२$$

इस प्रकार अव्यवस्थित आँकड़ों में मध्यमान ज्ञात करने का सूत्र

$$\bar{X} = \frac{\Sigma X}{N}$$

<u>व्यवस्थित आँकड़ों में मध्यमान</u> - (अ) बारम्बारता बंटन में

किसी कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न प्रकार है -

प्राप्तांक (x)	बारम्बारता (f)	बार० × प्राप्तांक (fx)
२	३	३ × २ = ६
३	४	४ × ३ = १२
४	२	२ × ४ = ८
	N = ९	Σfx = २६

(१) उपरोक्त सारणी में ३ छात्रों ने २-२ अंक प्राप्त किये। इन ३ छात्रों द्वारा प्राप्त अंकों का योग = ३ × २ = ६

(२) इसी प्रकार ४ छात्रों ने ३-३ अंक प्राप्त किये। इन ४ छात्रों द्वारा प्राप्त अंकों का योग = f = ४ × ३ = १२

(३) इसी प्रकार २ छात्रों द्वारा प्राप्त अंकों का योग = fx = २ × ४ = ८

सभी (३ + ४ + २ = ९) छात्रों द्वारा प्राप्त अंकों का कुल योग (Σfx) = ६ + १२ + ८ = २६

अंक निम्न प्रकार से दिये हैं इनका मध्यमान ज्ञात कीजिए ?

प्राप्तांक (x)	२	३	४
बारम्बारता (f)	३	४	२

(१) उपरोक्त सारणी में ३ छात्रों ने २-२ अंक प्राप्त किये। इन ३ छात्रों द्वारा प्राप्त अंकों का योग = ३ x २ =
= ६

(२) इसी प्रकार ४ छात्रों ने ३-३ अंक प्राप्त किये। इन ४ छात्रों द्वारा प्राप्त अंकों का योग = ४ x ३ = १२

(३) इसी प्रकार २ छात्रों द्वारा प्राप्त अंकों का योग fx =
२ x ४ = ८

सभी (३+४+२ = ९) छात्रों द्वारा प्राप्त अंकों का कुल योग (Σfx) = ६+१२+८
= २६

इन सब योगों को सारणी में निम्न प्रकार लिखते हैं :

२	३	३ x २	= ६
३	४	४ x ३	= १२
४	२	२ x ४	= ८
		Σfx	= २६

$$\text{क्योंकि मध्यमान} = \frac{\text{छात्रों द्वारा प्राप्त कुल अंकों का योग}}{\text{छात्रों की संख्या}}$$

$$= \frac{\Sigma X}{N} = \frac{२६}{९} = २.८८$$

उदाहरण - एक कक्षा के १० छात्रों के प्राप्तांकों का बंटन निम्न है। इनका मध्यमान ज्ञात कीजिये ?

प्राप्तांक (X)	७	८	९	१०
बारम्बारता (f)	४	३	१	२

हल :

प्राप्तांक	बारम्बारता	प्राप्त अंकों का योग
७	४	४ x ७ = २८
८	३	३ x ८ = २४
९	१	१ x ९ = ९
१०	२	२ x १० = २०

प्राप्त अंकों का कुल योग = ८१

अतः मध्यमान ($\bar{X}$) $= \frac{८१}{१०} = ८.१$

इस प्रकार यदि प्राप्तांक बारम्बारता बंटन में दिये हों तो

मध्यमान ($\bar{x}$) $= \frac{\Sigma X}{N}$

अभ्यास प्रश्न - एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न है। इनका मध्यमान ज्ञात कीजिए ?

प्राप्तांक (x)	१०	११	१२
बारम्बारता (f)	५	४	२

(ब) वर्ग अन्तराल में मध्यमान (दीर्घ विधि)

एक समूह के किसी विषय में प्राप्तांक वर्ग अन्तरालों में निम्न प्रकार दिये हैं :

वर्ग अन्तराल (C.I.)	१०- २०	२०-३०	३०-४०
बारम्बारता (f)	८	३	६

उपरोक्त बंटन में ८ छात्रों ने १० तथा २० के बीच, ३ छात्रों ने २० तथा ३० के बीच तथा ६ छात्रों ने ३० तथा ४० के बीच अंक पाये हैं। इस प्रकार जब प्राप्तांक वर्ग अन्तरालों में दिये हों तब हम यह नहीं बता सकते हैं कि कितने छात्रों ने कौन से अंक प्राप्त किया है। ऐसी स्थिति में हम वर्ग अन्तराल के मध्यमूल्य को उस वर्ग अन्तराल का प्रतिनिधि अंक मान लेते हैं। किसी वर्ग अन्तराल का मध्य मूल्य (x)

$$= \frac{\text{निम्न सीमा + उच्च सीमा}}{२}$$

उपरोक्त बंटन में, वर्ग अन्तराल १०-२० का मध्यमूल्य

$$= \frac{१० + २०}{२} = १५$$

वर्ग अन्तराल २०-३० का मध्यमूल्य

$$= \frac{२० + ३०}{२} = २५$$

वर्ग अन्तराल ३० -४० का मध्यमूल्य

$$= \frac{३० + ४०}{२} = ३५$$

दिए हुए वर्ग अन्तरालों का मध्यमूल्य निकालने के बाद हम प्रत्येक वर्ग अन्तराल के प्राप्तांकों का योग (x) ज्ञात करने के लिए उसके मध्यमूल्य (x) को बारम्बारता (f) से गुणा करके fx ज्ञात कर लेते हैं, क्योंकि यहाँ मध्यमूल्य (x) को पूर्ण अन्तराल का प्रतिनिधि मान लेते हैं। प्राप्तांकों का कुल योग प्रत्येक वर्ग अन्तराल के fx को जोड़कर ज्ञात कर लेते हैं।

००००००००००००००००००००००००००००००००००००००

दिए वर्ग अन्तरालों के बंटन में मध्य मूल्य (x) तथा प्रत्येक वर्ग अन्तराल के प्राप्तांकों का योग (fx) लिखने पर -

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	मध्यमुल्य (x)	प्राप्तांकों का योग (fx)
१०- २०	८	१५	८ × १५ = १२०
२०-३०	३	२५	३ × २५ = ७५
३०- ४०	९	३५	९ × ३५ = ३१५

हम जानते हैं कि मध्यमान $= \dfrac{\text{प्राप्तांकों का कुल योग}}{\text{छात्रों की संख्या}}$

$$= \frac{\Sigma fx}{N}$$

$$= \frac{५१०}{२०} = २५.५$$

उदाहरण - एक कक्षा के छात्रों के २० प्राप्तांक वर्ग अन्तरालों में निम्न प्रकार दिये हैं। इनका मध्यमान ज्ञात कीजिए ?

वर्ग अन्तराल (C.I.)	५-१५	१५-२५	२५-३५	३५-४५
बारम्बारता (f)	१५	१२	२०	५

हल :

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	मध्यमुल्य (x)	बारम्बारता मध्यमुल्य (fx)

५- १५	१५	१०	१५ x १०	= १५०
१५- २५	१२	२०	१२ x २०	= २४०
२५-३५	१०	३०	१० x ३०	= ३००
३५- ४५	५	४०	५ x ४०	= २००
	४२		fx	= ८६०

मध्यमान (x̄) $= \frac{८६०}{४२}$ = २१. १६

इस प्रकार वर्ग अन्तराल दिये होने पर मध्यमान (x̄) $= \frac{\Sigma fx}{N}$

अभ्यास प्रश्न : निम्न बारम्बारता बंटन में दिये वर्ग अन्तरालों का मध्यमान ज्ञात कीजिए ?

वर्ग अन्तराल (C.I.)	०-१०	१०-२०	२०-३०	३०-४०
बारम्बारता (f)	१५	१२	२०	२३

वर्ग अन्तराल में(मध्यमान लघुविधि से) - निम्न सारणी में दिये प्राप्तांकों का मध्यमान ज्ञात कीजिए ?

वर्ग अन्तराल (C.I.)	२०-३०	३०-४०	४०-५०	५०-६०	६०-७०
बारम्बारता (f)	८	६	१७	१४	१२
मध्यमूल्य (x̄)	२५	३५	४५	५५	६५

क्योंकि मध्यमान ($\bar{X}$) = $\frac{\Sigma fx}{N}$

अत: सर्वप्रथम हमें x का मान ज्ञात करना होगा। x का मान ज्ञात करने पर हम देखते हैं कि गुणा बहुत बड़ा हो जाता है। अत: जब बारम्बारता अधिक हो तो गुणा की इस कठिनाई से बचने के लिए हम एक अन्य विधि का प्रयोग करते हैं जिसे लघु विधि कहते हैं।

लघु विधि से मध्यमान निकालने के लिए हम किसी वर्ग अन्तराल के मध्यमूल्य को कल्पित मध्यमान मान लेते हैं। गणना की सुगमता के लिए श्रेणी के बीच के वर्ग अन्तराल के मध्य मूल्य को कल्पित मध्यमान मान लेते हैं। उपरोक्त वंटन में ४०-५० श्रेणी के लगभग बीच में है अत: इसके मध्यमूल्य को कल्पित मध्यमान माना है। कल्पित मध्यमान = ४५ जिस वर्ग अन्तराल के मध्य मूल्य को कल्पित मध्यमान माना जाता है उसके सामने स्तम्भ - ३ में शून्य लिख दिया जाता है। निम्न वंटन में कल्पित मध्यमान ४०-५० के मध्यमूल्य को माना है। अत: इसके आगे शून्य लिख दिए हैं। कल्पित मध्यमान वाले वर्ग अन्तराल से छोटे अन्तरालों की ओर क्रमश: -१, -२, -३ आदि तथा बड़े अन्तरालों की ओर क्रमश: +१, +२, +३ आदि लिख देते हैं। इन्हें विचलन मान कहते हैं तथा d से प्रदर्शित करते हैं। निम्न वंटन में वर्ग अन्तराल ३०-४० में d का मान -१ तथा उससे छोटे वर्ग अन्तराल २०-३० में -२ लिख दिया है। इसी प्रकार दूसरी ओर वर्ग अन्तराल ५०-६० में d का मान +१ लिख दिया है उससे अगले वर्ग अन्तराल ६०-७० में d का मान +२ लिख विचलन (d) का मान रखने के बाद f और d का गुणा करते हैं। fd का मान अगले स्तम्भ में लिख देते हैं। f और d का गुणा निम्न

सहित करते हैं। फिर प्रत्येक fd को चिन्हों का ध्यान रखते हुए जोड़कर ज्ञात करते हैं।

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	विचलन (d)	बारम्बारता विचलन (fd)
२०-३०	८	-२	-१६
३०-४०	६	-१	- ६
४०-५०	१७	०	०
५०-६०	१४	+ १	+ १४
६०-७०	१२	+ २	+ २४
	६०		fd = १६

का मान ज्ञात करने के बाद मध्यमान निम्न सूत्र से ज्ञात करते हैं

$$= \frac{\Sigma fd}{N} \times i + A$$

जहाँ A = कल्पित मध्यमान , i = वर्ग विस्तार, d = विचलन

f = बारम्बारता है।

$$= \frac{१६}{६०} \times १० + ४५ = २.६६ + ४५ = ४७.६६$$

उदाहरण : एक कक्षा के ४५ छात्रों के प्राप्तांक वर्ग अन्तरालों में निम्न हैं-

वर्ग अन्तराल (C.I.)	०-५	५-१०	१०-१५	१५-२०	२०-२५
बारम्बारता (f)	५	१०	१४	१२	४

हल : उपरोक्त वर्ग अन्तरालों में वर्गअन्तराल १०-१५ बीच का अन्तराल है अतः इसके मध्यमूल्य १२.५ को कल्पित मध्यमान मान लिया है।

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	विचलन (d)	बारम्बारता x विचलन (fd)
०-५	५	-२	-१०
५-१०	१०	-१	-१०
१०-१५	१४	०	०
१५-२०	१२	+ १	+ १२
२०-२५	४	+ २	+ ८
	४५		$\Sigma fd = 0$

अतः मध्यमान = $\frac{\Sigma fd}{N} \times i + A$ = $\frac{0}{45} \times$ ५ + १२.५ = १२.५

१७ प्रकार लघु विधि से मध्यमान ज्ञात करने का सूत्र

$$= \frac{\Sigma fd}{N} \times i + A$$

<u>अभ्यास प्रश्न</u> - (१) एक कक्षा के कुछ छात्रों द्वारा गणित की परीक्षा में प्राप्त अंक वर्ग अन्तराल के रूप में निम्न हैं। इनका लघु विधि से मध्यमान ज्ञात करो ?

वर्ग अन्तराल (C.I.)	०-१०	१०-२०	२०-३०	३०-४०	४०-५०	५०-६०	६०-७०
बारम्बारता (f)	८	१२	२२	१८	१६	११	६

I)

(२) निम्न वर्गअन्तरालों का लघुविधि से मध्यमान ज्ञात कीजिए ?

वर्गअन्तराल	३०-४०	४०-५०	५०-६०	६०-७०	७०-८०	८०-९०
बारम्बारता	३	५	६	१२	७	२

१८ प्रकार जब प्राप्तांक वर्ग अन्तराल में दिये हों तो मध्यमान ज्ञात करने की दो विधियां हैं-

(अ) दीर्घ विधि

(ब) लघु विधि

अध्याय -3 मध्यांक की गणना

अव्यवस्थित आंकड़ों में मध्यमान (१) जब N विषम हो- एक समूह ने किसी परीक्षा में निम्न अंक प्राप्त किये। इनका मध्यांक ज्ञात कीजिए ?

१ , १ , ४ , ७ , ९

हम जानते हैं कि मध्यांक समूह के बीच का अंक अर्थात् वह अंक होता है जिसके ऊपर ५० प्रतिशत तथा नीचे ५० प्रतिशत छात्रों के अंक हैं। यदि छात्रों की संख्या है तब बीच का अंक $= \frac{N+1}{2}$ वें छात्र का प्राप्तांक

अत: मध्यांक $= \frac{N+1}{2}$ वें छात्र का प्राप्तांक $= \frac{५+१}{२}$ वें छात्र का प्राप्तांक

= ३ वें छात्र का प्राप्तांक = ४

उदाहरण - एक कक्षा के कुछ छात्रों ने एक परीक्षा में निम्न अंक प्राप्त किये। इनका मध्यांक ज्ञात कीजिए ?

७, ९, १८, २४, २५, २७ व ३०

यहाँ N = ७ अत: मध्यांक $= \frac{७+१}{२}$ वें छात्र का प्राप्तांक = ४ वें छात्र का प्राप्तांक = २४

अभ्यास प्रश्न- एक कक्षा के कुछ छात्रों के प्राप्तांक निम्न हैं, इनका मध्यांक बताइये ?

३, ४, ५, ५, ६, ६, ७, ८, ९

(ब) जब N सम हो - एक कक्षा के ४ छात्रों के प्राप्तांक निम्न है।

इनका मध्यांक ज्ञात कीजिये ?

३, ४, ८, ९

हम जानते हैं कि मध्यांक = $\frac{N+1}{2}$ वें छात्र का प्राप्तांक

यहाँ N = ४ : मध्यांक = $\frac{४+१}{२}$ वें छात्र का प्राप्तांक =

= २.५ वें छात्र का प्राप्तांक, २.५ वें छात्र का प्राप्तांक २ वें तथा ३ वें छात्र का प्राप्तांक का औसत होगा। अतः

$$\text{मध्यांक} = \frac{\text{२ वें छात्र का प्राप्तांक} + \text{३ वें छात्र का प्राप्तांक}}{२}$$

$$= \frac{४+८}{२} = ६$$

उदाहरण - निम्न छात्रों के प्राप्तांकों का मध्यांक ज्ञात कीजिये ?

२, २, ६, ७, ८, ९

हम जानते हैं कि मध्यांक = $\frac{N+1}{2}$ वें छात्र का प्राप्तांक =

$$= \frac{६+१}{२} \text{ वें छात्र का प्राप्तांक}$$

= ३.५ वें छात्र का प्राप्तांक

$$= \frac{\text{३ वें छात्र का प्राप्तांक} + \text{४ वें छात्र का प्राप्तांक}}{२}$$

$$= \frac{६+७}{२} = ६.५$$

११	६	३ + ६ = १२
१२	८	३ + ६ + ८ = २०
१३	४	३ + ६ + ८ + ४ = २४

अभ्यास प्रश्न : निम्न बारम्बारता बंटन में संचयी बारम्बारता भरिये ?

प्राप्तांक	बारम्बारता	संचयी बारम्बारता
३	१	
४	२	
५	४	
६	६	
७	३	
८	१	

(२) मध्यांक - एक परीक्षा में १७ छात्रों द्वारा प्राप्त अंकों का बंटन उनकी संचयी बारम्बारता सहित निम्न प्रकार है। इनका मध्यांक ज्ञात कीजिए ?

प्राप्तांक (x)	३	४	५	६	७	८
बारम्बारता (f)	१	२	४	६	३	१
संचयी बारम्बारता (cf)		३	७	१३	१६	१७

हम जानते हैं कि मध्यांक $= \dfrac{n + १}{२}$ वें पद का मान

$= \dfrac{१७ + १}{२} =$ ९ वें छात्र का प्राप्तांक

अभ्यास प्रश्न- निम्न छात्रों द्वारा प्राप्त अंकों के मध्यांक ज्ञात कीजिए ?

५, ७, ८, ८, १०, ११, १२, १३, १८, २०

<u>व्यवस्थित आँकड़ों में मध्यांक</u> - (ब) बारम्बारता बंटन में-

(१) संचयी बारम्बारता- एक कक्षा के कुछ छात्रों के अंकों का बंटन निम्न प्रकार है- इसकी मध्यांक ज्ञात कीजिए ?

प्राप्तांक (x)	३,	५	६	७
बारम्बारता (f)	२,	८	५	२

बारम्बारता बंटन में मध्यांक ज्ञात करने के लिए सर्वप्रथम संचयी बारम्बारता ज्ञात करते हैं। उपरोक्त बंटन में प्राप्तांक ३, २ छात्रों ने, अंक ५, ८ छात्रों ने अंक ६, ५ छात्रों ने तथा अंक ७, २ छात्रों ने प्राप्त किये हैं। अंक ५ अथवा उससे कम ८ + २ = १० छात्रों ने प्राप्त किये। यही १० प्राप्तांक ५ की संचयी बारम्बारता कहलाती है। इसी प्रकार प्राप्तांक ६ की संचयी बारम्बारता = ५ + ८ + २ = १५ तथा प्राप्तांक ७ की संचयी बारम्बारता २ + ५ + ८ + २ = १७ होगी।

उदाहरण - निम्न सारणी में कुछ छात्रों के प्राप्तांक दिये हैं। संचयी बारम्बारता करिये ?

प्राप्तांक	-	१०	११	१२	१३
बारम्बारता	-	३	६	८	४

हल :

प्राप्तांक	बारम्बारता	संचयी बारम्बारता
१०	३	० + ३ = ३

संचयी बारम्बारताओं से हम देखते हैं कि १ वें छात्र तक के प्राप्तांक ३, ३ वें छात्र तक के प्राप्तांक ४ तथा ७ वें छात्र तक के प्राप्तांक ५ है, १३ वें छात्र तक के प्राप्तांक ६ है अत: ७ वें के बाद अर्थात् ८ वें से १३ वें तक के छात्रों ने ६-६ प्राप्तांक प्राप्त किये हैं।

मध्यांक = ६ वें छात्र का प्राप्तांक = ६

उदाहरण- निम्न बारम्बारता बंटन का मध्यांक ज्ञात कीजिए ?

प्राप्तांक (x)	२०	२१	२२	२३	२४	२५
बारम्बारता (f)	१	३	५	७	४	३
संचयी बारम्बारता (cf)	१	४	९	१६	२०	२३

$$\text{मध्यांक} = \frac{२३ + १}{२} \text{ वें छात्र का प्राप्तांक} = १२ \text{ वें छात्र का प्राप्तांक}$$

= १० वें से १६ वें छात्र का प्राप्तांक = २३

अभ्यास प्रश्न :

(१) एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न है। इसका मध्यांक ज्ञात करो ?

प्राप्तांक (x)	१८	१९	२०	२१	२२	२३	२४
बारम्बारता (f)	२	९	३	५	७	२	३
संचयी बारम्बारता (cf)	२	११	१४	१९	२६	२८	३१

(२) एक कक्षा २४ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न है। बहुलांक ज्ञात कीजिए ?

प्राप्तांक (x)	१०	११	१२	१३
बारम्बारता (f)	३	१०	७	४
संचयी बारम्बारता (cf)	३	१३	२०	२४

व्यक्तिगत आंकड़ों में बहुलांक (ब) वर्ग अन्तराल में

एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों को वर्ग अन्तरालों में निम्न प्रकार दिया है। इनका बहुलांक ज्ञात कीजिए ?

वर्ग अन्तराल (C.I.)	०–५	५–१०	१०–१५	१५–२०	२०–२५	२५–३०
बारम्बारता (f)	२	१०	६	७	३	२

किसी बारम्बारता बंटन में बहुलांक ज्ञात करने के लिए सर्वप्रथम हम संचयी बारम्बारता ज्ञात करते हैं। उपरोक्त प्रश्न में संचयी बारम्बारता लिखने पर –

वर्ग अन्तराल	बारम्बारता	संचयी बारम्बारता	
०–५	२	२	२
५–१०	१०	१०+२	१२
१०–१५	६	६+१०+२	१८
१५–२०	७	७+ ६+१०+२	२५
२०–२५	३	३+७+६+१०+२	२८
२५–३०	२	२+३+७+६+१०+२	३०

जहाँ N = 30 अतः N/2 = १५

उपरोक्त सारणी से हम देखते हैं कि २ छात्रों ने ० से ५ के बीच, ३ से १२ वें छात्रों ने ५ से १० के बीच १३ से १८ वें छात्रों ने १० से १५ के बीच, १८ से २५ वें छात्रों ने १५ से २० के बीच, २६ से २८ वें छात्रों ने २० से २५ के बीच तथा २९ से ३० वें छात्रों ने २५ से ३० के बीच अंक पाये हैं। अतः अर्थात् १५ वें छात्र के अंक १० से १५ के बीच होंगे। यही वर्ग अन्तराल १०-१५ मध्यांक वर्ग कहलाता है। इस वर्गअन्तराल में कौन सा अंक मध्यांक है। यह हम निम्न सूत्र से ज्ञात करते हैं

$$\text{मध्यांक} = l + \frac{N/2 - cf}{f} \times i$$

जहाँ Md = मध्यांक, l = वर्ग की निम्न सीमा, cf = मध्यांक वर्ग से पहले वर्ग अन्तराल की संचयी बारम्बारता, f = मध्यांक वर्ग की बारम्बारता है।

$$\text{अतः मध्यांक} = ५ + \frac{(१५-१०)}{६} \times ५ =$$

$$५ + \frac{५}{६} \times ५ = ५ + ४.१ = ६.१$$

उदाहरण - निम्न आंकड़ों का मध्यांक ज्ञात करो ?

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	संचयी बारम्बारता (cf)
१०-२०	८	८
२०-३०	४	१२
३०-४०	५	१७
४०-५०	१०	२७
५०-६०	१०	२७

६०-७०	५	३७
७०-८०	३	४०

उपरोक्त बंटन में N = ४०, N/2 = २०

अत: मध्यांक वर्ग = ४०-५०

अत: मध्यांक = $l + \left(\frac{\frac{N}{2} - cf}{f}\right) i$ = ४० + $\left(\frac{२०-१७}{१०}\right) \times १०$

= ४० + ३ = ४३

इस प्रकार किसी मध्यांक वर्ग में मध्यांक ज्ञात करने का सूत्र मध्यांक = $l + \frac{N/2 - cf}{f} \times i$ है।

अभ्यास प्रश्न

(१) एक कक्षा के [illegible] छात्रों ने निम्न प्रकार अंक प्राप्त किये। उनका मध्यांक ज्ञात कीजिए?

वर्ग अन्तराल	१-७	७-९	९-११	११-१३	१३-१५	१५-१७	१७-१९
बारम्बारता	१०	५	३	७	३	३	२

(२) एक कक्षा के छात्रों का बंटन निम्न है। उनका मध्यांक ज्ञात कीजिए?

वर्ग अन्तराल (C.I.)	०-५	५-१०	१०-१५	१५-२०	२०-२५
बारम्बारता (f)	४	७	९	३	१

अध्याय ४ बहुलांक की गणना

अवर्गीकृत आंकड़ों में बहुलांक

एक समूह ने निम्न अंक प्राप्त किये। इनका बहुलांक ज्ञात कीजिए ?

१, २, २, ५, ७

हम जानते हैं कि बहुलांक वह अंक है जो समूह में सबसे अधिक बार आया है। उपरोक्त समूह में अंक २ सबसे अधिक बार आया है। यह अंक २ इस समूह का बहुलांक है।

अभ्यास प्रश्न- एक कक्षा के कुछ छात्रों ने हिन्दी विषय में निम्न अंक प्राप्त किए। इनका बहुलांक ज्ञात कीजिए ?

१४, १६, १८, २१, २७, ३०, ३०, ४१, ५२

व्यवस्थित आंकड़ों में बहुलांक (अ) बारम्बारता बंटन (सामान्य विधि)

एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों के बंटन निम्न है। इनका बहुलांक बताओ ?

प्राप्तांक (X)	२	३	४	६	८
बारम्बारता (f)	१	५	३	४	२

हम जानते हैं कि बहुलांक वह अंक है जो समूह में सबसे अधिक बार आया हो अर्थात् जिस अंक की बारम्बारता सबसे अधिक हो। उपरोक्त बंटन में प्राप्तांक ३ की बारम्बारता सबसे अधिक ५ है। अतः बहुलांक = ३

अभ्यास प्रश्न- एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न है। बहुलांक बताओ ?

प्राप्तांक (X)	१	२	३	४	५
बारम्बारता (f)	२	२	१	७	३

(ब) बारम्बारता बंटन (समूहन विधि) - किसी कालेज में छात्रों द्वारा प्राप्त अंक नीचे दिये गये हैं। इनका बहुलांक ज्ञात करिये।

प्राप्तांक	१४	१५	१६	१७	१८	१९	२०	२१	२२	२३
बारम्बारता	१५	२८	४०	३६	४०	२८	[illegible]	१३	७	४

हम जानते हैं कि बहुलांक वह अंक है जिसकी बारम्बारता सबसे अधिक हो। उपरोक्त बंटन में सबसे अधिक बारम्बारता (४०) दो अंकों की है। अतः इनमें से कौन सा अंक बहुलांक होगा, यह निश्चित करना कठिन होगा। ऐसी स्थिति में हम समूहन विधि से बहुलांक ज्ञात करते हैं। इस विधि में सारणी में बारम्बारता के प्रथम स्तम्भ में प्रत्येक प्राप्तांक की बारम्बारता के प्रथम स्तम्भ में प्रत्येक प्राप्तांक की बारम्बारता दी हुई है। दूसरे स्तम्भ में २-२ बारम्बारता को जोड़कर समूह बनाये जाते हैं। तीसरे में प्रथम बारम्बारता को छोड़कर २-२ बारम्बारता को जोड़कर समूह बनाये जाते हैं। चौथे स्तम्भ में पहली बारम्बारता से शुरू करके ३-३ बारम्बारता को जोड़कर समूह बनाये जाते हैं। इसी प्रकार पांचवें व छठे स्तम्भ में क्रमशः पहली बारम्बारता को छोड़कर व पहली दो बारम्बारता को छोड़कर ३-३ बारम्बारता के समूह बनाते हैं। इस प्रकार उपरोक्त सारणी की समूहन सारणी निम्न प्रकार बन जाती है।

समूहन सारणी

प्राप्तांक	१-१ की	२-२ की			३-३ की	
१०	१५					
		४३		८३		
१५	८					
			८८			
					१०४	
१६	४०					
१७	३६	७६		११४		११६
			७६			
१८	४०					
१९	३८	७८				
					६८	७१
			५८			
२०	२०					
२१	१३	३३				
२२	७		२०	४०		
२३	४	११			२४	

अब हम इस संतुलन धारणा के आधार पर यह देखते हैं कि प्रत्येक स्तम्भ में किस प्राप्तांक की बारम्बारता सबसे अधिक है। इसको विश्लेषण धारणा में रख देते हैं।

विश्लेषण धारणी

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारम्बारता
स्तम्भ	१	२	३	४	५	६	७
१४							०
१५					|		१
१६	|				|	|	३
१७			|	|	|	|	४
१८	|	|	|	|		|	५
१९		|		|			२
२०							०
२१							०
२२							०
२३							०

उपरोक्त विश्लेषण सारणी से हम देखते हैं कि कि १८ की बारम्बारता सबसे अधिक है। अतः बहुलांक = १८

उदाहरण - कक्षा ६ के कुछ छात्रों के प्राप्तांक निम्नलिखित हैं। इनका बहुलांक ज्ञात कीजिए ?

प्राप्तांक (x)	५	६	७	८	९	१०	११	१२	१३	१४
बारम्बारता(f)	५	७	४	१३	१८	११	१८	७	६	८

उपर्युक्त बंटन में हम देखते हैं कि सबसे अधिक
रता वाले प्राप्तांक ९ व ११ हैं। ९ व ११ में से कौन सा प्राप्तांक ब
है। यह हम समूहन विधि से ज्ञात करते हैं। समूहन विधि में सबसे पहि
दिये बंटन में बारम्बारताओं के युग्म बनाकर समूहन सारणी बनाते है

समूहन सारणी

प्राप्तांक स्तम्भ	१-१ की १	२ की २	३	४	३-३ की ५	६
४	५	१२		१६		
६	७		११		२४	
७	४	१७				३५
८	१३		३१	४२		
९	१८	२९			४७	
१०	११		२९			३६
११	१८	२५		३४		
१२	७		१६		२४	
१३	९	१७				
१४	८					

अब उपरोक्त समूहन सारणी के आधार पर

सारणी बनाते हैं।

विश्लेषण सारणी

प्राप्तांक स्तम्भ	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह १	२	३	४	५	६	बारम्बारता ७
४							०
६							०
७							०
८			|	|			२
९	|	|	|	|	|		५
१०		|		|	|	|	४
११	|				|	|	३
१२	||					|	१
१३							०
१४							०

विश्लेषण सारणी के स्तम्भ ७ से हम देखते हैं कि प्राप्तांक ९ की बारम्बारता सबसे अधिक है। अतः बहुलांक = ९

इस प्रकार समूहन विधि से बहुलांक ज्ञात करने में सर्वप्रथम समूहन सारणी तथा फिर विश्लेषण सारणी बनाते हैं।

अभ्यास प्रश्न- एक कक्षा के ७० छात्रों ने वार्षिक परीक्षा में निम्न अंक प्राप्त किये। इनका बहुलांक ज्ञात कीजिए ?

प्राप्तांक (X)	१०	१२	१४	१६	२०	२१	२३	२८
बारम्बारता (f)	५	७	११	१३	८	१३	४	६

व्यवस्थित आंकड़ों में बहुलांक (ब) वर्ग अन्तराल में

किसी कक्षा के छात्रों का किसी विषय में प्राप्तांकों का वंटन निम्न है। इनका बहुलांक ज्ञात कीजिए।

वर्गअन्तराल	२०-२५	२५-३०	३०-३५	३५-४०	४०-४५
बारम्बारता	५	३	७	४	५

क्योंकि बहुलांक वह अंक होता है जिसकी बारम्बारता सबसे अधिक है। उपरोक्त सारणी में वर्ग अन्तराल ३०-३५ की बारम्बारता सबसे अधिक है। अतः बहुलांक का मान इसी वर्गान्तर में होगा। इस वर्ग अन्तराल में कौन सा मान बहुलांक होगा यह हम निम्न सूत्र से ज्ञात करते हैं।

$$\text{बहुलांक} = l + \frac{f - f_1}{2f - f_1 - f_2} \times i$$

जहाँ l = वर्गअन्तराल की निम्न सीमा, f = वर्गअन्तराल की बारम्बारता

f_1 = वर्गअन्तराल से पहिले वर्ग की बारम्बारता तथा f_2 = वर्ग अन्तराल से बाद के वर्गअन्तराल की बारम्बारता है।

उपरोक्त बंटन मे $l = ३०$, $f = ७$, $f_1 = ३$, $f_2 = ४$ तथा $i = ५$ है

अत: बहुलांक $= ३० + \dfrac{(७-३)}{(१४.३ - ४)} \times ५$

$$= ३० + \frac{४}{७} \times ५ = ३० + \frac{२०}{७} = ३२.८$$

उदाहरणत: निम्न आंकड़ों का बहुलांक ज्ञात कीजिए ?

वर्गान्तराल	५-१०	१०-१५	१५-२०	२०-२५	२५-३०	३०-३५
बारम्बारता	५	७	१३	२१	१६	८

उपरोक्त सारणी में वर्गान्तराल २०-२५ की बारम्बारता सबसे अधिक है। अत: बहुलांक वर्ग २०-२५

$$\text{बहुलांक} = l + \frac{f - f_1}{2f - f_1 - f_2} \times i$$

$$= २० + \frac{(२१-१३)}{(४२ - १३ - १६)} \times ५$$

$$= २० + \frac{८ \times ५}{१३} = २० + \frac{४०}{१३}$$

$$= २३.१$$

अभ्यास प्रश्न : निम्न बंटन का बहुलांक ज्ञात कीजिए ?

वर्ग अन्तराल (C.I.)	०-१०	१०-२०	२०-३०	३०-४०	४०-५०	५०-६०
बारम्बारता (f)	७	१०	१६	१५	८	६

APPENDIX-D

STATISTICS ACHIEVEMENT TEST

साँख्यिकी उपलब्धि परीक्षण

नाम.. प्राप्तांक..................................

उम्र..

कक्षा.................................. वर्ग...

स्कूल..

दिनांक..

निर्देश

1. जब तक आप से न कहा जाय, कृपया पन्ना न पलटिये ।

2. सम्पूर्ण प्रश्न पत्र को दो भागों में बांटदिया गया है । प्रत्येक भाग के प्रश्नों को हल करने के निर्देश उस भाग के प्रारम्भ में दिये गये हैं ।

3. प्रारम्भ से प्रश्न पढ़ते जाइये तथा उनके उत्तर देते जाइये ।

4. जो प्रश्न कठिन मालूम हो, उन पर समय नष्ट न कीजिये । पहले सरल प्रश्न कीजिये । यदि समय बचे तो कठिन प्रश्न बाद में कीजिये ।

5. आपको सभी प्रश्नों के उत्तर देने हैं । किन्तु बिना सोचे समझे केवल अनुभव से उत्तर मत दीजिये, क्योंकि गलत उत्तरों के अंक काट लिये जाते हैं ।

6. रफ कार्य के लिये अन्तिम पृष्ठ का प्रयोग कीजिये ।

7. यदि सभी प्रश्न करने के बाद भी समय बचे तो अपने उत्तरों को दोहराइये ।

8. इस प्रश्न पत्र के लिये केवल 60 मिनट हैं । अत: इन प्रश्नों का शीघ्र से शीघ्र उत्तर देने का प्रयत्न कीजिये ।

9. अब पन्ना पलटिये और कार्य प्रारम्भ कीजिये ।

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खण्ड-१

निर्देश :—अगले पृष्ठ पर कुछ प्रश्न दिये हुये हैं। प्रत्येक प्रश्न के दो या दो से अधिक संभावित उत्तर दिये हैं। उन में से केवल एक ही उत्तर सही है। इन प्रश्नों को ध्यान से पढ़िये और जिस उत्तर को आप सबसे अधिक सही समझे उस पर यह निशान (√) लगाइये।

उदाहरण प्रश्न–प्राप्तांक 10, 12, 18, 20 किस क्रम में हैं।

अ—आरोही
ब—अवरोही
स—किसी क्रम में नहीं

} इन उत्तरों में आरोही सही है। अत: यह (√) लगा दिया है।

केवल एक ही उत्तर पर निशान (√) लगाइये। यदि आप एक से अधिक उत्तरों पर निशान लगायेंगे तो उत्तर गलत समझा जावेगा।

(1) प्राप्तांक 15, 13, 8, 6, 4, 3, 2 व 1 किस क्रम में हैं ?

√अ-आरोही क्रम
ब-अवरोही क्रम
स-किसी क्रम में नहीं

(2) निम्नलिखित प्राप्तांक किस क्रम में हैं ?
47, 49, 50, 51, 54 55, 57, 59, 61

अ-आरोही क्रम
ब-अवरोही क्रम
स-किसी क्रम में नहीं

(3) निम्नलिखित प्राप्तांक किस क्रम में है ?
9, 8, 11, 16, 19, 9, 5, 4

अ-आरोही क्रम
ब-अवरोही क्रम
स-किसी क्रम मे नहीं

(4) +48, +16 तथा +22 का बीजगणितीय योग क्या होगा ?

अ-10
ब-86
स-उपरोक्त दोनों में से कोई नहीं।

(5) +68 और −44 का बीजगणितीय योग क्या होगा ?

अ- +112
ब- −112
स- +24
द- −24

(6) निम्न सख्याओं का बीजगणितीय योग बताइये ?
+14 +11, −9, −18, −26, −36

अ- +114
ब- −114
स- +64
द- −64

(7) वे संख्यायें जो दो से विभाजित हो जाती हैं कैसी संख्याएँ कहलाती हैं ?

अ- सम
ब- विषम
स- रुढ़

(8) 28 कैसी संख्या है ?

अ- सम
ब- विषम
स- रुढ़

(9) 15 कैसी संख्या है ?

अ- सम
ब- विषम
स- रुढ़

(10) चिन्ह Σ क्या प्रदर्शित करता है ?
अ- योग
ब- अन्तर
से- गुणनफल.

(11) चिन्ह ΣX क्या प्रदर्शित करता है ?
अ- एक पद का मान
ब- कुल पदों के मानो का योग
स- कुल पदों के मानों का गुणनफल

(12) अंकगणित में औसत ज्ञात करने का क्या सूत्र है ?
अ- $\frac{\Sigma X}{N}$
ब- ΣX
स- $\frac{X}{N}$

(13) एक कक्षा के तीन छात्रों ने क्रमश : 1, 5 व 6 अंक प्राप्त किये। इनका औसत क्या होगा ?
अ- 4
ब- 5
स- 12
द- उपरोक्त में से कोई नहीं।

(14) एक कक्षा के पाँच छात्रों ने क्रमश :
5, 10, 15, 20 व 30 अंक प्राप्त किये।
इनका औसत क्या होगा ?
अ- 15
ब- 16
स- 80
द- उपरोक्त में से कोई नहीं।

(15) एक परीक्षा में कुछ छात्रों के प्राप्तांक निम्नलिखित दो विधियों से प्रस्तुत किये गये हैं

विधि-I 5, 7, 7, 11, 16, 16,

विधि-II

प्राप्तांक	बारम्बारता
5	1
7	2
11	1
16	2

(ii) विधि-I के प्राप्तांक किस प्रकार हैं ?
अ- अव्यवस्थित
ब- व्यवस्थित

(ii) विधि-II के प्राप्तांक किस प्रकार हैं ?
अ- अव्यवस्थित
ब- व्यवस्थित

(iii) विधि-I में प्राप्तांक 9 की बारम्बारता कितनी हैं ?
अ- एक
ब- दो

(IV) विधि-II में प्राप्तांक 11 की बारम्बारता कितनी है ?
अ- एक
ब- दो

(16) एक परीक्षा में कुछ छात्रों के प्राप्तांक निम्न दो विधियों द्वारा प्रस्तुत किये गये हैं।

विधि- I 5, 5, 10, 10, 10, 19, 29, 29, 30, 40

विधि-II

प्राप्तांक	बारम्बारता
5	2
10	3
19	1
29	2
30	1
40	1

(i) विधि-I के प्राप्तांक किस प्रकार है ?
अ- अव्यवस्थित
ब- व्यवस्थित

(ii) विधि-II के प्राप्तांक किस प्रकार हैं ?
अ- अव्यवस्थित
ब- व्यवस्थित

(iii) विधि-I में प्राप्तांक 5 की बारम्बारता कितनी है ?

अ- एक

ब- दो

(IV) विधि-II में प्राप्तांक 10 की बारम्बारता कितनी है ?

अ- एक

ब- दो

स- तीन

(17) एक परीक्षा में कुछ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न प्रकार दिया हुआ है :

प्राप्तांक	बारम्बारता
9	6
11	5
12	8
15	2
20	7

(i) उपरोक्त बंटन में दिये गये प्राप्तांक किस प्रकार है ?

अ—अव्यवस्थित

ब—व्यवस्थित

(ii) उपरोक्त बंटन में कितने छात्रों के प्राप्तांक 12 हैं ?

अ- 8

ब- 9

स-11

(iii) उपरोक्त बंटन में N का मान क्या है ?

अ- 8

ब-20

स-28

(18) एक परीक्षा में कुछ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न है—

प्राप्तांक	बारम्बारता
11	3
12	6
14	7
16	9
18	2
20	1
21	4
22	3

(i) उपरोक्त प्राप्तांक किस प्रकार हैं ?

अ-अव्यवस्थित

ब-व्यवस्थित

(ii) उपरोक्त बंटन में कितने छात्रों के प्राप्तांक 14 हैं ?

अ- 7

ब- 9

स-19

(iii) उपरोक्त बंटन में N का मान क्या है ?

अ-22

ब-35

(19) एक परीक्षा में 23 छात्रों ने निम्न प्रकार अंक प्राप्त किये—

प्राप्तांक	0–10	10–20	20–30	30–40
बारम्बारता	4	2	9	8

(i) उपरोक्त बटन में प्राप्तांक किस प्रकार है ?

अ- अव्यवस्थित

ब- व्यवस्थित

स- व्यवस्थित-वर्ग अन्तराल

(ii) उपरोक्त बंटन में वर्ग अन्तराल 10–20 की निम्न सीमा क्या है ?

अ–10

ब–15

स–20

(iii) उपरोक्त बंटन में वर्ग अन्तराल 20–30 की उच्च सीमा क्या है ?

अ–20

ब–25

स–30

(iv) उपरोक्त बटन में वर्ग अन्तरालों का वर्ग विस्तार क्या है ?

अ- 9

ब–10

स–11

(v) उपरोक्त बंटन में कितने छात्रों ने 10 से 20 के बीच अंक पाये हैं ?

अ–2

ब–8

स–9

(20) एक परीक्षा में 65 छात्रों के प्राप्तांकों का बटन निम्न है—

प्राप्तांक	5–10	10–15	15–20	20–25
बारम्बारता	8	12	20	16

(i) उपरोक्त बंटन में दिये प्राप्तांक किस प्रकार है ?

अ- अव्यवस्थित

ब- व्यवस्थित

स- व्यवस्थित–वर्ग अन्तराल

(ii) उपरोक्त बंटन में वर्ग अन्तराल 10–15 की निम्न सीमा क्या है ?

अ- 10

ब- 12.5

स- 15

(iii) उपरोक्त बंटन में वर्ग अन्तराल 15–20 की उच्च सीमा क्या है ?

अ- 15

ब- 17.5

स- 20

(iv) उपरोक्त बंटन में वर्ग अन्तरालों का वर्ग विस्तार क्या है ?

अ- 4

ब- 5

स- 6

(v) उपरोक्त बटन में कितसे छात्रों ने 10–15 तक अंक प्राप्त किये हैं ?

अ- 8

ब- 12

स- 20

खण्ड २

भाग (1)

नीचे कुछ प्रश्न दिये हुये हैं। प्रत्येक प्रश्न के दो या दो से अधिक संभावित उत्तर दिये हैं। इनमें से केवल एक उत्तर ही सही है। इन प्रश्नों को ध्यान से पढ़िये और जिस उत्तर को आप सबसे सही समझे उस पर यह (√) निशान लगा दीजिये।

उदा० प्रश्न—प्राप्तांक 10, 18, 20 किस क्रम में है?

√अ- आरोही

ब- अवरोही

स- किसी क्रम में नहीं

इन उत्तरों में ....आरोही.... सही है। अतः इस पर निशान लगा दिया है। केवल एक ही उत्तर पर निशान (√) लगाइये। यदि आप एक से अधिक उत्तरों पर निशान लगायेंगे तो उत्तर गलत समझा जावेगा।

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(1) सांख्यिकी में औसत क्या कहलाता है?

अ- मध्यमान

ब- मध्यांक

स- बहुलांक

द- मध्य मूल्य

(2) अव्यवस्थित आंकड़ों में मध्यमान ज्ञात करने का क्या सूत्र है?

अ- $\frac{\Sigma X}{N}$ जहाँ X = प्राप्तांक तथा

N = छात्रों की संख्या

ब- ΣX

स- $\frac{X}{N}$

(3) एक परीक्षा में 5 छात्रों ने 11, 15, 18, 21 व 30 अंक प्राप्त किये। इनका मध्यमान क्या होगा?

अ- 18

ब- 19

स- 95

द- उपरोक्त में से कोई नहीं।

(4) एक परीक्षा में कुछ छात्रों के प्राप्तांक 5, 8, 15, 16, 20, 20 व 21 हैं। इनका मध्यमान क्या होगा?

अ- 15

ब- 16

स- 104

द- उपरोक्त में से कोई नहीं।

(5) किसी बारम्बारता बंटन में मध्यमान ज्ञात करने का सूत्र क्या है?

अ - $\frac{fX}{N}$

ब - $\frac{\Sigma fX}{N}$

स- ΣfX

(6) मध्य मूल्य (x) ज्ञात करने का क्या सूत्र है?

अ- उच्च सीमा+निम्न सीमा

ब- $\frac{\text{उच्च सीमा} - \text{निम्न सीमा}}{2}$

स- $\frac{\text{उच्च सीमा} + \text{निम्न सीमा}}{2}$

(7) प्राप्तांक वर्ग अन्तरालों में दिये होने पर मध्यमान ज्ञात करने का क्या सूत्र है?

अ- $\frac{\Sigma fX}{N}$

ब- $\frac{\Sigma fx}{N}$

स- $\frac{fx}{N}$

जहाँ X = प्राप्तांक तथा x = मध्यमूल्य

(8) जब f तथा × के गुणनफल बड़े-बड़े हो तो मध्यमान किस विधि से ज्ञात करते हैं ?

अ- दीर्घ विधि

ब-लघु विधि

(9) लघु विधि से मध्यमान ज्ञात करने का सूत्र क्या है ?

अ- $A + \frac{\Sigma fd}{N} xi$

ब- $1 + \left(\frac{N/2 - F}{f}\right) xi$

स- $1 + \left(\frac{f - f_1}{2f - f_1 - f_2}\right) xi$

जहाँ A = कल्पित मध्यमान

तथा 1 = निम्न सीमा है।

(10) सबसे अधिक उपयुक्त कल्पित मध्यमान किस वर्ग अन्तराल का मध्यमूल्य होता है ?

अ-कम बारम्बारता वाला सबसे ऊपर का वर्ग अन्तराल

ब-कम बारम्बारता वाला सबसे नीचे का वर्ग अन्तराल

स-अधिक बारम्बारता वाला बीच का वर्ग अन्तराल

(11)

वर्ग अन्तराल	बारम्बारता
0-10	2
10-20	3
20-30	6
30-40	3
40-50	2

उपरोक्त बटन में सबसे अधिक उपयुक्त कल्पित मध्यमान क्या होगा ?

अ- 5

ब-25

स-35

(12) किसी कक्षा या समूह का वह अंक जिसके ऊपर 50% छात्रों के अंक होते हैं क्या कहलाता है ?

अ-मध्यमान

ब-मध्यांक

स-बहुलांक

(13) निम्न प्राप्तांकों का मध्यांक क्या होगा ?
6, 6, 8, 12, 13

अ- 6

ब- 8

स- 9

(14) निम्न प्राप्तांकों का मध्यांक क्या होगा ?
2, 4, 6, 8, 9, 10, 13, 14, 15, 16, 19, 20

अ- 10

ब- 11

स- 15

(15) मध्यांक का मान बीच के दो छात्रों के प्राप्तांकों का औसत कब होता है ?

अ- जब N सम हो।

ब- जब N विषम हो।

स- जब N रुढ़ हो।

(16) राम ने प्रत्येक विषय में निम्न अंक प्राप्त किये—

7, 8, 9, 11, 13, 16 इनका मध्यांक क्या होगा ?

अ-9

ब-10

स-10.5

द-11

(17) एक कक्षा के कुछ छात्रों के प्राप्तांक निम्न हैं। इनका मध्यांक बताइयें ?

11, 12, 13, 15, 18, 19, 21, 25, 30, 31

अ–18
ब–18.5
स–19
द–19.5

(18) किसी बंटन में मध्यांक कौन से प्राप्तांक होते हैं ?

अ–$N+\frac{1}{2}$ वें

ब–$\frac{N+1}{2}$ वें

स–$\frac{N-1}{2}$ वें

(19) मध्यांक वर्ग ज्ञात होने पर मध्यांक निकालने का सूत्र क्या है ?

अ–$A+\frac{\Sigma fd}{N}\times i$

ब–$l+\left(\frac{N/_2-F}{f}\right)\times i$

स–$l+\left(\frac{f-f_1}{2l-f_1-f_2}\right)\times i$

जहाँ A=कल्पित मध्यमान
तथा l=निम्न सीमा है।

(20) किसी समूह या कक्षा में बहुलांक कौनसा अंक होता है ?

अ–जो समूह में सबसे बड़ा हो
ब–जो समूह में सब से अधिक बार आया हो।
स–जो समूह के बीच में हो।

(21) निम्न प्राप्तांकों का बहुलांक क्या होगा ?
9, 8, 7, 3, 3

अ–3
ब–6
स–7
द–9

(22) निम्न अंकों का बहुलांक क्या होगा ?
5, 7, 13, 13, 13, 17, 17, 29, 31, 35

अ–13
ब–17
स–18
द–35

(23) निम्न बंटन का बहुलांक क्या होगा ?

प्राप्तांक	बारम्बारता
7	5
9	4
10	8
15	3
16	1

अ–8
ब–10
स–16

(24) निम्न बंटन का बहुलांक क्या होगा ?

प्राप्तांक	बारम्बारता
5	3
6	7
7	5
8	4
9	10
10	15
11	12
12	6
13	9
14	2

अ–10
ब–14
स–15

(25) किसी बहुलांक वर्ग में बहुलांक ज्ञात करने का क्या सूत्र है ?

अ- $A+\frac{\Sigma fd}{N}\times i$

ब- $l+\left(\frac{N/2-F}{f}\right)\times i$

स- $l+\left(\frac{f-f_1}{2f-f_1-f_2}\right)\times i$

जहाँ A=कल्पित मध्यमान तथा l=निम्न सीमा

भाग (2)

नीचे कुछ प्रश्न दिये हैं। प्रत्येक प्रश्न के सामने रिक्त स्थान दिया है। इन प्रश्नों को ध्यान से पढ़िये तथा रिक्त स्थान पर उनका उत्तर लिखिये।

उदा० प्रश्न—14,20,25, 30 का योग क्या होगा ?

योग=..............

उपरोक्त प्रश्न में दो संख्यओं का योग 89 होगा अत: रिक्त स्थान में योग=89 लिख दिया जावेगा।

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(26) निम्न बटन में कुछ छात्रों के प्राप्तांक दिये हैं—

प्राप्तांक (×)	बारम्बारता (f)	प्राप्तांकों का योग (f×)
3	1	....
5	2	....
6	3	....
8	1	....
9	1	....
	N=8	Σf× =........

(i) उपरोक्त बंटन में f× के मान रिक्त स्थान में भरिये।

(ii) Σf× का मान भरिये।

(iii) उपरोक्त बंटन में मध्यमान क्या होगा ?

मध्यमान=..................

(27) निम्न बंटन में कुछ छात्रों के प्राप्तांक दिये हैं।

प्राप्तांक (×)	बारम्बारता (f)	प्राप्त अंकों का योग (f×)
7	3	....
11	2	....
15	4	....
17	6	....
20	4	...
23	5	....
25	1	....
	N=25	Σf× =....

(i) उपरोक्त बटन में f× तथा Σf× के मान भरिये ?

(ii) उपरोक्त बंटन में मध्यमान क्या होगा ?

मध्यमान=....

(28)

वर्ग अन्तराल	बार०	मध्यमूल्य	बार०×मध्यमूल्य
0–10	4	....	...
10–20	5	....	....
20–30	3	....	....

(i) उपरोक्त बंटन के स्तम्भ 3 में मध्यमूल्य भरिये ?

(ii) उपरोक्त बंटन में वर्ग अन्तराल 10-20 की बारम्बारता 5 है। इस वर्ग अन्तराल में इन 5 छात्रों के अंकों का योग क्या होगा ?

5 छात्रों द्वारा प्राप्त अंकों का योग=....

(iii) उपरोक्त बंटन में सभी 12 छात्रों द्वारा प्राप्त अंकों का कुल योग क्या होगा ?

प्राप्त अंकों का कुल योग=........

(29) कुछ छात्रों के द्वारा वार्षिक परीक्षा में प्राप्त अंकों का बंटन निम्न प्रकार है।

वर्ग अन्त० (C. I.)	बार० (f)	मध्य मूल्य (x)	बार० मध्य रूप (fx)
0–10	1	....	....
10–20	1	....	....
20–30	3	....	....
30–40	2	....	....
			Σfx=........

(i) उपरोक्त बंटन के स्तम्भ 3 व 4 को भरिये ?

(ii) उपरोक्त बंटन में रिक्त स्थान पर Σfx का मान भरिये ?

(iii) उपरोक्त बंटन से मध्यमान क्या होगा ?

मध्यमान=........

(30) एक समूह के प्राप्तांकों का बटन निम्न प्रकार है।

वर्ग अन्तराल (C. I.)	बार० (f)	मध्य मूल्य (x)	बार० × मध्य मूल्य (fx)
10–20	6	....	....
20–30	2	....	....
30–40	10	....	....
40–50	3	....	....
50-60	4	....	....
	N=25		Σfx=......

(i) उपरोक्त बंटन के स्तम्भ 3 व 4 में क्रमशः x तथा fx भरिये ?

(ii) उपरोक्त बंटन में Σfx का मान रिक्त स्थान में भरिये।

(iii) उपरोक्त बंटन का मध्यमान क्या होगा ?

मध्यमान=........

(31)

वर्ग अन्त० (C.I.)	बार० (f)	विचलन (d)	बार० × विचलन (fd)
0-10	2	....	....
10-20	4	....	....
20-30	9	...	....
30-40	5	....	....
			Σfd=........

उपरोक्त बटन में वर्ग अन्तराल 20-30 लगभग बीच का तथा सबसे अधिक बारम्बारता वाला वर्ग अन्तराल है। अतः कल्पित मध्यमान =25 होगा।

(i) उपरोक्त बंटन के स्तम्भ 3 व 4 में d तथा fd भरिये ?

(ii) उपरोक्त बटन में रिक्त स्थान पर Σfd का मान भरिये।

(iii) उपरोक्त प्राप्तांकों का मध्यमान क्या होगा ?

मध्यमान=.......

(23)

वर्ग अन्त० (C.I.)	बार० (f)	विचलन (d)	बार० × विचलन (fd)
0-5	6	....	...
5-10	8	....	....
10-15	14	...	....
15-20	6	....	....
20-25	1	...	....
25-30	10	....	....
30-35	5	....	....
	N=50		Σfd=.......

(i) उपरोक्त बटन में सबसे अधिक उपयुक्त कल्पित मध्यमान क्या होगा ?

कल्पित मध्यमान=............

(ii) उपरोक्त बटन में विचलन (d) तथा बार० एवं विचलन के गुणनफल (fd) के मान भरिये।

(iii) उपरोक्त बंटन Σfd का मान भरिये ?

(iv) उपरोक्त प्राप्तांकों का मध्यमान क्या होगा ?

मध्यमान=..................................

(33) एक समूह द्वारा प्राप्त अंकों का बंटन निम्न है :—

प्राप्तांक	बारम्बारता	संचयी बारम्बारता
5	3	....
6	8	....
7	9	....
8	4	....
9	2	....
	N=	

(i) उपरोक्त बंटन के स्तम्भ 3 मे संचयी बारम्बारता भरिये ?

(ii) उपरोक्त बंटन में मध्यांक क्या होगा ?
मध्यांक=........................

(34) एक समूह के प्राप्तांकों का बंटन निम्न है।

प्राप्तांक	बारम्बारता	सचयी बारम्बारता
10	4	....
13	6	....
15	5	....
18	8	....
21	4	....
23	1	....
25	2	....
	N=30	

(i) उपरोक्त बंटन के स्तम्भ 3 में संचयी बारम्बारता भरिये ?

(ii) उपरोक्त बंटन में मध्यांक क्या होगा ?
मध्यांक=..................

(35) एक परीक्षा में कुछ छात्रों ने निम्न प्रकार अंक प्राप्त किये—

वर्ग अन्तराल	बारम्बारता	संचयी बारम्बारता
0–5	4	....
5–10	6	....
10–15	7	....
15–20	3	....
	N=20	

(i) उपरोक्त बंटन में संचयी बारम्बारता भरिये ?

(ii) उपरोक्त बंटन में मध्यांक वर्ग कौन सा है ?
मध्यांक वर्ग= ....

(iii) उपरोक्त मध्यांक वर्ग में मध्यांक का मान क्या होगा ?
मध्यांक= ....

(36) निम्न बंटन में 50 छात्रों के प्राप्तांक दिये हैं—

वर्ग अन्तराल	बारम्बारता	सचयी बारम्बारता
10–20	4	....
20–30	6	....
30–40	15	....
40–50	17	...
50–60	5	....
60–70	3	....
	N=50	

(i) उपरोक्त बंटन के स्तम्भ 3 में संचयी बारम्बारता भरिये ?

(ii) उपरोक्त बंटन में मध्यांक वर्ग कौनसा है ?
मध्यांक वर्ग= ....

(iii) उपरोक्त बंटन का मध्यांक क्या होगा ?
मध्यांक= ....

(37) एक परीक्षा में छात्रों द्वारा प्राप्तांकों का बंटन निम्न है।

वर्ग अन्तराल	बारम्बारता
0-10	5
10-20	6
20-30	7
30-40	2

(i) उपरोक्त बंटन में बहुलांक वर्ग कौन सा होगा?
बहुलांक वर्ग= ....
(ii) उपरोक्त बंटन में बहुलांक क्या होगा?
बहुलांक= ....

(38)

वर्ग अन्तराल	बारम्बारता
5-10	7
10-15	4
15-20	1
20-25	9
25-30	3
30-35	2

(i) उपरोक्त बंटन में बहुलांक वर्ग क्या होगा?
बहुलांक वर्ग= ....
(ii) उपरोक्त बंटन में बहुलांक क्या होगा?
बहुलांक= ....

(39) कुछ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न प्रकार है—

प्राप्तांक	बारम्बारता
7	6
8	9
11	12
15	12
18	8

(i) उपरोक्त बंटन में बहुलांक किस विधि से ज्ञात किया जायेगा?
....विधि।

(ii) उपरोक्त बंटन से निम्न समूहन सारिणी पूर्ण कीजिये?

प्राप्तांक स्तम्भ →	एक-एक की	दो-दो की		तीन-तीन की		
	1	2	3	4	5	6
7 8 11 15 18						

(iii) उपरोक्त समूहन सारिणी से निम्न विश्लेषण सारिणी पूर्ण कीजिये?

प्राप्तांक स्तम्भ →	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारम्बारता
	1	2	3	4	5	6	7
7 8 11 15 18							

(IV) उपरोक्त विश्लोषण सारिणी से बहुलांक क्या होगा ?

बहुलांक=........

(40) कुछ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न है—

प्राप्तांक	बारम्बारता	प्राप्तांक	बारम्बारता
16	5	25	14
18	6	27	6
19	14	30	2
22	13		

(i) उपरोक्त बंटन में बहुलांक किस विधि से ज्ञात होगा ?

.................................विधि।

(ii) उपरोक्त विधि से बहुलांक ज्ञात करने के लिये निम्न समूहन सारिणी पूर्ण कीजिये ?

प्राप्तांक ↓ स्तम्भ →	एक एक की	दो दो की		तीन तीन की		
	1	2	3	4	5	6
16						
18						
19						
22						
25						
27						
30						

(iii) उपरोक्त समूहन सारिणी के आधार पर निम्न विश्लेषण सारिणी पूर्ण किजिये ?

प्राप्तांक ↓ स्तम्भ →	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारम्बारता
	1	2	3	4	5	6	7
16							
18							
19							
22							
25							
27							
30							

(iv) उपरोक्त विश्लेषण सारणी से बहुलांक क्या होगा ?

बहुलांक=

APPENDIX - E

Linear Style of Programming

(A) Deviation Values for Total and Within Subgroups:

y^2 total

$$y^2 = Y^2 - \frac{Y}{n}^2 = 147746 - \frac{2692}{58}^2 = 22800.07$$

y^2 within

$$y^2 = Y^2 - \frac{Y}{n_1} + \frac{I}{n} = 147746 - \frac{1478}{29} + \frac{1214}{29} = 21598.42$$

x_1^2 total

$$x_1^2 = X_1^2 - \frac{X_1}{n}^2 = 60506 - \frac{1666}{58}^2 = 12651.59$$

x_1^2 within

$$x_1^2 = X_1^2 - \frac{X_{12}^2}{n_{12}} + \frac{x_{14}}{n_{14}}^2 = 60506 - \frac{898}{29}^2 + \frac{768}{29}^2 = 12360.21$$

x_1y total

$$x_1y = X_1Y - \frac{(X_1)(Y)}{n} = 73507 - \frac{1666 \text{ X } 2692}{58} = -3818.38$$

x_1y within

$$x_1y = X_1Y - \frac{(X_{1H})(Y_H)}{n_H} + \frac{(X_{1L})(Y_L)}{n_L}$$

$$= 73507 - \frac{989 \text{ X } 1478}{29} + \frac{768 \text{ X } 1214}{29} = -9047.97$$

(B) Regression Equation

Total,

$$x_1 y = x_1^2$$

$$\therefore \quad a = \frac{x_1 y}{x_1^2} = -0.3018$$

Within,

$$x_1 y = a \quad x_1^2$$

$$\therefore \quad a = \frac{x_1 y}{x_1^2} = \frac{-9047.97}{12360.21} = -0.73$$

Thus regression equations are,

Total $Y = -0.3018\ X$ ------------(1)

Within $Y = -0.73\ X$ ------------(2)

(C) Sum of Squares of Residuals

Total sum of squares of residuals =

$$y^2 - \frac{(\ x_1 y)^2}{x_1^2} = 22800.07 - \frac{(-3818.38)^2}{12651.59} = 21647\text{-}64$$

Within sum of squares of residuals =

$$y^2 - \frac{(\ x_1 y)^2}{x_1^2} = 21598.42 - \frac{(-9047.97)^2}{12360.21} = 14975.69$$

(D) Adjustment Terms

$$\overline{Y}_H = (\ \overline{X}_H - \overline{X}_T)\ a = (30.97 - 28.735)\ (-0.73) = -1.63$$

$$\overline{Y}_L = (\ \overline{X}_L - \overline{X}_T)\ a = (26.48 - 28.735)(-0.73) = +1.65$$

(E) Adjusted Mean

High Group -- 50.97 - 1.63 = 48.34

Low Group --- 41.86 + 1.65 = 43.51

केन्द्रीय प्रवृत्ति के मान

(अभिक्रमित अध्ययन-लीनियर स्टाइल)

Measures of Central Tendencies

(A PROGRAMMED TEXT—LINEAR STYLE)

Developed by :

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नाम .... आयु ....

कक्षा .... समय ....

विद्यालय ....

12957

अध्यापकों से

स्व-अध्ययन की अनेक विधियां हैं जिसमें से अभिक्रमित अध्ययन विधि लीनियर स्टाइल (Programmed Learning-Linear Style) भी एक विधि है। इस विधि में बी० एफ० स्किनर द्वारा अपने क्रिया प्रसूत अनुबन्धन (Operant Conditioning) के सिद्धान्त में प्रतिपादित कुशल अध्ययन (Efficient Learning) के नियम का पूर्णरूप से उपयोग किया गया है। इस विधि में छात्र स्वयं सक्रिय रूप से भाग लेता है तथा उसे लगातार पुनर्बलन (Reinforcement) व तर्कसंगत प्रतिपुष्टि (Feedback) मिलता रहता है। उपरोक्त विधि में विषय वस्तु छोटे-छोटे खण्डों में इस प्रकार व्यवस्थित की जाती है कि छात्र द्वारा गलती करने के अवसर पूर्णरूप से समाप्त हो जाते हैं।

इस पुस्तक के अध्ययन के बाद छात्रः—

(१) केन्द्रीय प्रवृत्ति के विभिन्न मानों—मध्यमान, मध्यांक व बहुलांक के प्रत्यय तथा परिभाषा अच्छी प्रकार समझ जायेंगे।

(२) अव्यवस्थित तथा व्यवस्थित आँकड़ों में केन्द्रीय प्रवृत्ति के निम्नलिखित मानों की विभिन्न विधियों से गणना करना सीख जायेगें।

(अ) मध्यमान

(ब) मध्यांक

(स) बहुलांक

छात्रों से

प्रस्तुत पुस्तक मे हम आपको पढने के एक आधुनिक और आसान विधि के बारे में बतायेंगें । आगे आने वाले पृष्ठों में साँख्यिकी के 'केन्द्रीय प्रवृत्ति के मान' को इस तरह लिखा गया है जिससे आप स्वयं बिना किसी अध्यापक की सहायता से सुगमता से समझ जायेंगे । साथ-साथ यह आपको आसानी से याद भी हो जायगा । इस तरह अध्ययन करने की यह विधि अभिक्रमित अध्ययन विधि (लीनियर स्टाइल) कहलाती है ।

पूरे पाठ को छोटे-छोटे खण्डों में बाँट दिया गया है । प्रत्येक खण्ड में एक या अधिक रिक्त स्थान छोड़ दिये गये है जिसमें खण्ड को समझने के बाद सही उत्तर भर सकते है । प्रत्येक खण्ड का सही उत्तर उस पृष्ठ के पीछे पृष्ठ पर लिखा हुआ है । किन्तु यह उत्तर आप अपना उत्तर लिखने के बाद ही देखे । यदि आप ऐसा नहीं करेंगे तो आपको पाठ न तो समझ में ही आयेगा और न ही याद होगा । यदि आपका उत्तर पृष्ठ के पीछे लिखे सही उत्तर से नहीं मिलता तो उस खण्ड को पुनः समझने का प्रयत्न कीजिये और पहला उत्तर काटकर उसके ऊपर पुनः उत्तर लिखिये ।

यदि आप उपरोक्त विधि अपनायेगें तो पाठ निश्चित रूप से आपकी समझ में आ जायगा और साथ-साथ याद भी हो जायेगा ।

इस विधि से परिचय कराने के लिए एक उदाहरण दिया जा रहा है । अगले पृष्ठ पर 'औसत' को खण्डों में प्रस्तुत किया जा रहा है । खण्ड (१) को पढ़िये तथा उसमें दिये रिक्त स्थानों में उत्तर लिखिये । उत्तर लिखने के बाद अपने उत्तर को सही उत्तर जो कि पृष्ठ के पीछे दिया है से मिलाइये । जब आपका उत्तर पिछले पृष्ठ पर दिये उत्तर से मिल जाय तो अगले खण्ड के उत्तर भी इसी विधि से दीजिये ।

उदाहरण

(१) एक कक्षा के ६ छात्रों के प्राप्तांक क्रमशः ५, ७, १०, ११, १२ व १५ है।

उपरोक्त प्राप्तांकों का योग = ५ + ... + ... + ... + ...

= ....

iv]

(१) $= ५ + ७ + १० + ११ + १२ + १५$

$= ६०$

(२) बायें पृष्ठ से – ६ छात्रों के प्राप्तांकों का योग ६० है। अतः इनका

$$\text{ओसत} = \frac{\ldots\ \ldots\ldots\ \ldots}{\ldots\ldots\ldots\ldots}$$

$$= \ldots\ \ \ldots\ldots\ldots$$

प्रस्तावना

सांख्यकी में सामग्री संग्रहण का उद्देश्य किसी विषय के बारे में जानकारी प्राप्त करना होता है। पर संग्रहीत सामग्री की राशि अधिक होने के कारण उसे समझ सकना बहुत कठिन हो जाता है। अगर कोई ऐसी संख्या या प्राप्तांक जो इस समूह का स्तर प्रदर्शित करें, मालूम हो तो इस कठिनाई से बचा जा सकता है। सांख्यिकी में ऐसी संख्याओं को दिये गये समूह का 'केन्द्रीय प्रवृत्ति के मान कहते हैं।

अगले अध्याय में हम केन्द्रीय प्रवृत्ति के विभिन्न मानों—मध्यमान, मध्यांक व बहुलांक की परिभाषा के बारे में जानेंगे।

अब आप पृष्ठ १ के खण्ड (१) से अध्ययन करना शुरू कीजिये। इस पृष्ठ के पीछे पृष्ठ पर लिखी समीक्षा अध्याय-४ को आप अभी मत पढ़िये।

समीक्षा-अध्याय ४

उपरोक्त अध्याय में हमने विभिन्न प्रकार के आकड़ों में बहुलांक की गणना करना जाना है—

(अ) **अव्यवस्थित आकड़ों में**—जब प्राप्तांक अव्यवस्थित हो तो बहुलांक वह अंक होता है जो समूह में सबसे अधिक बार आता है।

(ब) **व्यवस्थित आकड़ों में**—

(i) जब प्राप्तांक बारम्बारता बंटन में दिये हों तो बहुलांक वह अंक होता है जिसकी बारम्बारता सबसे अधिक हो।

(ii) जब दिये बारम्बारता वटन मे बहुलांक दो अंक हो तो समूहन विधि से बहुलांक ज्ञात करते है। इस विधि में सर्व प्रथम समूहन तालिका बनाते हैं तथा फिर इसके आधार पर विश्लेषण सारणी बनाते हैं। विश्लेषण सारणी में जिस प्राप्तांक की बारम्बारता सबसे अधिक हो वही बहुलाँक होता है।

(iii) जब प्राप्ताँक वर्ग अन्तरालों में दिये हों तो पहले बहुलांक वर्ग ज्ञात करते हैं फिर निम्न सूत्र द्वारा बहुलांक की गणना कर लेते हैं

बहुलांक $= l + \left(\frac{f-f_1}{2f-f_1-f_2} \right) \times i$ जहाँ $l =$ बहुलांक वर्ग की निम्न सीमा, $f =$ बहुलांक वर्ग की बारम्बारता, $f_1 =$ बहुलांक वर्ग से पूर्व वर्ग की बारम्बारता तथा $f_2 =$ बहुलाँक वर्ग के बाद के वर्ग की बारम्बारता है।

इस प्रकार सम्पूर्ण पुस्तक में हमने केन्द्रीय प्रवृत्ति के तीनों मानों—मध्यमान, मध्याँक व बहुलाँक की परिभाषा तथा उनकी विभिन्न प्रकार के प्राप्ताँकों में गणना करना सीखा है।

अब यह ज्ञात करने के लिए कि आप केन्द्रीय प्रवृत्तियों के मानों की गणना सीखने में कितने सफल हुये हैं संलग्न प्रश्न पत्र के उत्तर ध्यान पूर्वक देने का प्रयत्न कीजिए।

अध्याय–1 केन्द्रीय प्रवृत्ति के विभिन्न मान

(१) किसी कक्षा के ५ छात्रों ने विज्ञान में क्रमशः १५, २०, ३०, ३५ व ३५ अंक प्राप्त किये ।

इन छात्रों द्वारा प्राप्त अंकों का औसत $= \dfrac{१५ + \cdots + \cdots + \cdots + \cdots}{५}$

$= \dfrac{\cdots\cdots}{५}$

$= \cdots\cdots$

प्रत्येक पृष्ठ के नीचे उलटे छपे हुए खण्ड को अभी आप न पढ़ें । इन खण्डों को खण्ड (१०) तक पढ़ लेने के बाद पुस्तक को उलट कर अध्ययन किया जायेगा ।

---

(१०८) २०–३०

बहुलाँक $= l + \left(\dfrac{f - f_1}{२f - f_1 - f_2}\right) \times i$

यहाँ $l = २०$, $f = १९$, $f_1 = १०$, $f_2 = १५$ तथा $i = १०$ है ।

अतः बहुलाँक $= २० + \left(\dfrac{१९ - १०}{३८ - १० - १५}\right) \times १०$

$= २० + \dfrac{९}{१३} \times १० = २० + ६.९$

$= २६.९$

(१००) निम्न श्रेणी में छात्रों के प्राप्तांक दिये हैं। इनका बहुलांक ज्ञात कीजिये ।

वर्ग अन्तराल (C.I.)	बारम्बारता (f)
०—१०	७
१०—२०	१०
२०—३०	१६
३०—४०	१५
४०—५०	८
५०—६०	६

(i) उपरोक्त बंटन में बहुलांक वर्ग=...............

(ii) बहुलांक=........ + $\left(\dfrac{\ldots\ldots\ldots\ldots}{\ldots\ldots\ldots\ldots}\right)\times$ ....

यहाँ l=......., f=......., f_1=......., f_2=....... तथा i=.......है।

अतः बहुलांक=....... + $\left(\dfrac{\ldots\ldots\ldots\ldots}{\ldots\ldots\ldots\ldots}\right)\times$ ....

=.................... = ...

=..........

---

(१) औसत $= \dfrac{१५+२०+३०+३५+३५}{५}$

$= \dfrac{१३५}{५}$

$=२७$

(२) पिछले खण्ड में २७ इन छात्रों के प्राप्तांकों का ············है । यह अंक २७ इन पाँचों छात्रो के प्राप्तांकों का स्तर प्रदर्शित करता है । ऐसा अंक जो कि समूह के कुल छात्रों के प्राप्तांकों का स्तर प्रदर्शित करता है, उस समूह का ····················कहलाता है ।

(६६) निम्नलिखित प्राप्तांकों का बहुलांक ज्ञात कीजिये ?

वर्ग अन्तराल (C.I.)	बारम्बारता (f)
५—१०	५
१०—१५	७
१५—२०	१३
२०—२५	२१
२५—३०	१६
३०—३५	८

उपरोक्त बंटन में वर्ग अन्तराल..............की बारम्बारता सबसे अधिक है। अत: बहुलांक वर्ग=............ ...

$$\text{बहुलांक वर्ग में बहुलांक} = l + \left(\frac{\cdots\cdots - \cdots\cdots}{२f - \cdots\cdots - \cdots\cdots}\right) \times i$$

उपरोक्त बंटन में,

$l=२०$, $f=२१$, $f_1=१३$, $f_2=१६$ तथा $i=५$

अत: बहुलांक $=२० + (\cdots\cdots\cdots\cdots\cdots\cdots) \times ५$

$=२० + \cdots\cdots\cdots\cdots\cdots$

$=\cdots\cdots\cdots\cdots\cdots\cdots$

४]

(२) औसत

औसत

(३) उपरोक्त विधि से ज्ञात किया हुआ औसत ही सांख्यिकी में मध्यमान कहलाता है। मध्यमान को M से प्रदर्शित करते है।

औसत ज्ञात करने के लिए समूह या कक्षा के छात्रों द्वारा प्राप्त अंकों के..................को उनकी..................से भाग देते है।

क्योकि औसत ही मध्यमान कहलाता है अतः मध्यमान ज्ञात करने का भी यही तरीका है।

अर्थात् मध्यमान $= \dfrac{\ldots\ldots\ldots\ldots\ldots\ldots}{\text{छात्रो की संख्या}}$

(९८) पिछले खण्ड (९७) मे बहुलांक वर्ग ३०–३५ है। इसमें कौन सा अंक बहुलांक होगा, यह हम निम्न सूत्र से ज्ञात करते हैं—

$$\text{बहुलांक} = l + \left(\frac{f - f_1}{२f - f_1 - f_2}\right) \times i$$

जहाँ l = बहुलांक वर्ग की निम्न सीमा

f = बहुलांक वर्ग की बारम्बारता

f_1 = बहुलांक वर्ग से पूर्व वर्ग की बारम्बारता

f_2 = बहुलांक वर्ग से बाद के वर्ग की बारम्बारता

i = वर्ग विस्तार

पिछले खण्ड से—

$l = ३०$, $f = ७$, $f_1 = ३$, $f_2 = ४$ तथा $i = ५$

$$\text{अत: बहुलांक} = l + \left(\frac{f - f_1}{२f - f_1 - f_2}\right) \times i$$

$$= ३० + \left(\frac{\ldots\ldots - \ldots\ldots}{\ldots\ldots - \ldots\ldots - \ldots\ldots}\right) \times ५$$

$$= ३० + \ldots\ldots\ldots\ldots$$

$$= \ldots\ldots\ldots\ldots$$

---

(३) योग, संख्या

$$\text{मध्यमान} = \frac{\text{छात्रों द्वारा प्राप्तांक अंको का योग}}{\text{छात्रो की संख्या}}$$

(४) नीचे कुछ छात्रों के प्राप्तांक दिये हैं—

१५, १८, २०, २२, २५, २७, २७, ३०, ३२

उपरोक्त प्राप्तांकों का मध्यमान

$$M = \frac{१५ + \cdots + \cdots + \cdots + \cdots + \cdots + \cdots + \cdots + \cdots}{९}$$

$$= \frac{\cdots\cdots\cdots}{९}$$

$$= \cdots\cdots\cdots\cdots$$

$$(४)\ M = \frac{१५+१८+२०+२२+२५+२७+२७+३०+३२}{९}$$

$$= \frac{२१६}{९}$$

$$= २४$$

---

(९७) किसी कक्षा के छात्रों का अंग्रेजी विषय में प्राप्तांकों का बंटन निम्न है। इनका बहुलांक ज्ञात कीजिये ?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)
२०—२५	३
२५—३०	३
३०—३५	७
३५—४०	४
४०—४५	५

बहुलांक वह अंक है जिसकी बारम्बारता.........................होती है।

उपरोक्त बंटन में वर्ग अन्तराल............की बारम्बारता सबसे अधिक है। अत: बहुलांक इसी वर्ग मे होगा । इस वर्ग को बहुलांक वर्ग कहते हैं। उपरोक्त बंटन में बहुलांक वर्ग=...........

(५) पिछले खण्ड में दिये निम्न प्राप्तांकों का पुनः अध्ययन कीजिये —

१५, १८, २०, २२, २५, २७, २७, ३०, ३२

इन प्राप्तांकों में बीच का अंक अर्थात् वह अंक जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के प्राप्तांक है, ज्ञात कीजिये।

यह अंक ....है।

(९६) द्वितीय सारणी में प्रत्येक स्तम्भ की ....
..................वाले प्राप्तांकों के सामने उस स्तम्भ के नीचे टैली चिन्ह लगा देते हैं।

फिर प्रत्येक प्राप्तांक की ....स्तम्भ–७ में लिख देते हैं। विश्लेषण सारणी में जिस प्राप्तांक की बारम्बारता............................ होती है वही..................होता है।

१०]

(५) २५

(६) साँख्यिकी में वह अंक जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के अंक होते है, मध्यांक कहलाता है। पिछले खण्ड (५) में दिये प्राप्ताँकों में अंक २५ के ऊपर ५०% या ४ तथा नीचे ५०% या ४ छात्रों के प्राप्ताँक है ।

अत: मध्याँक = ……………

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(६५) (१) समूहन सारणी

(२) विश्लेषण सारणी

प्रथम सारणी—

(१) बारम्बारता

(२) दो-दो, योग

(३) बारम्बारता, २–२, बारम्बारताओं का योग लिखते है ।

(४) प्रारम्भ से ३–३ प्राप्ताँको की बारम्बारताओं का योग लिखने हैं।

(५) प्रथम प्राप्ताँक की बारम्बारता को छोड़कर अगले ३–३ प्राप्तांकों की बारम्बारताओं का योग लिखते है ।

(६) प्रथम दो प्राप्ताँकों की बाम्बारताओ को छोड़कर अगले ३–३ प्राप्ताँको की बारम्बारताओं का योग लिखते है ।

(६५) जब बहुलांक दो अंकों में होते हैं तो इसमें समूहन विधि का प्रयोग होता है। इस विधि में दो सारणी तैयार की जाती हैं —

(१) ………………………………………………

(२) ………………………………………………

प्रथम सारणी तैयार करने में निम्न चरण होते हैं –

(१) स्तम्भ–१ में प्रत्येक प्राप्तांक को ……………… लिख देते हैं।

(२) स्तम्भ–२ में प्रारम्भ से ………… प्राप्तांकों की बारम्बारताओं का ……………… लिख देते हैं।

(३) स्तम्भ–३ में प्रथम प्राप्तांक की ………… छोड़कर अगले …… अंकों ……………………………… ।

(४) स्तम्भ–४ में ………………………………………… ……………………… ।

(५) स्तम्भ–५ में ………………………………………… ……………………… ।

(६) स्तम्भ–६ में ………………………………………… ……………………… ।

(८) एक कक्षा के कुछ छात्रो के प्राप्तांक निम्नलिखित हैं—

५, ७, १०, १८, २१, २५, २८

इनका मध्यांक =………….

(६४) बायें पृष्ठ की समूहन सारणी के आधार पर हम विश्लेषण सारणी बनाते है। निम्न विश्लेषण सारणी को बाये पृष्ठ की समूहन सारणी के आधार पर भरिये--

विश्लेषण सारणी --

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारंबारता
↓ स्तम्भ→	१	२	३	४	५	६	७
१०							
१२							
१६							
१९							
२०							
२१							
२३							
२८							

उपरोक्त विश्लेषण सारणी से हम देखते हैं कि प्राप्तांक.................. की बारम्बारता सबसे अधिक है।

अतः बहुलांक=.......

(२) जो अंक समूह में सबसे अधिक बार आता है। वह उस समूह का बहुलांक कहलाता है। खण्ड (४) के प्राप्तांकों १५, १८, २०, २२, २५, २७, २७, ३०, ३२ में अंक ………………सबसे अधिक बार आया है।

अत: बहुलांक =…………

(६३) समूहन सारणी —

प्राप्तांक	१–१ की	२–२ की		३–३ की		
↓ स्तम्भ →	१	२	३	४	५	६
१०	५	१२		२३		
१२	७		१८			
१६	११	२४			३१	३२
१९	१३		२१			
२०	८	२१		३४		
२१	१३		१७		२५	
२३	४	१३				२६
२८	६					

(९३) समूहन विधि से बहुलांक ज्ञात करने के लिए सर्वप्रथम समूहन सारणी बनाते है। आप भी पिछले खण्ड के बारम्बारता बंटन से निम्न समूहन सारणी भरिये--

समूहन सारणी--

प्राप्तांक	१-१ की	२-२ की		३-३ की		
↓ स्तम्भ →	१	२	३	४	५	६
१०						
१२						
१६						
१९						
२०						
२१						
२३						
२८						

(८) २७
बहुलांक

(६) एक कक्षा के कुछ छात्रो ने विज्ञान में निम्नलिखित अंक प्राप्त किये।

२१, २५, २५, २७, २६, ३०, ३७

उपरोक्त प्राप्तांको का बहुलांक..................है क्योंकि यह समूह में.................................आया है

---

(६२) १६ तथा २१, १३
१६ व २१
समूहन

(९२) एक कक्षा के ७० छात्रों ने वार्षिक परीक्षा में निम्न अंक प्राप्त किये। इनका बहुलांक ज्ञात कीजिये ?

प्राप्तांक (X)	बारम्बारता (f)
१०	५
१२	७
१६	११
१९	१३
२०	८
२१	१३
२३	४
२८	९

उपरोक्त बंटन में प्राप्तांक·········तथा·········की बारम्बारता··························है। अतः बहुलांक·········या·········होगा।

इन दोनों में से कौन सा अंक बहुलांक होगा यह हम························ विधि से ज्ञात करते है।

(९) २५
सबसे अधिक बार

(१०) इस प्रकार

(i) मध्यमान वह मान है जो अंकों के कुल योग को………………… ………से भाग देकर प्राप्त होता है।

(ii) मध्यांक वह अंक है जिसके ऊपर ५०% तथा नीचे ……… …% छात्रों के अंक होते हैं।

(iii) बहुलांक वह अंक है जो समूह में……………………आता है। उपरोक्त तीनों मान ……………,…………………तथा……… …………सांख्यिकी में केन्द्रीय प्रवृत्ति के मान कहलाते हैं।

(१०) छात्रों की संख्या
 ५०%—५०%
 सबसे अधिक
 मध्यमान, मध्यांक तथा बहुलांक

---

(११) विश्लेषण सारणी में प्राप्तांकों के सामने टैली चिन्ह लगाने के बाद हम प्रत्येक प्राप्तांक की बारम्बारता स्तम्भ—७ में लिखते हैं। प्राप्तांक १५ की बारम्बारता १ है। इसी प्रकार बायें पृष्ठ की विश्लेषण सारणी में अन्य प्राप्ताँकों की बारम्बारतायें लिखिये।

बायें पृष्ठ की विश्लेषण सारणी से हम देखते हैं कि अंक……………………की बारम्बारता सबसे अधिक है।

क्योंकि अंक………की बारम्बारता सबसे अधिक है अत: बहुलांक = ……

(११) एक परीक्षा में ७ छात्रों ने क्रमशः निम्नलिखित अंक प्राप्त किये–

०, १, १, २, ३, ५, ६

इनका मध्यमान =.....................

मध्यांक =.....................

बहुलांक =.....................

(१०) ४०, १६ व १८

विश्लेषण सारणी--

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारंबारता
↓ स्तम्भ→	१	२	३	४	५	६	७
१४							०
१५					I		१
१६	I				I	I	....
१७			I	I	I	I	....
१८	I	I	I	I		I	..
१९		I		I			...
२०							....
२१							....
२२							...
२३							....

(६०) समूहन सारणी के आधार पर विश्लेषण सारणी बनाते हैं। इसमें प्रत्येक स्तम्भ की सबसे अधिक बारम्बारता वाले प्राप्ताँकों के सामने स्तम्भों में टैली चिन्ह लगा दिया जाता है।

समूहन सारणी के स्तम्भ–१ में हम देखते हैं कि सबसे अधिक बारम्बारता········है जो कि प्राप्ताँक··· ····व········की है। इन प्राप्ताँकों के सामने निम्न विश्लेषण सारणी के स्तम्भ–१ के नीचे टैली चिन्ह लगा दिये है। आप भी अन्य स्तम्भों के नीचे सबसे अधिक बारम्बारता वाले प्राप्ताँकों के सामने टैलीचिन्ह लगाइये।

विश्लेषण सारणी––

प्राप्तांक	१–१ की	२–२ की		३–३ की		
↓ स्तम्भ →	१	२	३	४	५	६
१४						
१५						
१६	।					
१७						
१८	।					
१६						
२०						
२१						
२२						
२३						

(११) ३
२
१

(८६) (i) १५, २८ व ४०, योग—८३

(ii) १५; १५, १६ व १७ की बारम्बारताओं २८, ४० व २६ का योग=१०४

(iii) १५, २८; १६, १७ व १८ की बारम्बारताओ ४०, ३६ व ४० का योग=११६

समूहन सारणी —

प्राप्तांक ↓ स्तम्भ →	१-१ की	२-२ की		३-३ की		
	१	२	३	४	५	६
१४	१५	३८		८३		
१५	२८		६८		१०४	
१६	४०	७६				११६
१७	३६		७६	११४		
१८	४०	७८			९८	
१९	३८		५८			७१
२०	२०	३३		४०		
२१	१३		२०		४०	
२२	७	११				
२३	४					

(१२) केन्द्रीय प्रवृत्तियों के मान व नि कौन से है—

(अ)……………

(ब)……………

(स)……………

(८६) (i) समूहन सारणी के स्तम्भ–४ मे प्रारम्भ से प्रत्येक ३-३ प्राप्तांको की बारम्बारताओ का योग लिखने है। वाये पृष्ठ की समूहन सारणी के स्तम्भ–४ मे पहले तीन प्राप्तांको १४, १५ व १६ की बारम्बारताओ ....,........ व........ का योग........ लिख दिया है। इसी प्रकार अगले ३–३ प्राप्तांको की बारम्बारताओ का योग बाये पृष्ठ की समूहन सारणी के स्तम्भ–४ मे लिखिये।

(ii) स्तम्भ–५ मे प्रथम प्राप्ताक की बारम्बारता को छोडकर अगले ३-३ प्राप्तांको की बारम्बारताओ का योग लिखने हे। बाये पृष्ठ की सारणी के स्तम्भ–५ मे प्रथम प्राप्तांक १४ की बारम्बारता.... को छोडकर अगले तीन अको १५, १६ व १७ की बारम्बारताओ ....,........व........का योग लिख दिया है। इसी प्रकार स्तम्भ–५ को पूर्ण कीजिये।

(iii) स्तम्भ–६ मे प्रथम दो प्राप्तांको की बारम्बारता छोडकर अगले ३–३ प्राप्तांको की बारम्बारताओ का योग लिखा जाता है। बाये पृष्ठ की समूहन सारणी के स्तम्भ–६ मे प्रथम दो प्राप्तांको १४ व १५ की बारम्बारताओ........व ....को छोडकर अगले तीन अको १६, १७ व १८ की बारम्बारताओ........,........व .... का योग स्तम्भ–६ मे लिख दिया है। इसी प्रकार स्तम्भ–६ को पूर्ण कीजिये।

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(१२) (अ) मध्यमान
(ब) मध्यांक
(स) बहुलांक

(१३) केन्द्रीय प्रवृत्ति के मानों की अलग-२ परिभाषा दीजिये—

केन्द्रीय प्रवृत्ति के मान **परिभाषा**

(अ) **मध्यमान** ....
....
....।

(ब) **मध्यांक** ....
....।

स) **बहुलांक** ....
....।

(८८) (i) १४ व **१५ की** बारम्बारताओं १५ व २८, योग=४३

(ii) १५, **१५** व १६ की बारम्बारता २८ व ४०, योग—६८

समूहन सारणी—

प्राप्तांक	१-१ की	२-२ की		३-३ की		
↓ स्तम्भ→	१	२	३	४	५	६
१४	१५	३८		८३		
१५	२८		६८		१०४	
१६	४०	७६				११६
१७	३६		७६	....		
१८	४०	७८			....	
१९	३८		५८			....
२०	२०	३३		....		
२१	१३		२०		....	
२२	७	११				
२३	४					

(१३) (अ) छात्रों द्वारा प्राप्त अंकों के कुल योग को उनकी संख्या से भाग देकर प्राप्त मान मध्यमान कहलाता है ।

(ब) वह अंक जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के अंक होते हैं ।

(स) वह अंक जो समूह में सबसे अधिक बार आया हो ।

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(८८) समूहन सारणी बनाने के निम्न चरण (Step) हैं --

(i) समूहन सारणी के स्तम्भ-१ में प्रत्येक प्राप्तांक की बारम्बारता लिखी जाती है । आप भी बायें पृष्ठ की समूहन सारणी के स्तम्भ-१ में प्रत्येक प्राप्तांक की बारम्बारता लिखिये ।

(ii) स्तम्भ-२ में शुरू से २-२ प्राप्तांकों की बारम्बारताओं के समूह का योग करके लिखते है । पहले दो प्राप्ताँकों·········तथा·········की बारम्बारतायें क्रमशः·········व ·······है । इनका योग·········है । इसको बायें पृष्ठ की समूहन सारणी के स्तम्भ-२ में लिख दिया है । इसी प्रकार शेष २-२ प्राप्ताँकों की बारम्बारताओं के समूह का योग स्तम्भ-२ में लिखिये ।

(iii) स्तम्भ-३ में प्रथम प्राप्ताँक की बारम्बारता को छोड़कर अगले २-२ प्राप्ताँकों की बारम्बारताओं का योग लिखते हैं । अतः प्रथम प्राप्ताँक १४ की बारम्बारता·········को छोड़कर अगले दो प्राप्ताँकों·········तथा·········की बारम्बारताओं·· ···· तथा·········का योग ···· लिख दिया है । इसी प्रकार अगले २-२ प्राप्ताँकों की बारम्बारताओं का योग बायें पृष्ठ पर दी समूहन सारणी के स्तम्भ-३ में लिखिये ?

समीक्षा–अध्याय १

उपरोक्त अध्याय मे हमने विभिन्न केन्द्रीय प्रवृत्ति के मानों अर्थात् मध्यमान, मध्यांक व बहुलांक की परिभाषा के बारे में जाना है ।

(i) मध्यमान वह मान है जो प्राप्त अंकों के कुल योग को उनकी संख्या से भाग देकर प्राप्त होता है ।

(ii) मध्यांक वह अंक है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के प्राप्तांक हैं ।

(iii) बहुलांक वह अंक है जो समूह में सबसे अधिक बार आया हो ।

अगले अध्याय में हम अव्यवस्थित आँकड़ों तथा व्यवस्थित आँकड़ों में दीर्घ विधि तथा लघुविधि से मध्यमान ज्ञात करना सीखेंगे ।

अध्याय–२—मध्यमान की गणना

(१४) अव्यवस्थित आंकड़ो में मध्यमान

एक परीक्षा मे ५ छात्रों ने क्रमश: १६, १८, १६, २१, २२ अंक प्राप्त किये। किसी समूह का मध्यमान प्राप्त अंकों के……………को उनकी…… ……से भाग देकर प्राप्त होता है।

उपरोक्त प्राप्तांको मे अंकों का कुल योग=……+……+……+……+……=……

अंको के कुल योग को ΣX से प्रदर्शित करते है। अत: ΣX =……

छात्रों की संख्या को N से प्रदर्शित करते है। उपरोक्त प्राप्तांकों मे N=……।

अत: $M = \dfrac{\text{प्राप्त अंकों का कुल योग}}{\text{छात्रों की संख्या}} = \dfrac{\ldots\ldots}{N}$

$= \dfrac{\ldots\ldots}{\ldots\ldots} = \ldots\ldots\ldots$

(८७) १६ व १८
१६ व १८
समूहन विधि

समूहन सारणी—

प्राप्तांक ↓ स्तम्भ→	१–१ की: १	२–२ की: २	२–२ की: ३	३–३ की: ४	३–३ की: ५	३–३ की: ६
१४	….	२८				
१५	….		६८			
१६	….	….				
१७	….		….			
१८	….	….				
१९	….		….			
२०	….	….				
२१	….		….			
२२	….	….				
२३	….					

(१४) कुल योग, संख्या

अंकों का कुल योग $=$ १६ + १८ + १९ + २१ + २२ $=$ ९६

अतः $\Sigma X =$ ९६

$N =$ ५

$$\therefore M = \frac{\Sigma X}{N} = \frac{९६}{५} = १९.२$$

---

(८७) किसी कक्षा में छात्रों द्वारा प्राप्त अंकों का बंटन निम्न प्रकार दिया है। इनका बहुलांक ज्ञात कीजिये ?

प्राप्तांक (X)	बारम्बारता (f)
१४	१५
१५	२८
१६	४०
१७	३६
१८	४०
१९	३८
२०	२०
२१	१३
२२	७
२२	४

उपरोक्त सारणी में अंक.................. ...तथा..................की बारम्बारता सबसे अधिक है। अतः बहुलांक .... या .... होगा।

उपरोक्त दोनों में से कौन सा प्राप्तांक बहुलांक होगा यह हम सामान्य विधि या समूहन विधि में से किस विधि से निश्चित करते हैं।

समूहन विधि से बहुलांक ज्ञात करने के लिए सर्व प्रथम समूहन सारणी बनाते हैं। समूहन सारणी का खाका इस खण्ड के उत्तरों के साथ दिया है।

(१५) कुछ छात्रों ने एक परीक्षा में क्रमशः १, ७, ८, ९, ९, ११, १६, १८ व २० अंक प्राप्त किये।

उपरोक्त प्राप्तांकों का मध्यमान,

$$M = \frac{\cdots\cdots}{N} = \frac{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots}{\cdots\cdots} \text{———}$$

$$= \frac{\cdots\cdots\cdots\cdots}{\cdots\cdots\cdots} = \cdots\cdots\cdots\cdots$$

(८६) विश्लेषण सारणी—

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारंबारता
↓ स्तम्भ →	१	२	३	४	५	६	७
४							०
६							०
७							०
८			।	।			२
९	।	।	।	।	।		५
१०		।		।	।	।	४
११	।				।	।	३
१२						।	१
१३							०
१४							०
	९						

बहुलांक = ९

(८६) प्रत्येक स्तम्भ मे टैली चिन्ह लगाने के बाद विश्लेषण सारणी के स्तम्भ-७ में प्रत्येक प्राप्तॉक की बारम्बारता ज्ञात करते है। प्राप्ताँक ८, दो बार आया है अत: इसकी बारम्बारता २ होगी । प्राप्ताँक ४, ६ व ७ एक भी बार नही आये है अत: इनकी बारम्बारता ० होगी। इसी प्रकार अन्य प्राप्तॉको की बारम्बारतायें भी लिखिये।

स्तम्भ-७ मे बारम्बारताओ से हम देखते है कि प्राप्तॉक..................
की बारम्बारता सबमे अधिक है।

अत: बहुलॉक=............

$$(१५)\quad M = \frac{\Sigma X}{N} = \frac{१ + ७ + ८ + ९ + ९ + ११ + १६ + १८ + २०}{९}$$

$$= \frac{९९}{११} = ९$$

(१६) इस प्रकार अव्यवस्थित आंकड़ों में मध्यमान ज्ञात करने का सूत्र

$$M = \frac{\cdots\cdots\cdots\cdots}{\cdots\cdots\cdots}$$

(८५) ४७

९, १० व ११ की बारम्बारता १८, ११ व १८

विश्लेषण सारणी—

प्राप्तांक ↓ स्तम्भ →	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह १	२	३	४	५	६	बारंबारता ७
४							
६							
७							
८			I	I			
९	I	I	I	I	I		
१०		I		I	I	I	
११	I				I	I	
१२						I	
१३							
१४							

(८१) खण्ड (८०) में दी हुई समूहन सारणी के स्तम्भ-५ में अधिकतम बारम्बारता………है। यह बारम्बारता प्राप्तांक……,……व……की बारम्बारताओं……,……व……का योग है। बायें पृष्ठ की विश्लेषण सारणी में उपयुक्त स्थान पर टैलीचिन्ह लगाइये।

इसी प्रकार स्तम्भ-६ में अधिकतम बारम्बारता वाले प्राप्तांकों के सामने टैलीचिन्ह लगाइये।

---

(१६) $M = \frac{\Sigma X}{N}$

व्यवस्थित आंकड़ों में मध्यमान —(अ) बारम्बारता बंटन में

(१७) किसी कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न प्रकार है—

प्राप्तांक (X)	बारम्बारता (f)
२	३
३	४
४	२

(i) प्राप्तांक २ कितने छात्रों ने प्राप्त किये है= ....छात्र ।
इन छात्रों द्वारा प्राप्त अंकों का योग $= f \times X = ३ \times २ =$ ......

(ii) प्राप्तांक ३ कितने छात्रों ने प्राप्त किये हैं= ....छात्र ।
इन छात्रों द्वारा प्राप्त अंकों का योग $= f \times X = ... \times ३ =$ ......

(iii) प्राप्तांक ४ कितने छात्रों ने प्राप्त किये हैं= ....छात्र ।
इन छात्रों द्वारा प्राप्त अंकों का योग $= f \times X = ... \times ... =$ ......

(iv) उपरोक्त सभी छात्रों द्वारा प्राप्त अंकों
का योग $= \Sigma fX =$ .... = ...

(८४) ४२
८, ९ व १० की बारम्बारता १३ व १८ व

विश्लेषण सारणी—

प्राप्तांक ↓ स्तम्भ→	सबसे अधिक बारंबारता वाले प्राप्तांकों के टैली चिन्ह १	२	३	४	५	६	बारंबारता
४							
६							
७							
८			|	|			
९	|	|	|	|			
१०		|		|			
११	|						
१२							
१३							
१४							

(८४) खण्ड (८०) में दी समूहन सारणी के स्तम्भ-४ में अधिकतम बारम्बारता ………है। यह प्राप्तांक ८, ६ व १० की बारम्बारताओं ……, …… तथा…… का योग है। बायें पृष्ठ की विश्लेषण सारणी मे इन प्राप्तांकों के सामने स्तम्भ-४ के नीचे उपयुक्त स्थान पर टैलीचिह्न लगाइये ?

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(१७) (i) ३

$३ \times २ = ६$

(ii) ४

$४ \times ३ = १२$

(iii) २

$२ \times ४ = ८$

(iv) $\Sigma fX = २६$

(१८) पिछले खण्ड के बारम्बारता वँटन में-

छात्रो की संख्या (N)- ... .

$$\text{क्योंकि मध्यमान} = \frac{\text{छात्रों द्वारा प्राप्त अंकों का कुलयोग}}{\text{छात्रा की संख्या}}$$

$$= \frac{\text{.......}}{N} = \frac{\text{............}}{\text{.......}}$$

$$= \text{..............}$$

(८३) ३१

८ व ९ की बारम्बारता १३ व १८

विश्लेषण सारणी—

प्राप्तांक ↓ स्तम्भ →	१	२	३	४	५	६	बारंबारता
४							
६							
७							
८			I				
९	I	I	I				
१०		I					
११	I						
१२							
१३							
१४							

$$(१८)\quad M = \frac{\Sigma fX}{N}$$

$$= \frac{२६}{९}$$

$$= २.८८$$

(८३) खण्ड (८०) में दी समूहन सारणी के स्तम्भ–३ में अधिकतम बारम्बारता ·······है । यह बारम्बारता प्राप्तांक······ तथा·······की बारम्बारता ······· तथा·········का योग है । बायें पृष्ठ की विश्लेषण सारणी में इन प्राप्तांकों के सामने उपयुक्त स्थान पर टैलीचिन्ह लगाइये ?

(१६) एक कक्षा के १० छात्रों के प्राप्तांकों का बारम्बारता बंटन निम्न प्रकार है—

प्राप्तांक (X)	बारम्बारता (f)
७	४
८	३
९	१
१०	२
	N = १०

४ छात्रों द्वारा प्राप्त अंकों का योग $= fX =$ ४ × ७ = ……………

३ छात्रों द्वारा प्राप्त अंकों का योग $= fX =$ ३ × …… = ……………

१ ,, ,, ,, $= fX =$ …… × …… = ……………

२ ,, ,, ,, $= fX =$ ………… = ……………

सभी…… ,, ,, ,, $= \Sigma fX$ = ……………

(८२) २६, १८ व ११

प्राप्तांक—९ व १०

विश्लेषण सारणी—

प्राप्तांक ↓ स्तम्भ →	सबसे अधिक बारंबारता वाले प्राप्तांकों के टैली चिन्ह १	२	३	४	५	६	बारंबारता
४							
६							
७							
८							
९	|	|					
१०		|					
११	|						
१२							
१३							
१४							

(१९)

$$fX = ४ \times ७ = २८$$
$$fX = ३ \times ८ = २४$$
$$fX = १ \times ९ = ९$$
$$fX = २ \times १० = २०$$

सभी १० छात्रों द्वारा
प्राप्त अंकों का योग $= \Sigma fX =$ ८१

(२०) बायें पृष्ठ से–

$N = \cdots\cdots\cdots$

$\Sigma fX = \cdots\cdots\cdots$

अतः $M = \dfrac{\cdots\cdots\cdots}{\cdots\cdots\cdots}$

$= \dfrac{\cdots\cdots\cdots}{\cdots\cdots\cdots}$

$= \cdots\cdots\cdots$

(८१) १८

६ व ११

विश्लेषण सारणी––

प्राप्तांक ↓ स्तम्भ →	सबसे अधिक बारम्बारता वाले प्राप्तांकों व टैली चिन्ह						बारंबारता
	१	२	३	४	५	६	
४							
६							
७							
८							
९	I						
१०							
११	I						
१२							
१३							
१४							

(२०) $N=१०$

$\Sigma fX=८१$

अतः $M=\frac{\Sigma fX}{N}$

$=\frac{८१}{१०}$

$=८\cdot१$

(८१) समूहन विधि से बहुलांक निकालने के लिए समूहन सारणी पूर्ण हो जाने के बाद इसके आधार पर विश्लेषण सारणी बनाते हैं। इसमें प्रत्येक स्तम्भ की अधिकतम बारम्बारता के प्राप्तांकों के सामने टैली-चिन्ह लगा दिया जाता है। बायें पृष्ठ की समूहन सारणी से हम देखते हैं कि स्तम्भ-१ में अधिकतम बारम्बारता……है यह बारम्बारता प्राप्तांक…… तथा……की है। अतः विश्लेषण सारणी में स्तम्भ-१ के नीचे प्राप्तांक ९ व ११ के सामने टैलीचिन्ह लगाइये ?

विश्लेषण सारणी—

प्राप्तांक ↓ स्तम्भ→	सबसे अधिक बारंबारता वाले प्राप्तांकों के टैली चिन्ह						बारंबारता
	१	२	३	४	५	६	
४							
६							
७							
८							
९							
१०							
११							
१२							
१३							
१४							

(२१) एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न है—

प्राप्तांक	बारम्बारता	अंकों का योग
X	f	f × X = fX
१०	५	५ × १० = ………
११	४	४ × …. = ………
१२	२	…. × …. = ………

कुल छात्रों द्वारा प्राप्त अंकों का योग = Σfx = ………

छात्रों की संख्या = N = ………

अतः मध्यमान(M) = $\frac{\dots\dots}{N} = \frac{\dots\dots}{\dots\dots}$

= …………

(८०) ५ व ७

४, १३ व १८, ३५

समूहन सारणी—

प्राप्तांक ↓ स्तम्भ→	१–१ की: १	२–२ की: २	२–२ की: ३	३–३ की: ४	३–३ की: ५	३–३ की: ६
४	५	१२		१६		
६	७		११		२४	
७	४	१७				३५
८	१३		३१	४२		
९	१८	२९			४७	
१०	११		२९			३६
११	१८	२५		३४		
१२	७		१६		२४	
१३	९	१७				
१४	८					

(२१) $५ \times १० = ५०$

$४ \times ११ = ४४$

$२ \times १२ = २४$

$\Sigma fX = ११८$

$N = ११$

अतः मध्यमान $(M) = \frac{\Sigma fX}{N} = \frac{११८}{११}$

$= १०.७२$

---

(८०) समूहन सारणी के स्तम्भ–६ में पहले दो अंकों की बारम्बारताओं को छोड़कर ३–३ अंकों की बारम्बारताओं के समूह का योग लिखा जाता है। अतः बायें पृष्ठ की समूहन सारणी में ३–३ के समूह बनाते समय प्राप्तांक ४ व ६ की बारम्बारता.........व.........को छोड़ दिया गया है। अगले प्राप्तांक ७, ८ तथा ९ की बारम्बारता.........,.........तथा......... हैं। इनका योग.........हैं। इसे बायें पृष्ठ पर स्तम्भ–६ में लिख दिया है। *इसी प्रकार इससे अगले ३ अंकों की बारम्बारताओं के समूह का योग स्तम्भ-६ में लिखिये ?*

इस प्रकार हमारी समूहन सारणी पूर्ण हो जायेगी।

(२२) इस प्रकार यदि प्राप्तांकों का बारम्बारता बंटन दिया हो तो उनका मध्यमान निकालने का सूत्र होता है--

$$M = \frac{\ldots\ \ldots\ldots}{\ldots}$$

(७६) ५

७, ४, १३, योग = २४

समूहन सारणी—

प्राप्तांक	१-१ की	२-२ की		३-३ की		
↓ स्तम्भ→	१	२	३	४	५	६
४	५	१२		१६		
६	७		११		२४	
७	४	१७				३५
८	१३		३१	४२		
९	१८	२९			४७	
१०	११		२९			....
११	१८	२५		३४		
१२	७		१६		२४	
१३	९	१७				
१४	८					

(८६) पहले प्राप्ताक की बारम्बारता को छोड़कर ३-३ प्राप्तांको की बारम्बारताओं के समूह का योग स्तम्भ-५ मे लिखा जाता है। बायें पृष्ठ की समूहन सारणी में हमने प्राप्ताक ४ की बारम्बारता……… को छोड दिया है। तथा उमसे अगले प्राप्तांक ६, ७ व ८ की बारम्बारताओ………,………तथा………का योग……… स्तम्भ-५ मे लिख दिया है। इसी प्रकार ३-३ के अगले समूह बनाकर उनका योग स्तम्भ-५ मे लिखिये ?

---

(९२) $M = \frac{\Sigma fX}{N}$

(ब) **वर्गअन्तरालों में मध्यमान—दीर्घ विधि**

(२३) एक समूह के अंग्रेजी विषय मे प्राप्तांक वर्गअन्तरालों के रूप में निम्न प्रकार दिये हुए है—

वर्गअन्तराल (C.I.)	बारम्बारता (f)
१०–२०	८
२०–३०	३
३०–४०	६

उपरोक्त बंटन मे—

(i) ८ छात्रो ने······ तथा······ के बीच अंक पाये है।

(ii) ३ छात्रो ने·········तथा·········के बीच अक पाये हैं।

(iii) ६ छात्रो ने······ तथा·········के बीच अंक पाये हैं।

इस प्रकार जब प्राप्तांक वर्गअन्तरालों में दिये हो तब हम यह नही बता सकते हैं कि कितने छात्रो ने कौन सा अंक प्राप्त किया है। ऐसी स्थिति में हम निम्न मे से किसको उस वर्गअन्तराल में शामिल अंकों का प्रतिनिधि अक मानना चाहिये ?

उच्च सीमा या निम्न सीमा या मध्यमूल्य

(७८) ४, ६ व ७ की बारम्बारताओं ५, ७ व ४

समूहन सारणी—

प्राप्तांक	१-१ की	२-२ की		३-३ की		
↓ स्तम्भ →	१	२	३	४	५	६
४	५	१२		१६		
६	७		११		२४	
७	४	१७				
८	१३		३१	४२		
९	१८	२९			....	
१०	११		२९			
११	१८	२५		३४		
१२	७		१६		....	
१३	९	१७				
१४	८					

(२३) (i) १० तथा २०

(ii) २० तथा ३०

(iii) ३० तथा ४०

मध्यमूल्य

(२४) किसी वर्गअन्तराल का मध्यमूल्य

$$= \frac{\text{निम्न सीमा} + \text{उच्च सीमा}}{२}$$

मध्यमूल्य को छोटे एक्स (X) से प्रदर्शित किया जाता है ।

(i) वर्गअन्तराल १०–२० की निम्नसीमा............तथा उच्चसीमा ....है ।

अत: मध्यमूल्य(X) $= \frac{\text{.......} + \text{.......}}{२} =$ ....

(ii) वर्गअन्तराल २०–३० की निम्नसीमा .... तथा उच्चसीमा ....है ।

अत: X $= \frac{\text{.......}}{२} =$ ....

(iii) वर्गअन्तराल ३०–४० का मध्यमूल्य (X) = ...

(७७) ४

६ व ७ की बारम्बारता ७ व ४

योग = ११

समूहन सारणी—

प्राप्तांक ↓ स्तम्भ →	१-१ की	२-२ की		३-३ की		
	१	२	३	४	५	६
४	५	१२		१६		
६	७		११			
७	४	१७				
८	१३		३१	....		
९	१८	२९				
१०	११		२९			
११	१८	२५		....		
१२	७		१६			
१३	९	१७				
१४	८					

(२४) (i) १०, २०

$$x = \frac{१० + २०}{२}$$

$$= \frac{३०}{२}$$

$$= १५$$

(ii) २०, ३०

$$x = \frac{२० + ३०}{२}$$

$$= \frac{५०}{२}$$

$$= २५$$

(iii) $x = ३५$

---

(७७) समुचित मानों के स्थान-३ में पहले तालिका की बारम्बारता को छोड़कर विभिन्न वर्ग (७१) की विधि से ही १-१ तालिकाओं की बार-म्बारताओं का योग लिख देते है।

सही पुष्ठ की मानों में प्रथम तालिका.......है। इसकी बारम्बारता को छोड़ दिया है। अतः तालिका.......और.......की बारम्बारताओं क्रमशः.......और.......का योग.......है। इस योग को स्तम्भ-३ में लिख दिया है। इसी प्रकार इसके बाद तालिका ८ व ९ की बारम्बार-ताओं १३ व १८ का योग ३१ स्तम्भ-३ में लिख दिया गया है। बाद की सही पुष्ठ की समुचित मानों के स्तम्भ-३ में उसके बाद ६-६ तालिकाओं की बारम्बारताओं का योग इसी प्रकार लिखिये। यदि अन्त में केवल एक बारम्बारता बचे तो उसे छोड़ दीजिये।

(२५) इस प्रकार जब प्राप्तांक वर्गअन्तराल में दिये हो तब उस वर्गअन्तराल का प्रतिनिधि अंक..................होता है। मध्यमूल्य निकालने का सूत्र

$$X = \frac{\text{.......} + \text{.......}}{२}$$

(७६) ५, ७; योग = १२

समूहन सारणी—

प्राप्तांक ↓ / स्तम्भ →	१–१ की: १	२–२ की: २	३	४	३–३ की: ५	६
४	५	१२				
६	७		११			
७	४	१७				
८	१३		....			
९	१८	२९				
१०	११		....			
११	१८	२५				
१२	७		....			
१३	९	१७				
१४	८					

(२५) मध्यमूल्य

$$X = \frac{\text{निम्न सीमा + उच्च सीमा}}{२}$$

---

(७६) समूहन सारणी के स्तम्भ–२ में प्रारम्भ से प्रत्येक २–२ अंकों की बारम्बारताओं को जोड़कर समूह बनाये जाते हैं। बायें पृष्ठ पर दी सारणी में हमने प्रथम दो प्राप्तांकों ४ तथा ६ की बारम्बारताओं……… तथा………के योग………को स्तम्भ–२ के प्रारम्भ में लिख दिया है। इसी प्रकार अगले २–२ अंकों की बारम्बारताओं के समूह का योग स्तम्भ–२ में लिखिये ?

(२६) निम्नलिखित सारणी मे दिये वर्गअन्तरालो के मध्यमूल्य निकालिये ?

वर्गअन्तराल (C I)	बारम्बारता (f)	मध्यमूल्य (x)
०– ५	४	२·५
५-१०	५	....
१०–१५	१	....
१५–२०	५	....
२०–२५	३	....

वर्गअन्तराल ०–५ का मध्यमूल्य $(X) = \frac{० + ५}{२} = २·५$ । इसी प्रकार अन्य वर्गअन्तरालों का मध्यमूल्य (X) निकालिये ?

(७५) **समूहन सारणी**—

प्राप्तांक ↓ स्तम्भ →	१–१ की: १	२–२ की: २	२–२ की: ३	३–३ की: ४	३–३ की: ५	३–३ की: ६
४	५	} १२				
६	७					
७	४	}				
८	१३					
९	१८	}				
१०	११					
११	१८	}				
१२	७					
१३	९	}				
१४	८					

(२६)

C.I.	f	x
०– ५	४	२.५
५–१०	५	७.५
१०–१५	१	१२.५
१५–२०	५	१७.५
२०–२५	३	२२.५

---

(७५) बाय पृष्ठ में दी हुई समूहन सारणी के स्तम्भ–१ में प्रत्येक प्राप्तांक को बारम्बारता लिख देते है । आप भी स्तम्भ–१ में खण्ड (७०) से प्रत्येक प्राप्तांक की बारम्बारतायें लिखिये ?

(२७) कुछ छात्रों के प्राप्तांकों का बंटन उनके मध्यमूल्य के साथ निम्न है—

वर्गअन्तराल (C.I)	बारम्बारता (f)	मध्यमूल्य (x)
१०–२०	८	१५
२०–३०	३	२५
३०–४०	९	३५

क्योंकि मध्यमूल्य को प्रत्येक वर्गअन्तराल का प्रतिनिधि अंक मान लेते हैं अतः उपरोक्त बंटन से —

(i) अंक १५ कितने छात्रों ने प्राप्त किये हैं = ……छात्र ।
इन छात्रों द्वारा प्राप्त अंकों का योग $= f \times x$
$= ८ \times १५ \quad = \ldots\ldots$

(ii) अंक २५ कितने छात्रों ने प्राप्त किये हैं = ……छात्र ।
इन छात्रों द्वारा प्राप्त अंकों का योग $= f \times x$
$= \ldots\ldots \times २५ \quad = \ldots\ldots$

(iii) अंक ३५ कितने छात्रों ने प्राप्त किये है = ……छात्र ।
इन छात्रों द्वारा प्राप्त अंकों का योग $= f \times x$
$= \ldots\ldots \times \ldots\ldots = \ldots\ldots$

(iv) इन सभी २० छात्रों द्वारा प्राप्त अंकों का कुल योग $= \Sigma fx = \ldots$

(७४) सबसे अधिक ९ व ११
समूहन सारिणी–

प्राप्तांक	१-१ की	२-२ की		३-३ की		
↓ स्तम्भ →	१	२	३	४	५	६
४						
६						
७						
८						
९						
१०						
११						
१२						
१३						
१४						

(२७) $f \times x = ८ \times १५ = १२०$

$f \times x = ३ \times २५ = \ ७५$

$f \times x = ९ \times ३५ = ३१५$

$\Sigma fx \quad = ५१०$

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(८४) **बारम्बारता बंटन में बहुलांक—समूहन विधि**

एक कालिज में कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न है इसका बहुलांक ज्ञात कीजिये ?

प्राप्तांक (X)	बारम्बारता (f)
४	५
६	७
७	४
८	१३
९	१८
१०	११
११	१८
१२	७
१३	९
१४	८

बहुलांक वह अंक है जिसकी बारम्बारता..............................होती है। उपरोक्त बंटन में सबसे अधिक बारम्बारता दो अंकों क्रमश:.........तथा ....की है। अत: यह निश्चित करना कठिन है कि इनमें से कौन सा अंक बहुलांक है। ऐसी स्थिति में बहुलाँक निकालने के लिए समूहन विधि का प्रयोग करते हैं।

समूहन विधि से बहुलांक ज्ञात करने के लिए सर्व प्रथम समूहन सारणी बनाते हैं। इसका खाका इस खण्ड के उत्तरों के साथ पृष्ठ पर दिया है।

(२८) हम जानते है कि किन्हीं प्राप्तांकों का मध्यमान

$$M = \frac{\text{छात्रों द्वारा प्राप्त अंकों का कुल योग}}{\text{छात्रों की संख्या}}$$

वायें पृष्ठ से—

छात्रों द्वारा प्राप्त अंकों का कुल योग $= \Sigma fx$ = ...

छात्रों की संख्या $= N$ = ...

अतः मध्यमान $(M) = \frac{\ldots\ldots}{N} = \frac{\ldots\ldots}{\ldots\ldots}$ = ...

इस प्रकार खण्ड (२७) के बारम्बारता वंटन का मध्यमान = ...

(२८) $\Sigma fx = ५१०$

$N = २०$

$$M = \frac{\Sigma fx}{N} = \frac{५१०}{२०} = २५.५$$

मध्यमान $= २५.५$

---

(७३) एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न है–

प्राप्तांक (X)	बारम्बारता (f)
१	२
२	३
३	१
४	७
५	३

उपरोक्त बंटन में किस अंक की बारम्बारता सबसे अधिक है $=$ ……

अतः बहुलांक $=$ ……………

(२६) एक कक्षा के छात्रों के प्राप्तांक वर्गअन्तराल में निम्न प्रकार दिये हैं। इन प्राप्तांकों का मध्यमान ज्ञात कीजिये ?

वर्गअन्तराल (C. I)	बारम्बारता (f)	मध्यमूल्य (x)
५—१५	१५	....
१५—२५	१२	....
२५—३५	१०	....
३५ ४५	५	....
	N = ४२	

मध्यमान ज्ञात करने के लिए सबसे पहले........................ ....या X ज्ञात करते है। उपरोक्त बारम्बारता बंटन के स्तम्भ–३ में X के मान लिखिये।

(३६)

C. I.	f	x
५—१५	१५	१०
१५—२५	१२	२०
२५—३५	१०	३०
३५—४५	५	४०
	N=४२	

मध्यमूल्य

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(७२) **बारम्बारता बंटन में बहुलांक—सामान्य विधि**

एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न प्रकार है—

प्राप्तांक (X)	बारम्बारता (f)
२	१
३	५
४	३
६	४
८	२

उपरोक्त बंटन में अंक २ कितने छात्रों ने प्राप्त किया है=.......छात्र

,, ,, ,, ३ ,, ,, ,, =.......छात्र

इसी प्रकार ,, ६ ,, ,, ,, = ...छात्र

सबसे अधिक छात्रों ने कौन सा अंक प्राप्त किया है =..........

अतः उपरोक्त बंटन का बहुलांक =..........

(३०) बायें पृष्ठ के बारम्बारता बंटन से—

(i) १५ छात्रों द्वारा प्राप्त अंकों का योग $= fx = १५ \times १० =$ ....

(ii) १२ छात्रों „ „ „ $= fx = १२ \times$ .... $=$ ....

(iii) १० „ „ „ $= fx =$ .... $\times$ .... $=$ ....

(iv) ५ „ „ „ $= fx =$ .... $=$ ....

सभी........ „ „ „ „ $= \Sigma fx$ $=$ ....

(v) कुल छात्रों की संख्या $= N$ $=$ ....

(३०)	१५ छात्रों द्वारा	प्राप्त अंकों	का योग	$= fx =$	१५०
	१२ ,,	,,	,,	$= fx =$	२१०
	१० ,,	,,	,,	$= fx =$	३००
	५ ,,	,,	,,	$= fx =$	२००

सभी ४२ छात्रों द्वारा प्राप्त अंकों का योग $= \Sigma fx =$ ८६०

छात्रों की संख्या $N =$ ४२

(३१) वायें पृष्ठ से —

खण्ड (३०) के बारम्बारता बंटन का मध्यमान

$$M = \frac{\cdots\cdots\cdots}{N} = \frac{\cdots\cdots}{\cdots\cdots}$$

$$= \cdots\cdots\cdots\cdots\cdots\cdots$$

---

(७०) २
२, बहुलांक

अध्याय–३ बहुलांक

(६०) ***अव्यवस्थित आंकड़ों में बहुलांक—***

एक समूह ने निम्न अंक प्राप्त किये । इनका बहुलांक ज्ञात कीजिये ?

१, २, २, ५, ७

हम जानते है कि बहुलांक वह अंक है जो समूह में सबसे अधिक बार आया हो ।

उपरोक्त समूह में अंक..................सबसे अधिक बार आया है । यह अंक.........इस समूह के अंकों का.....................है ।

---

(५९) $M=\frac{\Sigma fx}{N}=\frac{८९०}{४२}$

$=२१.१९$

समीक्षा–अध्याय ३

(३) अध्याय–३ में हमने विभिन्न प्रकार के आंकडों में मध्यांक ज्ञात करना जाना है ।

(अ) अव्यवस्थित आंकड़े में—

(i) जब छात्रों की संख्या विषम हो तो मध्यांक $\left(\frac{N+१}{२}\right)$ वें छात्र का प्राप्तांक होता है ।

(ii) जब छात्रो की संख्या सम हो तो मध्यांक बीच के दो प्राप्तांकों का औसत होता है ।

(ब) व्यवस्थित आंकड़ों में—

(i) जब प्राप्तांक बारम्बारता बंटन में दिये हो तब मध्यांक वह प्राप्तांक होता है जहाँ बारम्बारता $\frac{N+१}{२}$ हो जाती है ।

(ii) जब प्राप्तांक वर्ग अन्तराल में हो तो हम सर्व प्रथम मध्यांक वर्ग ज्ञात करते है फिर मध्यांक वर्ग में मध्यांक की गणना निम्न सूत्र से करते है ।

$$\text{मध्यांक} = l + \left(\frac{N/२ - F}{f}\right) \times i$$

जहाँ l=मध्यांक वर्ग की निम्न सीमा

F=मध्याक वर्ग से पूर्व वर्ग की संचयी बारम्बारता,

f=मध्याक वर्ग की बारम्बारता ।

अगले अध्याय में हम अव्यवस्थित तथा व्यवस्थित आंकड़ों में सामान्य विधि, समूहन विधि तथा वर्ग अन्तरालों में बहुलांक की गणना करना सीखेगे ।

(३२) निम्नलिखित बारम्बारता वंटन का मध्यमान ज्ञात कीजिये ?

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	मध्यमूल्य (x)	बार० × मध्यमूल्य fx
०–१०	१५	....	....
१०–२०	१२	....	....
२०–३०	२०	....	....
३०–४०	२३	....	...
	N = ...		Σfx = ...

उपरोक्त बारम्बारता वंटन के स्तम्भ–३ तथा स्तम्भ–४ में क्रमशः मध्यमूल्य (X) तथा बार० × मध्यमूल्य (fX) का मान भरिये।
उपरोक्त वंटन से, N = ... तथा Σfx = ... । अतः

$$M = \frac{....}{N} = \frac{....}{....} = ...$$

(३२)

C.I.	f	x	fx
०-१०	१५	५	७५
१०-२०	१२	१५	१८०
२०-३०	२०	२५	५००
३०-४०	२३	३५	८०५
	N=७०		Σfx=१५६०

$$\therefore M=\frac{\Sigma fx}{N}=\frac{१५६०}{७०}=२२.२८$$

---

(१६) इस प्रकार किसी मध्यांक वर्ग में मध्यांक ज्ञात करने का सूत्र

मध्यांक=...............

जहाँ l=...............

F=...............

f=...............

i=...............

N=...............

(३३) यदि प्राप्तांक वर्गअन्तरालो मे दिये हो तो मध्यमान ज्ञात करने का सूत्र

$$M = \frac{\cdots\cdots}{\cdots\cdots} \quad \text{जहाँ } X = \cdots\cdots\cdots\cdots\cdots\cdots \text{ है ।}$$

---

(६८) १०, १४, १८, २४, २८, ३१, ३३

$N = ३३, \qquad N/२ = १६\cdot५$

मध्यांक वर्ग $= ९—११$

$$\text{मध्यांक} = l + \left(\frac{N/२ - F}{f}\right) \times i$$

$$= ९ + \left(\frac{१६\cdot५ - १५}{३}\right) \times २$$

$$= ९ + \frac{१\cdot५}{३} \times २ = १०$$

(६८) एक कक्षा के ३३ छात्रों ने निम्न प्रकार अंक प्राप्त किये ।

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	संचयी बारम्बारता (cf)
५– ७	१०	....
७– ९	५	....
९-११	३	....
११-१३	७	....
१३-१५	३	....
१५-१७	३	....
१७-१९	२	....
	N = ३३	

उपरोक्त बंटन में संचयी बारम्बारता भरिये ?

यहाँ N = ... अत: $\frac{N}{२}$ = ...

अत: मध्यांक वर्ग = ...

∴ मध्यांक = ... + $\frac{(..........)}{.........}$ × ....

= .... + ...

= ....

---

६३) $M = \frac{\Sigma fx}{N}$

वर्ग अन्तराल में मध्यमान—(ब) लघु विधि

(३४) एक कक्षा के छात्रों के प्राप्तांकों को वर्गअन्तराल में निम्न प्रकार दिया गया है। मध्यमान ज्ञात कीजिये ?

वर्गअन्तराल (C. I.)	बारम्बारता (f)	मध्यमूल्य (x)	बार० × मध्यमूल (fx)
२०—३०	८	२५	....
३०—४०	९	३५	....
४०—५०	१७	४५	....
५०—६०	१४	५५	....
६०—७०	१२	६५	....
	N = ६०		

क्योंकि मध्यमान $(M) = \dfrac{\cdots\cdots\cdots}{N}$

अत: सर्वप्रथम हमें fx का मान ज्ञात करना होगा। उपरोक्त सारणी में fx ज्ञात कीजिये। fx का मान ज्ञात करने में क्या कठिनाई होती है ?——.. ....। इस कठिनाई को कम करने के लिए ही हम लघुविधि का प्रयोग करते है।

---

(६७) N/२=२०

४०—५०

मध्यांक $= l + \left(\dfrac{N/२ - F}{f}\right) \times i$

$= ४० + \left(\dfrac{२० - १७}{१०}\right) \times १० = ४० + ३$

$= ४३$

(३४)

C. I.	f	x	fx
२०–३०	८	२५	२००
३०–४०	९	३५	३१५
४०–५०	१७	४५	७६५
५०–६०	१४	५५	७७०
६०–७०	१२	६५	७८०
	N=६०		

fx के गुणा बहुत बड़े-बड़े हो जाते हैं या अन्य कोई मिलता-जुलता उत्तर ।

---

(६७) *निम्न प्राप्तांकों का मध्यांक ज्ञात कीजिये ?*

वर्गअन्तराल (C.I.)	बारम्बारता (f)	संचयी बारम्बारता (cf)
१०–२०	८	८
२०–३०	४	१२
३०–४०	५	१७
४०–५०	१०	२७
५०–६०	५	३२
६०–७०	५	३७
७०–८०	३	४०
	N=४०	

उपरोक्त बटन में N/२=·······

संचयी बार०(cf) स्तम्भ से हम देखते हैं कि N/२ अर्थात २० वाँ छात्र २७ में शामिल है जिसका कि वर्गअन्तराल ४०–५० है। अतः मध्यांक वर्ग=·······

अतः मध्यांक $= l + \left(\frac{N/२ - F}{f}\right) \times i$

$= ४० + \frac{(\cdots - \cdots)}{\cdots\cdots} \times १०$

$= ४० + \cdots\cdots\cdots\cdots = \cdots\cdots\cdots\cdots$

(३५) लघुविधि मे मध्यमान निकालने के लिए हम किसी वर्गअन्तराल के मध्यमूल्य को कल्पित मध्यमान मान लेते है। गणना की सुगमता के लिए श्रेणी के बीच के वर्ग अन्तराल के मध्यमूल्य को कल्पित मध्यमान मान लेते है।

बाये पृष्ठ के बारम्बारता बटन से हम देखते है कि वर्गअन्तराल ४०–५० श्रेणी क लगभग बीच मे है अतः इसके मध्यमूल्य को कल्पित मध्यमान माना है।

अतः बाये पृष्ठ के बटन मे कल्पित मध्यमान = ·· · ·········

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(६६) मध्यांक $= १० \left(\frac{१२-११}{९}\right) \times ५$

$= १० + \frac{५}{९} = १० + .५५$

$= १०.५५$

(६६) क्योंकि मध्यांक वर्ग १०–१५ है। अत: इसमें १० से लेकर १५ तक अंक शामिल है। इनमें कौन सा अंक मध्यांक होगा, यह हम निम्न सूत्र से ज्ञात करते हैं—

$$\text{मध्यांक} = l + \left(\frac{N/२-F}{f}\right) \times i$$

जहाँ l=मध्यांक वर्ग की निम्न सीमा ; f=मध्यांक वर्ग की बारम्बारता F=मध्यांक वर्ग से पहले वर्ग की संचयी बारम्बारता; i=वर्ग विस्तार

पिछले खण्ड के बंटन से—

$l=१०; f=६, F=११, i=५, N/२=१२$

$$\text{अत: मध्यांक} = l + \left(\frac{N/२-F}{f}\right) \times i$$

$$= १० + \left(\frac{१२-\cdots\cdots}{\cdots\cdots}\right) \times ५$$

$$= \cdots\cdots + \cdots\cdots$$

$$= \cdots\cdots + \cdots\cdots$$

$$= \cdots\cdots\cdots\cdots$$

---

(३५) कल्पित मध्यमान=४५

(३६) जिस वर्ग अन्तराल के मध्यमूल्य को कल्पित मध्यमान माना जाता है उसके सामने स्तम्भ–३ में शून्य लिख दिया जाता है। निम्न बंटन में कल्पित मध्यमान ४०–५० के मध्यमूल्य को माना है अत: इसके आगे शून्य लिख दिया है। कल्पित मध्यमान वाले वर्ग अन्तराल से छोटे अन्तरालों की ओर क्रमश: –१,–२,–३ आदि तथा बड़े अन्तरालों की ओर क्रमश: +१, +२, +३ आदि लिख देते हैं। इन्हें विचलन मान कहते है तथा d से प्रदर्शित करते है। निम्न बंटन में वर्ग अन्तराल ३०–४० में d का मान–१ तथा उससे छोटे वर्ग अन्तराल २०–३० मे–२ लिख दिया है। इसी प्रकार दूसरी ओर वर्ग अन्तराल ५०–६० मे d का मान +१ लिख दिया है उससे अगले वर्ग अन्तराल ६०–७० में d का मान भरिये ?

C. I.	f	d
२०—३०	८	–२
३०—४०	९	–१
४०—५०	१७	०
५०—६०	१४	१
६०—७०	१२	+....
	N=६०	

(३६)

C. I.	f	d
२०—३०	८	−२
३०—४०	६	−१
४०—५०	१७	०
५०—६०	१४	+१
६०—७०	१२	+२
	N=६०	

---

(६५) एक कक्षा के छात्रों का बारम्बारता बंटन निम्न हैं–

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	संचयी बारम्बारता (cf)
०— ५	४	४
५—१०	७	११
१०—१५	९	२०
१५—२०	३	२३
२०—२५	१	२४

उपरोक्त बंटन में N=………अत: N/२=…………

४ वे छात्र के अंक किस वर्ग अन्तराल में है=…………

११ वें छात्र के अंक ,, ,, =…………

२० वें ,, ,, ,, =…………

$\frac{N}{२}$ अर्थात् १२ वें छात्र के अंक किस वर्ग अन्तराल में है

अत: मध्यांक वर्ग=……………… =…………

(३७) विचलन (d) का मान रखने के बाद f और d का गुणा करते हैं। यह fd का मान अगले स्तम्भ मे लिख देते है। f और d का गुणा चिन्ह सहित करते है। फिर प्रत्येक fd को चिन्हों का ध्यान रखते हुए जोडकर Σfd ज्ञात करते है। निम्न तालिका मे fd तथा Σfd का का मान लिखिये–

C. I.	f	d	fd
२०–३०	८	–२	–१६
३०-४०	६	–१	....
४०-५०	१७	०	०
५०–६०	१४	+१	....
६०-७०	१२	+२	....
	N=६०		Σfd=........

(३७)

C.I.	f	d	fd
२०–३०	८	–२	–१६
३०–४०	३	–१	– ३
४०–५०	१७	०	०
५०–६०	१४	१	+ १४
६०–७०	१२	+ २	+ २४
	N = ६०		Σfd = १९

(६४) एक कक्षा के कुछ छात्रों के प्राप्तांक वर्गअन्तरालो मे निम्न प्रकार है—

वर्गअन्तराल (C.I)	बारम्बारता (f)	सचयी बारम्बारता (cf)
०–१०	४	४
१०–२०	६	१०
२०–३०	८	१८
३०–४०	१०	२८
४०–५०	२	३०
	N = ३०	

४ वे छात्र के अंक किस वर्गअन्तराल मे है = ...

१० वे ,, ,, ,, = ...

वर्गअन्तराल (१०–२०) मे ५ वे से लेकर.........वे तक छात्र सम्मलित हैं।

१८ वे छात्र के अक किस वर्गअन्तराल मे है = ...

वर्गअन्तराल (२०–३०) मे.........वे से लेकर.........वें तक छात्र सम्मलित है।

$\frac{N}{2}$ अर्थात् १५ वे छात्र के अक किस वर्गअन्तराल मे सम्मलित है = ...

अत: मध्याक वर्ग = ...

(३८) Σfd निकालने के बाद मध्यमान निम्न सूत्र से ज्ञात करते है—

$$M = \frac{\Sigma fd}{N} \times i + A,$$

जहाँ A = कल्पित मध्यमान i = वर्गविस्तार

d = विचलन   ; f = बारम्बारता

उपर्युक्त सूत्र मे बायें पृष्ठ से मान रखने पर—

$$M = \frac{\Sigma fd}{N} \times i + A$$

$$= \frac{\cdots\cdots}{६०} \times \cdots\cdots + ४५$$

$$= \cdots\cdots + \cdots\cdots$$

$$= \cdots\cdots\cdots\cdots$$

(६३) वाये पृष्ठ की सारणी से,

छात्रो की संख्या N=……, N/२=……

२ वे छात्र के अक किस वर्ग अन्तराल में है=……

१२ वे ,, ,, ,, =……

वर्ग अन्तराल ५–१० मे ३ वें से लेकर १२ वे छात्र तक के अंक शामिल है ।

१८ वे छात्र के अक किस वर्गअन्तराल मे है=……

वर्गअन्तराल १०–१५ मे १३ वे से लेकर…… वे छात्र तक के अक शामिल है ।

अत: N/२ अर्थात् १५ वे छात्र के अंक किस वर्गअन्तराल मे है=……

इसी वर्गअन्तराल को मध्याक वर्ग कहते है ।

---

(३८) $M = \frac{१३}{६०} \times १० + ४५$

$= २.१६ + ४५$

$= ४७.१६$

(३९) एक कक्षा के ४५ छात्रों के प्राप्तांक वर्गअन्तरालों में निम्न हैं—

वर्गअन्तराल (C.I.)	बारम्बारता (f)
०– ५	५
५–१०	१०
१०–१५	१४
१५–२०	१२
२०–२५	४
	N=४५

लघुविधि से मध्यमान ज्ञात करने के लिए सबसे पहले सारणी के बीच के वर्गअन्तराल के मध्यमूल्य को कल्पित मध्यमान मान लेते हैं। उपरोक्त बारम्बारता वंटन में कल्पित मध्यमान (A)=……………

(६२)

(C.I.)	(f)	(cf)
०– ५	२	२
५–१०	१०	१२
१०–१५	६	१८
१५–२०	७	२५
२०–२५	३	२८
२५–३०	२	३०
	N=३०	

वर्गअन्तराल में मध्यांक—

(६२) एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों को वर्गअन्तरालों में निम्न प्रकार दिया है—

वर्गअन्तराल (C.I)	बारम्बारता (f)	संचयी बारम्बारता (cf)
०– ५	२	....
५–१०	१०	....
१०–१५	६	....
१५–२०	७	....
२०–२५	३	....
२५–३०	२	....

किसी बारम्बारता बंटन में मध्यांक ज्ञात करने के लिए सर्व प्रथम हम..............................ज्ञात करते हैं।

उपरोक्त सारणी के रिक्त स्थानों में प्रत्येक वर्गअन्तराल की संचयी बारम्बारता (cf) भरिये ? यह cf भी पिछली विधि के अनुसार ही निकाली जाती है।

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(३) कल्पित मध्यमान (A)=१२.५

(४०) कल्पित मध्यमान ज्ञात करने के बाद d तथा fd का मान चिन्हों का ध्यान रखते हुए सारणी के स्तम्भ–३ व ४ में रखा जाता है। निम्न सारणी के स्तम्भ ३ व ४ में क्रमश: d तथा fd का मान रखिये ?

C.I.	f	d	fd
०– ५	५	....	....
५–१०	१०	....	....
१०–१५	१४	०	....
१५–२०	१२	+१	....
२०–२५	४	....	....
	N=४५		Σfd=........

(६१) $N = २५, \frac{N+१}{२} = १३$

मध्यांक = १४

(४०)

C.I.	f	d	fd
०– ५	५	–२	–१०
५–१०	१०	–१	–१०
१०–१५	१४	०	०
१५–२०	१२	+१	+१२
२०–२५	४	+२	+८
	N = ४५		Σfd = ०

(६१) कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न है—

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
७	१	१
८	३	४
९	८	१२
१४	७	१९
१५	६	२५
	N = २५	

उपरोक्त सारणी से, N = ...,

$$\therefore \frac{N+1}{2} = \ldots\ldots$$

अतः मध्यांक = ...

(४१) Σfd का मान निकालने के बाद मध्यमान निम्न सूत्र से ज्ञात कर लेते हैं ।

$$M = \frac{\Sigma fd}{N} \times i + \cdots\cdots$$

बायें पृष्ठ से,

$\Sigma fd = \cdots\cdots$, $N = \cdots\cdots$, $i = \cdots\cdots$ तथा $A = \cdots\cdots$

$$\therefore M = \frac{\cdots\cdots}{४५} \times \cdots + \cdots\cdots$$

$$= \cdots\cdots \cdot \cdots\cdots$$

$$= \cdots\cdots\cdots$$

---

(६०) मध्यांक $= \frac{N+१}{२}$ वें छात्र के प्राप्तांक

$= \frac{२२+१}{२}$ " "

$= ११·५$ " "

मध्यांक $= ८$

(४१) $M = \frac{\Sigma fd}{N} \times i + A$

$\Sigma fd = ०$, $N = ४५$, $i = ५$ तथा $A = १२.५$

$\therefore M = \frac{०}{४५} \times ५ + १२.५$

$= ० + १२.५$

$= १२.५$

---

(६०) निम्नलिखित बारम्बारता बंटन का मध्यांक ज्ञात कीजिये ?

प्राप्तांक	बारम्बारता	संचयी बारम्बारता
X	f	cf
३	२	२
४	१	३
५	६	९
६	८	१७
७	३	२०
८	२	२२
	N=२२	

मध्यांक=…………वें छात्र के प्राप्तांक

=…………वें छात्र के प्राप्तांक

=………… ,, ,,

संचयी बारम्बारता बंटन से हम देखते है कि उपरोक्त छात्र के प्राप्तांक संचयी बारम्बारता १७ में शामिल है ।

अत: मध्यांक=…………

(४२) एक कक्षा के कुछ छात्रों द्वारा गणित की परीक्षा में प्राप्त अंक वर्ग-अन्तराल के रूप में निम्न हैं। इनका लघुविधि से मध्यमान ज्ञात कीजिये ?

वर्गअन्तराल (C.I.)	बारम्बारता (f)	विचलन (d)	बार० × विचलन (f×d)
०–१०	८	....	....
१०–२०	१३	....	....
२०–३०	२२	....	....
३०–४०	१८	०	....
४०–५०	१६	....	....
५०–६०	११	....	....
६०–७०	६	....	....

$\Sigma fd =$ ....

उपरोक्त बंटन में कल्पित मध्यमान $(A) =$ ....

(५६) एक कक्षा के २४ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न है। मध्यांक ज्ञात कीजिये ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
१०	३	३
११	१०	१३
१२	७	२०
१३	४	२४
	N = २४	

हम जानते हैं कि मध्यांक $= \dfrac{\text{........} + १}{२}$ वें छात्र के प्राप्तांक

= ....

= १२·५ वें छात्र के प्राप्तांक

संचयी बारम्बारता स्तम्भ से हम देखते हैं कि—

३ वें छात्र तक द्वारा प्राप्त अंक = ...

१३ वें „ „ = ...

अत: ४ वें से १३ वें छात्र तक के प्राप्तांक = ...

इस ४ वें से १३ वें छात्र वें तक के प्राप्तांक में $\dfrac{N+१}{२}$ अर्थात् १२·५ वें छात्र के प्राप्तांक भी शामिल हैं।

अत: मध्यांक = ...

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(५२)

(C. I.)	(f)	(d)	(fd)
०–१०	८	−३	−२४
१०–२०	१३	−२	−२६
२०–३०	२२	−१	−२२
३०–४०	१८	०	०
४०–५०	१६	+१	१६
५०–६०	११	+२	२२
६०–७०	९	+३	२७
	N = १००		Σfd = −४

A = ३५

(४३) बायें पृष्ठ से,

$\Sigma fd = \cdots\cdots$, $N = \cdots\cdots$, $i = \cdots\cdots$ तथा $A = \cdots\cdots$

अतः $M = \dfrac{\cdots\cdots}{N} \times i + \cdots\cdots$

$= \dfrac{\cdots\cdots}{\cdots\cdots} \times \cdots + \cdots\cdots$

$= \cdots\cdots\cdots\cdots + \cdots\cdots$

$= \cdots\cdots\cdots\cdots$

---

(५८) मध्यांक $= \dfrac{N+१}{२}$ वें छात्र के प्राप्तांक

$= \dfrac{३१+१}{२}$,, ,,

$= १६$,, ,,

१६ वे छात्र तक के प्राप्तांक $= २१$

१५ वें से १६ वे छात्र तक के प्राप्तांक $= २१$

१६ वें

मध्यांक $= २१$

(५८) एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न है—

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
१८	२	२
१९	९	११
२०	३	१४
२१	५	१९
२२	७	२६
२३	२	२८
२४	३	३१

हम जानते हैं कि मध्यांक = ...वें छात्र का प्राप्तांक

= ....वें छात्र का प्राप्तांक

= ....वें छात्र का प्राप्तांक

संचयी बारम्बारता स्तम्भ से हम देखते है कि—

१९ वें छात्र तक के प्राप्तांक = ...

१५ वें से १९ वें छात्र तक के प्राप्तांक = ...

१५ वें से १९ वें छात्र तक के प्राप्तांकों में $\frac{N+१}{२}$ वें अर्थात् ....वें छात्र के प्राप्तांक भी शामिल हैं।

अतः मध्यांक = ...

---

(४३) $\Sigma fd = -४$, $N = १००$, $i = १०$, $A = ३५$

$$\therefore M = \frac{\Sigma fd}{N} \times i + A$$

$$= \frac{-४}{१००} \times १० + ३५$$

$$= -.३३ + ३५$$

$$= ३४.६७$$

(४४) निम्न वर्गअन्तरालों का लघुविधि से मध्यमान ज्ञात कीजिये ?

वर्गअन्तराल (C. I)	बारम्बारता (f)	विचलन (d)	बारम्बारता × विचलन (fd)
३०-४०	३	....	....
४०-५०	५	....	....
५०-६०	६	....	....
६०-७०	१२	०	....
७०-८०	७	....	....
८०-९०	२	....	....
	N = ३५		Σfd = ...

(i) उपरोक्त सारणी मे रिक्त स्थान भरिये ?

(ii) उपरोक्त सारणी से—

A = ..., Σfd = ..., N = ..., i = ...

अत: $M = \dfrac{\cdots\cdots}{३५} \times \cdots + \cdots\cdots$

$= \cdots\cdots\cdots\cdots + \cdots\cdots$

$= \cdots\cdots$

---

(५७) मध्यांक $= \dfrac{N+१}{२}$ वें छात्र के प्राप्तांक

$= \dfrac{२३+१}{२}$ वें छात्र के प्राप्तांक

= १२ वे छात्र के प्राप्तांक

९ वे छात्र तक के प्राप्तांक = २२

१६ वें ,, ,, = २३

९ वें से १६ वें छात्र तक के प्राप्तांक = २३

मध्यांक = २३

	C.I.	f	d	fd
(४४)	३०–४०	३	–३	– ९
	४०–५०	५	–२	–१०
	५०–६०	६	–१	– ६
	६०–७०	१२	०	०
	७०–८०	७	+ १	७
	८०–९०	२	+ २	४
		N = ३५		Σfd = –१४

उपरोक्त सारणी से,

$A = ५५,\ \Sigma fd = -१४,\ N = ३५,\ i = १०$

$\therefore M = \frac{-४}{३५} \times १० + ५५ = ५१$

---

(५७) निम्न बारम्बारता बंटन का मध्यांक ज्ञात कीजिये ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
२०	१	१
२१	३	४
२२	५	९
२३	७	१६
२४	४	२०
२५	३	२३
	N = २३	

मध्यांक = $\frac{\cdots + १}{२}$ वें छात्र के प्राप्तांक

= $\frac{\cdots + १}{२}$ वे छात्र के प्राप्तांक = ········वें छात्र के प्राप्तांक

उपरोक्त संचयी बारम्बारता स्तम्भ से हम देखते है कि—

९ वें छात्र तक के प्राप्तांक = ········

१६ वें छात्र तक के प्राप्तांक = ········

इस प्रकार·······वे से ·······वें छात्र तक के प्राप्तांक = ········

इसमें $\frac{N + १}{२}$ वें छात्र के प्राप्तांक भी शामिल है।

अत: मध्यांक = ··········

(४५) इस प्रकार जब प्राप्तांक वर्गअन्तरालों में दिये हों तो मध्यमान ज्ञात करने की दो विधियाँ हैं—

(अ) दीर्घविधि ; M=……………………………

(ब) लघुविधि ; M=……………………………

(५६) बाये पृष्ठ के बारम्बारता वंटन से --

$N = १७$

हम जानते है कि मध्यॉंक $= \dfrac{N - १}{२}$ वे पद का मान

$= \dfrac{\cdots\cdots + १}{२}$ वे पद का मान

$=$ ९ वें पद का मान

संचयी बारम्बारता स्तम्भ से हम देखते है कि

१ वे छात्र तक के प्राप्तांक $= \cdots\cdots$

३ वे ,, ,, $= \cdots\cdots$

अत: २ वे से ३ वे छात्र तक द्वारा प्राप्त अंक $=$ ४

७ वे छात्र तक के प्राप्तांक $= \cdots\cdots$

अत: ४ वे से $\cdots\cdots$ वे छात्र तक द्वारा प्राप्त अंक $= \cdots\cdots$

१३ वे छात्र तक के प्राप्तांक $= \cdots\cdots$

अत: $\cdots\cdots$ वे से $\cdots\cdots$ वे छात्र तक द्वारा प्राप्त अंक $= \cdots\cdots$

उपरोक्त विवरण से हम देखते है कि प्राप्तांक ६, ८ वे से १३ वे छात्र तक ने प्राप्त किया है जिसमे कि $\dfrac{N + १}{२}$ वे अर्थात ९ वें छात्र के प्राप्तांक भी हैं।

अत: मध्यॉंक $=$

$$= \frac{N + १}{२} \text{ वे छात्र के प्राप्तांक}$$

$= \cdots\cdots\cdots$

(५५) (अ) $M = \dfrac{\Sigma fx}{N}$

(ब) $M = \dfrac{\Sigma fd}{N} \times i + A$

समीक्षा–अध्याय २

उपरोक्त अध्याय मे हमने विभिन्न आंकड़ों में मध्यमान ज्ञात करना सीखा है ।

(अ) अव्यवस्थित आंकड़ों —जब प्राप्तांक अव्यवस्थित हो तब

मध्यमान $M = \frac{\Sigma X}{N}$, जहाँ ΣX =प्राप्तांकों का कुल योग तथा

N = छात्रो की सख्या

(ब) व्यवस्थित आंकड़ों—

(i) जब प्राप्तांक बारम्बारता बंटन में हो तो, $M = \frac{\Sigma fX}{N}$

जहाँ f =प्राप्तांक की बारम्बारता तथा X = प्राप्तांक

(ii) जब प्राप्तांक वर्गअन्तराल में हो तो मध्यमान निकालने की दो विधियाँ हैं ।

दीर्घविधि—$M = \frac{\Sigma fx}{N}$ जहाँ f =प्राप्तांक की बारम्बारता तथा x = वर्गअन्तराल का मध्यमूल्य

लघुविधि—$M = \frac{\Sigma fd}{N} \times i + A$ जहाँ f =प्राप्तांक की बारम्बारता, d =कल्पित मध्यमान से विचलन तथा A =कल्पित मध्यमान है ।

अगले अध्याय मे हम अव्यवस्थित आंकड़ों में, जब छात्रों की संख्या विषम व सम हो तथा व्यवस्थित आँकड़ों में मध्यांक की गणना करना सीखेगे ।

अध्याय–३ मध्यांक की गणना

(४६) **अव्यवस्थित आंकड़ों में मध्यांक—**

एक समूह ने किसी परीक्षा में निम्न अंक प्राप्त किये—

१, १ ४, ७, ६

उपरोक्त प्राप्तांकों का मध्यांक ज्ञात कीजिये ?

हम जानते है कि मध्यांक समूह के बीच का अंक अर्थात् वह अंक होता है जिसके ऊपर........०० तथा नीचे........०० छात्रों के अंक है।

यदि छात्रों की संख्या N है

तब बीच का अंक $= \frac{N+१}{२}$ वें छात्र का प्राप्तांक

अतः मध्यांक $= \frac{N+१}{२}$ वें छात्र का प्राप्तांक

दिये समूह में,

N = ...,

∴ मध्यांक $= \frac{N+१}{२}$ वे छात्र का प्राप्तांक

$= \frac{\text{.......} + १}{२}$ वे छात्र का प्राप्तांक

= ....३ वें छात्र का प्राप्तांक

= ....

---

(५५)

X	f	cf
३	१	१.
४	२	३.
५	४	७
६	६	१३
७	३	१६
८	१	१७
	N = १७	

(४६) ५०%—५०%

$N = ५$

मध्यांक $= \frac{५ + १}{२}$ वें छात्र का प्राप्तांक

$=$ ३ वे छात्र का प्राप्तांक

$=$ ४

(४७) एक कक्षा के कुछ छात्रों ने एक परीक्षा में निम्न अंक प्राप्त किये। इनका मध्यांक ज्ञात कीजिये ?

७, ६, १८, २४, २५, २७, ३०

यहां N = ……………

प्राप्तांकों का मध्यांक $= \dfrac{N+१}{२}$ वें छात्र का प्राप्तांक

अत: मध्यांक $= \dfrac{\cdots\cdots + १}{२}$ वें छात्र का प्राप्तांक

= ……………वें छात्र का प्राप्तांक

= ……………

---

(५४)

(X)	(f)	(cf)
१०	३	३
११	६	१२
१२	८	२०
१३	४	२४
	N=२४	

३ , ६

संचयी बारम्बारता = ६ + ३ = १२

(४७) N=७

मध्यांक $= \frac{७ + १}{२}$ वें छात्र का प्राप्तांक

= ४ वें छात्र का प्राप्तांक

= २४

---

(५४) निम्न सारणी में कुछ छात्रों के प्राप्तांकों का बारम्बारता बंटन दिया हुआ है —

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
१०	३	३
११	९	....
१२	८	....
१३	४	....

(i) उपरोक्त बंटन में प्राप्तांक १० की बारम्बारता........है तथा यह पहला प्राप्तांक है अतः इसकी संचयी बारम्बारता = ३

(ii) प्राप्तांक १० की बारम्बारता........है तथा प्राप्तांक ११ की बारम्बारता........है ।

अतः प्राप्तांक ११ की संचयी बारम्बारता = ... + ... = ...

इसी प्रकार उपरोक्त बंटन में प्राप्तांक १२ व १३ की संचयी बारम्बारता भरिये ?

(१८) एक कक्षा के कुछ छात्रों के प्राप्तांक निम्नलिखित हैं। इनका मध्यांक बताइये ?

३, ४, ५, ५, ६, ६, ७, ८, ६

उपरोक्त समूह में,

N = ...

∴ मध्यांक = ...वें छात्र का प्राप्तांक

= ....वें छात्र का प्राप्तांक

= ....वें छात्र का प्राप्तांक

= ....

---

(५३) (i) ५, २

(ii) ५ + ८ + २ = १५ ; सं० बा० = १५

(iii) २ + ५ + ८ + २ = १७ ; सं० बा० = १७

(५३) **बारम्बारता बंटन में मध्यांक—(संचयी बारम्बारता)**

एक कक्षा के कुछ छात्रों के अंकों का बारम्बारता बंटन निम्न प्रकार है—

प्राप्तांक (X)	बारम्बारता (f)
३	२
५	८
६	५
७	२
	N=१७

(i) उपरोक्त बंटन में प्राप्तांक ३, २ छात्रों ने तथा प्राप्तांक ५, ८ छात्रों ने प्राप्त किये। इसी प्रकार प्राप्तांक ६,·······छात्रों ने तथा प्राप्तांक ७,·······छात्रों ने प्राप्त किये है।

(ii) प्राप्तांक ५ अथवा उससे कम, ८ + २ = १० छात्रों ने प्राप्त किये क्योंकि प्राप्तांक ५, ८ छात्रों ने तथा उससे कम प्राप्तांक ३, २ छात्रों ने प्राप्त किये है।

यह १० ही प्राप्तांक ५ की संचयी बारम्बारता कहलाती हैं।

(iii) इसी प्रकार प्राप्तांक ६ अथवा उससे कम, ५ + ······· + ·······
= ·······छात्रों ने प्राप्त किये है।
अत: प्राप्तांक ६ की संचयी बारम्बारता = ·······

(iv) प्राप्तांक ७ अथवा उससे कम,······· + ······· + ······· + ·······
= ·······छात्रों ने प्राप्त किये हैं।
अत: प्राप्तांक ७ की संचयी बारम्बारता = ··········

---

(४८) N = ९

$\therefore$ मध्यांक $= \dfrac{N+१}{२}$ वें छात्र का प्राप्तांक

$= \dfrac{९+१}{२}$ वें छात्र का प्राप्तांक

= ५ वें छात्र का प्राप्तांक

= ६

(४६) एक कक्षा के ४ छात्रो के प्राप्तांक निम्नलिखित हैं। इनका मध्यांक ज्ञात कीजिये ?

३, ४, ८, ६

हम जानते है कि मध्यांक $= \frac{\text{.........} + १}{२}$ वें छात्र का प्राप्तांक

उपरोक्त प्राप्तांकों मे, $N = ४$

$\therefore$ मध्यांक $= \frac{४ + १}{२}$ वे छात्र का प्राप्तांक

$=$ २·५ वे छात्र का प्राप्तांक

२·५ वें छात्र का प्राप्ताक २ वे तथा ३ वें छात्र का प्राप्तांक का औसत होगा। अतः उपरोक्त प्राप्तांकों का

मध्यांक $= \frac{\text{२ वें छात्र का प्राप्तांक} + \text{३ वें छात्र का प्राप्तांक}}{२}$

$= \frac{\text{................} + \text{........} + \text{........}}{२}$

$=$ ....

(४६) मध्यांक $= \frac{N + १}{२}$ वें छात्र का प्राप्तांक

मध्यांक $= \frac{४ + ८}{२}$

$= ६$

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(५२) बारम्बारता बंटन में मध्यांक

अभी तक हमने अव्यवस्थित आँकड़ों में मध्याँक निकालना जाना है ।

किसी समूह का मध्याँक $= \left(\frac{N + १}{२}\right)$ वें छात्र का प्राप्तांक

बारम्बारता बंटन में मध्याँक ज्ञात करने के लिए सर्वप्रथम हमको संचयी बारम्बारता ज्ञात करनी होगी । अगले खण्डो में हम आपको संचयी बारम्बारता तथा फिर संचयी बारम्बारता की सहायता से मध्याँक निकालना बतलायेंगे ।

(५०) निम्नलिखित छात्रों के प्राप्तांकों का मध्यांक ज्ञात कीजिये ?

२, २, ६, ७, ८, ६

हम जानते है कि मध्यांक $= \dfrac{\cdots\cdots + \cdots\cdots}{२}$ वें छात्र का प्राप्तांक

उपरोक्त प्राप्तांकों में N = ६

∴ मध्यांक $= \dfrac{६ + १}{२} =$ ………………वें छात्र का प्राप्तांक

यह पद का मान……………वें तथा……………वें छात्र का प्राप्तांक का……………होगा। अतः

मध्यांक $= \dfrac{\text{……वें छात्र का प्राप्तांक + ……वें छात्र का प्राप्तांक}}{२}$

$= \dfrac{\cdots\cdots + \cdots\cdots}{२}$

= ……………

---

(५१) N = ६

मध्यांक $= \dfrac{N + १}{२}$ वें छात्र का प्राप्तांक

$= \dfrac{१० + १}{२}$ वें छात्र का प्राप्तांक

= ५.५ वें छात्र का प्राप्तांक

इसलिए मध्यांक ५ वें तथा ६ वे छात्र का प्राप्तांक का औसत होगा।

मध्यांक $= \dfrac{\text{५ वें छात्र का प्राप्तांक + ६ वें छात्र का प्राप्तांक}}{२}$

$= \dfrac{१० + ११}{२}$

= १०.५

(५०) मध्याँक $= \dfrac{N+१}{२}$ वें छात्र का प्राप्ताँक

$= \dfrac{६+१}{२}$ वें छात्र का प्राप्ताँक $=$ ३·५ वें छात्र का प्राप्ताँक

यह पद का मान ३ वें तथा ४ वें छात्र का प्राप्ताँक का औसत होगा। अतः

मध्याँक $= \dfrac{\text{३ वें छात्र का प्राप्ताँक} + \text{४ वें छात्र का प्राप्ताँक}}{२}$

$= \dfrac{६+७}{२}$

$=$ ६·५

अगला खण्ड (५१) नीचे उल्टा छपा हुआ है इसे पढ़ने के लिए अब आप पुस्तक को उलट लीजिये और पहले की ही तरह अगले खण्डों का अध्ययन कीजिये।

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(५१) निम्नलिखित छात्रों द्वारा प्राप्त अंकों का मध्यांक ज्ञात कीजिये ?

५, ७, ८, ८, १०, ११, १२, १३, १८, २०

यहां N = ...

∴ मध्यांक $= \dfrac{N+८}{२}$ वें छात्र का प्राप्ताँक

इसलिए मध्यांक...........वें तथा...........वें छात्र का प्राप्ताँक.........होगा।

अतः मध्यांक $= \dfrac{\ldots\ldots + \ldots\ldots}{२}$

$= \dfrac{\ldots\ldots + \ldots\ldots}{२}$

$=$ ....

केन्द्रीय प्रवृत्ति के मान

(अभिक्रमित अध्ययन-शाखायित प्रकार)

Measures of Central Tendencies

(A PROGRAMMED TEXT-BRANCHING STYLE)

Developed by :

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*

1978

नाम .... आयु ....

कक्षा .... समय ....

विद्यालय ....

शिक्षकों से

स्व–अध्ययन की अनेक विधियाँ है। जिनमें से अभिक्रमित अध्ययन-शाखायित प्रकार (Programmed Learning - Branching Style) भी एक विधि है। इस विधि को सर्वप्रथम नार्मन ए० क्राउडर ने प्रस्तुत किया था। इस विधि में सम्पूर्ण विषय वस्तु को खण्डों में विभक्त कर दिया जाता है। इसके प्रत्येक खण्ड मे लीनियर स्टाइल की अपेक्षा अधिक विषय वस्तु का समावेश होता है। प्रत्येक खण्ड मे एक प्रत्यय (Concept) को विभिन्न प्रकार के उदाहरणों के द्वारा पूर्ण रूप से समझाने का प्रयत्न किया जाता है। इसके लिए कुशल अध्ययन के अनेक तरीके प्रयुक्त होते हैं। प्रत्येक खण्ड मे एक प्रत्यय को समझाने के पश्चात उस पर आधारित बहु विकल्प प्रश्न होता है। यदि छात्र शुद्ध उत्तर चयन करता है तो उसे उस उत्तर से सबंधित पृष्ठ देखने पर ज्ञात होता है कि उसका उत्तर सही है और उसे अगले खण्ड की विषय वस्तु को पढ़ने का निर्देश होता है। यदि वह गलत उत्तर चयन करता है तो उसको उस खण्ड की ओर अग्रसरित कर दिया जाता है जहाँ उसकी गलती सुधारने का समुचित प्रबन्ध होता है। गलती का स्पष्टीकरण हो जाने पर वह पुनः मूल खण्ड की पाठ्यवस्तु को पढ़कर प्रश्न का सही उत्तर चुनता है। यदि उसका चुना हुआ उत्तर सही होता है तो वह अगले खण्ड पर जाता है। इस विधि मे भी छात्र को निरन्तर प्रतिपुष्टि मिलती रहती है।

छात्रों से

प्रस्तुत पुस्तक मे हम आपको सीखने के एक आधुनिक व आसान विधि के बारे में बतायेंगे। आगे आने वाले पृष्ठों पर सांख्यिकी के एक अध्याय को इस तरह लिखा गया है कि आप इसे स्वयं बिना किसी अध्यापक की सहायता से आसानी से समझ जायेगें तथा साथ-साथ यह आपको याद भी हो जायेगा।

पूरे पाठ को छोटे-छोटे खण्डों मे बाँट दिया गया है। प्रत्येक खण्ड के अन्त में एक प्रश्न दिया हुआ है जिसके दो या दो से अधिक संभावित उत्तर दिये हुए है। आप उपयुक्त उत्तर छाटिये तथा उसके सामने दिये हुए निर्देश के अनुसार अगले खण्ड को पढ़िये।

उदाहरण के लिए खण्ड (१) के अन्त में तीन संभावित उत्तर दिये हैं। यदि आप प्रथम उत्तर चुनते है तो सामने दिये निर्देश के अनुसार आपको खण्ड (२९) पढ़ना है। इसी प्रकार यदि आप द्वितीय उत्तर चुनते हैं तो आपको खण्ड (३०) पढ़ना होगा। तृतीय उत्तर चुनने पर खण्ड (३१) पढ़ना होगा। प्रत्येक खण्ड को पढ़ने के बाद आपको यह निर्देश दिया होगा कि आपको उसके बाद कौन सा खण्ड पढ़ना है।

यदि आपका उत्तर गलत भी है तो भी परेशान होने की आवश्यकता नही है। खण्डों में यह बताया गया है कि आपने कहाँ गलती की है तथा उसे सुधारने का समुचित प्रबन्ध है।

यदि आप उपरोक्त विधि से पाठ पढ़ेगें तो पाठ निश्चत रूप से आपको समझ में आ जायेगा।

प्रस्तावना

साँख्यिकी में सामग्री संग्रहण का उद्देश्य किसी विषय के बारे में जानकारी प्राप्त करना होता है। पर संग्रहीत सामग्री की राशि अधिक होने के कारण उसे समझ सकना बहुत कठिन हो जाता है। अगर कोई ऐसी संख्या या प्राप्तांक जो इस समूह का स्तर प्रदर्शित करें, मालूम हो तो इस कठिनाई से बचा जा सकता है। साँख्यिकी में ऐसा संख्या या प्राप्तांक को दिये हुए समूह के 'केन्द्रीय प्रवृत्ति के मान' कहते है ।

अध्याय-१ में हम केन्द्रीय प्रवृत्ति के विभिन्न मानों—मध्यमान, मध्यांक बहुलांक की परिभाषा के बारे में जानेगे।

अब आप पृष्ठ २ पर खण्ड (१) से अध्ययन करना शुरू कीजिये ?

अध्याय १

केन्द्रीय प्रवृत्ति के विभिन्न मान

खण्ड (१)— **प्रस्तावना से—**

एक कक्षा के ५ छात्रों के प्राप्तांक १५, २०, ३०, ३५, व ३५ है।

किसी कक्षा या समूह के छात्रों का स्तर ज्ञात करने के लिए हमें किसी ऐसे अंक की आवश्यकता होती है जो उस कक्षा या समूह के सम्पूर्ण छात्रों का प्रतिनिधित्व करता हो। अंकगणित में यही अंक औसत होता है। किसी समूह का

$$\text{औसतमान} = \frac{\text{अंको का कुल योग}}{\text{छात्रों की संख्या}}$$

उपरोक्त विधि से ज्ञात किया हुआ औसत मान ही सांख्यिकी में मध्यमान कहलाता है। इस प्रकार किसी कक्षा या समूह का

$$\text{मध्यमान} = \frac{\text{अंको का कुल योग}}{\text{छात्रों की संख्या}}$$

मध्यमान को M से प्रदर्शित करते है। उपरोक्त ५ छात्रों के प्राप्तांकों का

$$\text{मध्यमान} = \frac{१५ + २० + ३० + ३५ + ३५}{५} = \frac{१३५}{५} = २७$$

इस प्रकार किसी कक्षा का मध्यमान छात्रों द्वारा प्राप्त अंकों के कुल योग को उन छात्रों की संख्या से भाग देकर प्राप्त करते हैं।

प्रश्न : नीचे कुछ छात्रों के प्राप्तांक दिये है। इनका मध्यमान ज्ञात कीजिये ?

१५, १८, २०, २२, २५, २७, २७, ३०, ३२

यदि आपका उत्तर है—

(अ) २१६ तो पृष्ठ ५१ पर खण्ड (२९) देखिये।

(ब) २४ पृष्ठ ५२ पर खण्ड (३०) देखिये।

(स) उपरोक्त दोनों में से कोई नहीं। पृष्ठ ५३ पर खण्ड (३१) देखिये।

खण्ड (२)-- **खण्ड (३०) से**

खण्ड (१) में दिये ५ छात्रों के प्राप्तांक १५, २०, ३०, ३५, ३५ हैं। इन प्राप्तांकों में बीच का अंक या वह अंक जिसके ऊपर ५०% छात्रों (अर्थात २ छात्रों) के तथा नीचे भी ५०% छात्रों के (अर्थात २ छात्रों के) अंक है, ३० है। यही बीच का अंक ३० मध्यांक कहलाता है।

अतः मध्यांक किसी समूह का वह अंक है जिसके ऊपर ५०% छात्रों के अंक तथा नीचे ५०% छात्रों के अंक होते हैं।

प्रश्न:--एक कक्षा के कुछ छात्रों के प्राप्तांक क्रमशः ५, ७, १०, १८, २१, २५ व २८ हैं। इनका मध्यांक ज्ञात कीजिये ?

यदि आपका उत्तर है—

(अ) १६ तो पृष्ठ ५४ पर खण्ड (३२) देखिये।

(ब) १८ पृष्ठ ५५ पर खण्ड (३३) देखिये।

(स) १६ व १८ मे से कोई नहीं। पृष्ठ ५६ पर खण्ड (३४) देखिये।

खण्ड (३)— **खण्ड(३३)से**

वह अंक जो समूह में सबसे अधिक बार आता है उस समूह का बहुलांक कहलाता है।

खण्ड (१) में दिये ५ छात्रों के प्राप्तांकों १५, २०, ३०, ३५, ३५ में प्राप्तांक ३५ दो बार तथा शेष सभी प्राप्तांक एक-एक बार आये हैं।

अत: बहुलांक = ३५

प्रश्न—निम्नलिखित प्राप्तांकों का बहुलांक ज्ञात कीजिये ?

यदि आपका उत्तर है—

(अ) २५ तो पृष्ठ ५७ पर खण्ड (३५) देखिये।

(ब) २७ पृष्ठ ५८ पर खण्ड (३६) देखिये।

(स) २७.७ पृष्ठ ५९ पर खण्ड (३७) देखिये।

(द) २७ पृष्ठ ६० पर खण्ड (३८) देखिये।

खण्ड (४)— **खण्ड (३५) से**

अब तक हमने मध्यमान, मध्यांक तथा बहुलांक की **परिभाषा के** बारे में जाना है। उपरोक्त तीनों मान केन्द्रीय प्रवृत्ति **के मान** कहलाते है।

प्रश्न—वह अंक जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के अंक होते हैं केन्द्रीय प्रवृत्ति का कौन सा मान होता है।

यदि आपका उत्तर है—

(अ) मध्यमान तो पृष्ठ ६१ पर खण्ड (३९) देखिये।

(ब) मध्यांक पृष्ठ ६२ पर खण्ड (४०) देखिये।

(स) बहुलांक पृष्ठ ६३ पर खण्ड (४१) देखिये।

अध्याय १ ‖

समीक्षा

खण्ड (४०) से

पिछले अध्याय में हमने विभिन्न केन्द्रीय प्रवृत्ति के मानों अर्थात मध्यमान, मध्यांक व बहुलांक की परिभाषा के बारे में जाना है।

(i) मध्यमान वह मान है जो प्राप्त अंकों के कुल योग को उनकी संख्या से भाग देकर प्राप्त होता है।

(ii) मध्यांक वह अंक है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के प्राप्तांक हैं।

(iii) बहुलांक वह अंक है जो समूह में सबसे अधिक बार आया हो।

अगले अध्याय–२ में हम अव्यवस्थित आंकड़ों तथा व्यवस्थित आंकड़ों में दीर्घ व लघु विधि से मध्यमान की गणना करना सीखेंगे।

अब आप अव्यवस्थित आँकड़ों में मध्यमान की गणना जानने के लिए अगले पृष्ठ ६ पर खण्ड (५) का अध्ययन कीजिये।

अध्याय २ || मध्यमान की गणना

खण्ड (५)—— समीक्षा अध्याय १ से

अव्यवस्थित आँकड़ों में मध्यमान

एक परीक्षा में ५ छात्रों ने क्रमश: १६, १८, १९, २१, व २२ अंक प्राप्त किये। इनका मध्यमान ज्ञात कीजिये ?

हम जानते हैं कि किन्हीं प्राप्तांकों का मध्यमान

$$M = \frac{\text{प्राप्त अंकों का कुल योग}}{\text{छात्रों की संख्या}} \quad)$$

प्राप्त अंको के कुल योग को ΣX तथा छात्रों की संख्या को N से प्रदर्शित करते हैं। अत: $M = \frac{\Sigma X}{N}$)

उपरोक्त प्रश्न में—

प्राप्त अंकों का कुल योग $(\Sigma X) = १६ + १८ + १९ + २१ + २२$

$= ९६$

छात्रों की संख्या $(N) = ५$

अत: मध्यमान $M = \frac{\Sigma X}{N} = \frac{९६}{५} = १९.२$

इस प्रकार जब प्राप्तांक अव्यवस्थित आंकड़ों में हो तो मध्यमान ज्ञात करने का सूत्र होता है :—

$$M = \frac{\Sigma X}{N}$$

प्रश्न–कुछ छात्रो ने एक परीक्षा में क्रमश: १, ७, ८, ९, ९, ११, १६, १८, २० अंक प्राप्त किये। इनका मध्यमान क्या होगा ?

यदि आपका उत्तर है—

(अ) ११ तो पृष्ठ ६४ पर खण्ड (४२) देखिये।

(ब) ९९ पृष्ठ ६५ पर खण्ड (४३) देखिये।

(स) उपरोक्त दोनों में से कोई नही पृष्ठ ६६ पर खण्ड (४४) देखिये।

व्यवस्थित आँकड़ों में मध्यमान- (अ) बारम्बारता बंटन में

किसी कक्षा के कुछ छात्रों के प्राप्तांकों का बारम्बारता बंटन निम्नलिखित है–

प्राप्तांक (X)	बारम्बारता (f)
२	३
३	४
४	२

(i) उपरोक्त बंटन में हम देखते है कि ३ छात्रों ने २–२ अंक प्राप्त किये। इस प्रकार इन ३ छात्रों द्वारा प्राप्त कुल अंक $= ३ \times २ = f \times X = ६$

(ii) इसी प्रकार ४ छात्रों ने ३–३ अंक प्राप्त किये हैं। इस प्रकार इन ४ छात्रों द्वारा प्राप्त कुल अंक $= ४ \times ३ = f \times X = १२$

(iii) इसी प्रकार २ छात्रों द्वारा प्राप्त कुल अंक $= ४ \times २ = f \times X = ८$

इस प्रकार हम बंटन में ३, ४ व २ छात्रों द्वारा प्राप्त अंकों का अलग-२ योग f को X से गुणा करके ज्ञात कर सकते है।

क्योंकि $\text{मध्यमान} = \dfrac{\text{प्राप्त अंकों का कुल योग}}{\text{छात्रो की संख्या}}$

प्राप्त अंकों का कुल योग प्रत्येक fX को जोड़कर ज्ञात कर लेते हैं इसे ΣfX से प्रदर्शित करते हैं।

$$\therefore \text{मध्यमान} = \frac{\Sigma fX}{N}$$

उपरोक्त बंटन में, $\Sigma fX = fX + fX + fX = ६ + १२ + ८ = २६$

$$N = ९$$

$$\text{अतः मध्यमान} = \frac{२६}{९}$$

$$= २.८८$$

उदाहरण - एक कक्षा के १० छात्रों के प्राप्तांकों का बारम्बारता बंटन निम्न है। इनका मध्यमान ज्ञात कीजिये ?

प्राप्तांक	बारम्बारता	प्राप्त कुल अंक
(X)	(f)	(f×X)=(fX)
७	४	४× ७=२८
८	३	३× ८=२४
९	१	१× ९= ९
१०	२	२×१०=२०
	N=१०	(ΣfX)=८१

उपरोक्त बटन में प्राप्त कुल अंक (fX) प्रत्येक f को X से गुणा करके ज्ञात किया है तथा फिर प्राप्त अंकों का कुल योग (ΣfX) प्रत्येक fX को जोड़कर प्राप्त कर लेते है।

$$\therefore M = \frac{\Sigma fX}{N}$$

$$= \frac{८१}{१०} \text{(यहां } N=१० \text{)}$$

$$= ८.१$$

प्रश्न--एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न है। इनका मध्यमान ज्ञात कीजिये ?

प्राप्तांक	बारम्बारता	प्राप्त कुल अंक
(X)	(f)	(fX)
१०	५	....
११	४	....
१२	२	....

यदि आपका उत्तर है--

(अ) ३ तो पृष्ठ ६७ पर खण्ड (४५) देखिये।

(ब) १०.७२ पृष्ठ ६८ पर खण्ड (४६) देखिये।

वर्ग अन्तरालों में मध्यमान— दीर्घविधि

एक समूह के अंग्रेजी विषय में प्राप्तांक वर्ग अन्तराल के रूप मे निम्न हैं

वर्गअन्तराल	बारम्बारता
१०—२०	८
२०—३०	३
३०—४०	६

उपरोक्त बंटन में, ८ छात्रों ने १० से २० के बीच अंक प्राप्त किये है।
३ ,, २० से ३० ,, ,,
६ ,, ३० से ४० ,, ,,

इस प्रकार जब प्राप्तांक वर्ग अन्तरालों में दिये हो तो हम यह नहीं कह सकते हैं कि कितने छात्रों ने सही रूप से कौन सा अंक प्राप्त किया है। ऐसी स्थिति में वर्गअन्तराल के मध्यमूल्य को उस वर्ग अन्तराल का प्रतिनिधि अंक मान लेते है तथा यह मान कर चलते है कि उन छात्रों ने मध्यमूल्य के बराबर अंक प्राप्त किये हैं। मध्यमूल्य निम्न प्रकार ज्ञात करते है।

$$\text{किसी वर्ग अन्तराल का मध्यमूल्य (X)} = \frac{\text{निम्न सीमा} + \text{उच्च सीमा}}{२}$$

$$\text{वर्गअन्तराल १०—२० का मध्यमूल्य} = \frac{१० + २०}{२} = १५$$

इसी प्रकार वर्गअन्तराल २०–३० व ३०–४० के मध्यमूल्य क्रमशः २५ व ३५ होंगे।

इस प्रकार किसी वर्ग अन्तराल का मध्य मूल्य उसकी उच्च सीमा तथा निम्न सीमा के योग को दो से भाग देकर प्राप्त करते हैं।

प्रश्न— छात्रो के प्राप्तांक निम्न वर्ग अन्तरालों मे दिये हैं। रिक्त स्थान भरिये?

वर्ग अन्तराल	बारम्बारता	मध्यमूल्य
५—१५	१५	....
१५—२५	१२	....
२५—३५	१०	....
३५—४५	१३	....

अपने उत्तर की जाँच के लिए पृष्ठ ६६ पर खण्ड (४७) देखिये।

एक कक्षा के कुछ छात्रों का बंटन उनके मध्यमूल्य सहित निम्नलिखित है–

वर्गअन्तराल (C.I.)	बारम्बारता (f)	मध्यमूल्य (x)
१०—२०	८	१५
२०—३०	३	२५
३०—४०	९	३५

जब प्राप्तांक वर्गअंतराल में दिये हों तो मध्यमूल्य को ही हम प्रतिनिधि अंक मान लेते है। अतः हम कह सकते हैं कि पहले ८ छात्रों ने १५-१५ अंक प्राप्त किये है। अर्थात ८ छात्रों द्वारा प्राप्त अंकों का योग $= f \times x = ८ \times १५ = १२०$। इसी प्रकार प्रत्येक अन्तराल में प्राप्त अंकों का योग ($f \times x$) उस वर्गअन्तराल के मध्यमूल्य (x) को उसकी बारम्बारता (f) से गुणा करके प्राप्त कर सकते हैं। उपरोक्त बंटन में–

वर्गअन्तराल (C.I.)	बारम्बारता (f)	मध्यमूल्य (x)	अंकों का योग (fx)
१०—२०	८	१५	१२०
२०—३०	३	२५	७५
३०—४०	९	३५	३१५
	N=२०	कुल योग	(Σfx)=५१०

$$\text{क्योंकि मध्यमान} = \frac{\text{प्राप्त अंकों का कुल योग}}{\text{छात्रों की संख्या}} = \frac{\Sigma fx}{N} = \frac{५१०}{२०}$$

$$= २५.५$$

उदाहरण—एक कक्षा के छात्रों के प्राप्तांक निम्न वर्गअन्तरालों में दिये हैं। इनका मध्यमान ज्ञात कीजिए?

वर्गअन्तराल (C.I.)	बारम्बारता (f)
५—१५	१५
१५—२५	१२
२५—३५	१०
३५—४५	५
	N=४२

बंटन में मध्यमान ज्ञात करने से पहले प्रत्येक वर्गअन्तराल का मध्यमूल्य निकालना होगा। फिर प्राप्तांकों का कुल योग ज्ञात करने के लिए प्रत्येक वर्गअन्तराल में अंकों का योग (fX) ज्ञात करना होगा। पिछले पृष्ठ बंटन में मध्यमूल्य (X) तथा fX ज्ञात करने पर—

वर्गअन्तराल (C.I.)	बारम्बारता (f)	मध्यमूल्य (X)	प्राप्त अंकों का योग (fX)
५—१५	१५	१०	१५०
१५—२५	१२	२०	२६०
२५—३५	१०	३०	३००
३५—४५	५	४०	२००
	N=४२	कुल योग (ΣfX)	=८६०

अतः मध्यमान $(M) = \frac{\Sigma fX}{N} = \frac{८६०}{४२} = २१\cdot१६$

जब प्राप्तांक वर्गअन्तरालों में हो तब $M = \frac{\Sigma fX}{N}$ **जहाँ X मध्यमूल्य है।**

प्रश्न—निम्न बंटन का मध्यमान ज्ञात कीजिये ?

वर्गअन्तराल (C.I.)	बारम्बारता (f)	मध्यमूल्य (X)	बार० × मध्यमूल्य (fX)
०—१०	१५	५	....
१०—२०	१२	१५	....
२०—३०	२०	२५	....
३०—४०	२३	३५	....
	N=७०		

यदि आपका उत्तर है—

(अ) १·१४ तो पृष्ठ ७२ पर खण्ड (५०) देखिये।

(ब) २२·२८ पृष्ठ ७३ पर खण्ड (५१) देखिये।

(स) उपरोक्त दोनों के अलावा कोई। पृष्ठ ७४ पर खण्ड (५२) देखिये।

वर्ग अन्तरालों में मध्यमान-लघु विधि

अभी तक हमने अव्यवस्थित आंकड़ों में मध्यमान $\left(M=\frac{\Sigma X}{N}\right)$, व्यवस्थित आंकड़ों में मध्यमान $\left(M=\frac{\Sigma f X}{N}\right)$ तथा जब प्राप्तांक वर्ग अन्तराल में दिये हों तब मध्यमान $\left(M=\frac{\Sigma fx}{N}\right)$ ज्ञात करना जाना है।

निम्नलिखित सारणी में प्रत्येक वर्ग अन्तराल की बारम्बारता तथा उसका मध्यमूल्य दिया है। इनका मध्यमान ज्ञात कीजिए ?

वर्ग अन्तराल (C. I)	बारम्बारता (f)	मध्यमूल्य (x)
२०-३०	८	२५
३०-४०	९	३५
४०-५०	१७	४५
५०-६०	१४	५५
६०-७०	१२	६५

जब प्राप्तांक वर्ग अन्तरालों में दिए हों तब मध्यमान $M=\frac{\Sigma fx}{N}$

उपरोक्त सारणी में—

वर्ग अन्तराल　२०-३०　का　fx = ८ × २५ = २००

वर्ग अन्तराल　३०-४०　का　fx = ९ × ३५ = ३१५

वर्ग अन्तराल　४०-५०　का　fx = १७ × ४५ = ७६५

वर्ग अन्तराल　५०-६०　का　fx = १४ × ५५ = ७७०

वर्ग अन्तराल　६०-७०　का　fx = १२ × ६५ = ७८०

उपरोक्त fx ज्ञात करने में हम देखते हैं कि f तथा x के गुणा बहुत बड़े-बड़े हो जाते हैं तथा गलती होने की संभावनायें अधिक हो जाती हैं। इस कठिनाई को कम करने के लिये हम मध्यमान ज्ञात करने की एक दूसरी विधि का प्रयोग करते हैं जिसे लघु विधि कहते हैं।

प्रश्न—लघुविधि का प्रयोग हम उस समय करते हैं जब पहली विधि (दीर्घविधि) में मध्यमान ज्ञात नहीं किया जा सकता हो। क्या यह कथन सत्य है ?

यदि आपका उत्तर है—

(अ) हाँ　　　तो पृष्ठ ७५ पर खण्ड (५३) देखिये।

(ब) नहीं　　　पृष्ठ ७६ पर खण्ड (५३) देखिये।

एक कक्षा के कुछ छात्रों के प्राप्तांक वर्ग अन्तराल में निम्न प्रकार दिए हुये हैं। इनका लघु विधि से मध्यमान ज्ञात कीजिए?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	विचलन (d)
२०-३०	८	—२
३०-४०	९	—१
४०-५०	१७	०
५०-६०	१४	+१
६०-७०	१२	+२
	N=६०	

(i) लघु विधि से मध्यमान ज्ञात करने के लिए सर्व प्रथम हम बंटन के बीच के या लगभग बीच के वर्ग अन्तराल के मध्यमूल्य को कल्पित मध्यमान मान लेते है। इसे A से प्रदर्शित करते हैं। उपरोक्त बटन में बीच का वर्ग अन्तराल ४०-५० है।

अत: कल्पित मध्यमान (A)=वर्ग अन्तराल ४०-५० का मध्यमूल्य =४५

(ii) जिस वर्गअन्तराल के मध्यमूल्य को कल्पित मध्यमान माना गया है उसके सामने शून्य लिख देते हैं। उपरोक्त बंटन में वर्ग अन्तराल ४०-५० के सामने अगले स्तम्भ में ० लिखा है।

(iii) कल्पित मध्यमान वाले वर्ग अन्तराल से छोटे वर्ग अन्तरालों की ओर क्रमश: +१, +२, +३ आदि लिख देते हैं। इन्हें विचलन मान कहते हैं तथा d से प्रदर्शित करते हैं।

उपरोक्त बटन में वर्ग अन्तराल ४०-५० से छोटे वर्गअन्तराल क्रमश: ३०-४० व २०-३० हैं। अत: इन वर्ग अन्तरालों के विचलन-मान (d) क्रमश: —१, —२ होगे। वर्ग अन्तराल ४०-५० से बड़े वर्ग अन्तराल क्रमश: ५०-६० व ६०-७० हैं अत: इनमें d का मान क्रमश: +१, +२ होगा। उपरोक्त बंटन में स्तम्भ—३ में विचलन (d) का मान इसी प्रकार लिखा हुआ है।

उदा०—एक कक्षा के छात्रों के प्राप्तांक वर्ग अन्तरालों में निम्न प्रकार दिये हुए हैं—

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	विचलन (d)
०- ५	५	—२
५-१०	१०	—१
१०-१५	१४	०
१५-२०	१२	+१
२०-२५	४	+२
	N=४५	

(i) लघु विधि से मध्यमान ज्ञात करने के लिए हम किसी वर्ग अन्तराल के मध्यमूल्य को कल्पित मध्यमान मान लेते है उपरोक्त बटन मे बीच का वर्ग अन्तराल १०–१५ है।
अतः कल्पित मध्यमान=१२·५।

(ii) कल्पित मध्यमान वाले वर्ग अन्तराल १०-१५ में विचलन मान (d) ० तथा उससे छोटे वर्ग अन्तरालो की ओर क्रमश: —१, —२ तथा बड़े वर्ग अन्तरालो की ओर क्रमशः +१ व +२ रख देते है। उपरोक्त बंटन मे विचलन (d) का मान इसी प्रकार भर दिया गया है।

प्रश्न—एक कक्षा के १०० छात्रों द्वारा गणित मे प्राप्त अंकों का बंटन निम्न है। इसके अगले स्तम्भ—३ मे d का मान भरिये ?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	विचलन (d)
०-१०	८	....
१०-२०	१३	....
२०-३०	२२	....
३०-४०	१८	....
४०-५०	१९	....
५०-६०	११	....
६०-७०	९	....
	N=१००	

अपने उत्तर की जाँच के लिए पृष्ठ ७७ पर खण्ड (५५) देखिये।

(i) लघु विधि से मध्यमान ज्ञात करने के लिए d का मान रखने के बाद f से गुणा करके fd का मान ज्ञात कर लेते है। गुणा करते समय चिन्हों का विशेष ध्यान रखते हैं।

(ii) प्रत्येक वर्ग अन्तराल का fd ज्ञात करने के बाद उनका बीज-गणितीय योग ज्ञात कर लेते है। यह योग Σ f d द्वारा प्रदर्शित किया जाता है।

निम्न बारम्बारता बंटन में fd तथा Σ fd का मान ज्ञात कीजिए ?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	विचलन (d)	बार × विचलन f × d
२०-३०	८	—२	—१६
३०-४०	९	—१	—९
४०-५०	१७	०	०
५०-६०	१४	+१	+१४
६०-७०	१२	+२	+२४
			Σ fd = +१३

(i) f तथा d के गुणा करते समय चिन्हों का विशेष ध्यान रखते हैं। जैसे उपरोक्त बंटन मे f = ८ तथा d = —२ है।
अतः fd = ८ × (—२) = —१६।

(ii) इसी प्रकार अन्य f d के गुणा उपरोक्त बंटन में लिखे गए हैं Σfd, प्रत्येक fd के मानों का बीजगणितीय योग है इसे प्रत्येक धनात्मक fd को एक साथ तथा ऋणात्मक fd के मानों को एक साथ जोड़कर ज्ञात करते है। तथा चिन्ह सहित Σ fd ज्ञात कर लेते हैं।
उपरोक्त बंटन में

—fd = —१६ + (—९) = —२५
+fd = +१४ + २४ = +३८
Σ fd = +१३

उदा० – कुछ छात्रों द्वारा प्राप्त अंको का बंटन निम्न है--

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	विचलन (d)	f d
			--१०
०-५	५	—२	--१०
५-१०	१०	--१	—१०
१०-१५	१४	०	०
१५-२०	१२	०	+१२
२०-२५	४	+२	+ ८

Σ fd = (−२०) + (२०) = ०

लघु विधि से मध्यमान ज्ञात करने के लिए पहिले कल्पित मध्यमान मानते है। मान लीजिए बंटन मे कल्पित मध्यमान वर्ग अन्तराल १०-१५ के मध्यमूल्य १२.५ को माना है अत: वर्ग अन्तराल १०-१५ में विचलन (d) का मान शून्य, छोटे वर्ग अन्तरालों ५-१० व ०-५ में क्रमश: —१ व—२ तथा बड़े वर्ग अन्तरालो १५-२० व २०-२५ की ओर क्रमश: १ व + २ होगा। बंटन मे d का मान रखने के बाद f तथा d का चिन्ह सहित गुणा करके fd तथा फिर Σ fd ज्ञात कर लेते है।

प्रश्न —एक कक्षा के १०० छात्रों द्वारा गणित मे प्राप्त अंकों का बंटन निम्न है। इनका Σ fd ज्ञात कीजिए।

वर्ग अन्तराल (C.I.)	बारम्बारता ( )	विचलन (d)	बार० × विचलन f d
०–१०	८		
१०–२०	१३		
२०–३०	२५		
३०–४०	१८		
४०–५०	१६		
५०–६०	१४		
६०–७०	६		
	N = १००		Σ fd =

यदि आपका उत्तर है--

(अ) −४ तो पृष्ठ ८० पर खण्ड (५८) देखिए।

(ब) १४० पृष्ठ ८१ पर खण्ड (५६) देखिए।

Σfd का मान ज्ञात करने के पश्चात लघुविधि से मध्यमान ज्ञात करने के लिए हम निम्न सूत्र का प्रयोग करते है:—

$$M = \frac{\Sigma fd}{N} \times i + A$$

जहाँ Σfd = बारम्बारता तथा विचलन मानो के गुणनफलो का चिन्ह सहित योग

i = वर्ग विस्तार

N = छात्रों की संख्या

तथा A = कल्पित मध्यमान है।

पिछले खण्ड (११) के प्रथम बारम्बारता बंटन से—

$\Sigma fd = +$ १३, $N =$ ६०, $i =$ १० तथा $A =$ ४५ है। इनका मध्यमान ज्ञात कीजिये ?

अतः मध्यमान, $M = \frac{\Sigma fd}{N} \times i + A$

$$= \frac{१३}{६०} \times १० + ४५$$

$$= २\cdot१६ + ४५$$

$$= ४७\cdot१६$$

याद रखिये Σfd का मान चिन्ह सहित रखते है।

उदाहरण — पिछले खण्ड (११) के द्वितीय बारम्बारता बंटन से—

$\Sigma fd = ०$, $N = ४५$, $i = ५$ तथा $A = १२\cdot५$ है। इनका मध्यमान ज्ञात कीजिये ?

$$\therefore \text{मध्यमान, } M = \frac{\Sigma fd}{N} \times i + A$$

$$= \frac{०}{४५} \times ५ + १२\cdot५$$

$$= ० + १२\cdot५$$

$$= १२\cdot५$$

प्रश्न — पिछले खण्ड (११) के तृतीय बारम्बारता बंटन से—

$\Sigma fd = -४$, $N = १००$, $i = १०$, तथा $A = ३५$ है। इनसे मध्यमान की गणना कीजिये ?

यदि आपका उत्तर है—

(अ) ३४·६ तो पृष्ठ ५१ पर खण्ड (६०) देखिये।

(ब) ३५ ४ पृष्ठ ५२ पर खण्ड (६१) देखिये।

(स) उपरोक्त दोनों में से कोई नहीं। पृष्ठ ५३ पर खण्ड (६२) देखिये।

अध्याय 2

समीक्षा

खण्ड (६०) से

पिछले अध्याय में हमने विभिन्न आंकड़ों में मध्यमान ज्ञात करना सीखा है।

(अ) अव्यवस्थित आंकड़ों—जब प्राप्तांक अव्यवस्थित हो तब मध्यमान

$M=\frac{\Sigma X}{N}$ जहाँ ΣX = प्राप्तांकों का कुल योग तथा

N = छात्रों की संख्या है।

(ब) व्यवस्थित आंकड़ों—

(i) जब प्राप्तांक बारम्बारता बंटन में हो तो मध्यमान, $M=\frac{\Sigma fX}{N}$

जहाँ f = प्राप्तांक की बारम्बारता तथा X = प्राप्तांक है।

(ii) जब प्राप्तांक वर्ग अन्तराल में हो तो मध्यमान निकालने की दो विधियाँ हैं—

दीर्घ विधि—$M=\frac{\Sigma fX}{N}$; जहाँ f = प्राप्तांक की बारम्बारता तथा X वर्ग अन्तराल का मध्य मूल्य है।

लघु विधि—$M=\frac{\Sigma fd}{N}\times i+A$; जहाँ f = प्राप्तांक की बारम्बारता, d = कल्पित मध्यमान से मध्यमूल्य का विचलन तथा A = कल्पित मध्यमान है।

अगले अध्याय में हम अव्यवस्थित आंकड़ों में जब छात्रों की संख्या विषम तथा सम हो तथा व्यवस्थित आंकड़ों में मध्यांक की गणना करना सीखेंगे।

अब आप अव्यवस्थित आंकड़ों में मध्यांक की गणना जानने के लिए अगले पृष्ठ २० पर अध्याय ३ के खण्ड (१३) का अध्ययन कीजिये।

अध्याय 3

मध्यांक की गणना

खण्ड (१३) **समीक्षा अध्याय—२ से**

अव्यवस्थित आंकड़ों में मध्यांक—

एक समूह ने किसी परीक्षा में १, १, ४, ७, ६ अंक प्राप्त किये। इनका मध्यांक ज्ञात कीजिये ?

मध्यांक वह बीच का प्राप्तांक है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के प्राप्तांक हों। यदि छात्रों की संख्या N हो तो

बीच का प्राप्तांक $= \frac{(N+१)}{२}$ वें छात्र का प्राप्तांक

$\therefore$ मध्यांक $= \frac{N+१}{२}$ वें छात्र का प्राप्तांक

उपरोक्त समूह में,

मध्यांक $= \left(\frac{५+१}{२}\right)$ वें छात्र का प्राप्तांक (क्योंकि N=५ है)

$=$ ३ वे छात्र का प्राप्तांक $=$ ४

उदाहरण—एक कक्षा के कुछ छात्रों ने एक परीक्षा में

७, ६, १८, २४, २५, २७ व ३०

अंक प्राप्त किये। इनका मध्यांक ज्ञात कीजिये।

यहाँ N=७ है अतः मध्यांक $= \left(\frac{N+१}{२}\right)$ वें छात्र का प्राप्तांक

$= \frac{७+१}{२}$ वें छात्र का प्राप्तांक

$=$ ४ वे छात्र का प्राप्तांक $=$ २४

प्रश्न—एक कक्षा के कुछ छात्रों के प्राप्तांक निम्नलिखित हैं। इनका मध्यांक बताइये—३, ४, ५, ५, ६, ६, ७, ८, ६

यदि आपका उत्तर है—

(अ) ५ तो पृष्ठ ५४ पर खण्ड (६३) देखिये।

(ब) ५.८६ पृष्ठ ५५ पर खण्ड (६४) देखिये।

(स) ६ पृष्ठ ५६ पर खण्ड (६५) देखिये।

एक कक्षा के ४ छात्रों के प्राप्तांक ३, ४, ८ व ६ है। इनका मध्यांक ज्ञात कीजिये ?

हम जानते है कि मध्यांक $=\left(\frac{N+१}{२}=\frac{४+१}{२}\right)$वें छात्र का प्राप्तांक

$=$२·५ वें छात्र का प्राप्तांक (क्योंकि यहाँ $N=४$)

अर्थात जब छात्रों की संख्या (N) सम हो तो मध्यांक दोनों छात्रों के प्राप्तांकों के बीच का प्राप्तांक होगा। उपरोक्त प्राप्तांकों का

मध्यांक $=\frac{\text{२ वें छात्र का प्राप्तांक + ३ वें छात्र का प्राप्तांक}}{२}=\frac{४+८}{२}$

$=६$

उदाहरण—प्राप्तांकों २, २, ६, ७, ८, व ६ का मध्यांक ज्ञात कीजिये ?

मध्यांक $=\frac{६+१}{२}$वें छात्र का प्राप्तांक (क्योंकि $N=६$)

$=$३·५ वें छात्र का प्राप्तांक

३.५वें छात्र काप्राप्तांक ३वें तथा ४वें छात्र के प्राप्तांकों का औसत होगा।

अतः मध्यांक $=\frac{\text{३वें छात्र का प्राप्तांक + ४वें छात्र का प्राप्तांक}}{२}=\frac{६+७}{२}$

$=६·५$

प्रश्न—निम्नलिखित छात्रों द्वारा प्राप्त अंकों का मध्यांक ज्ञात कीजिये ?
५, ७, ८, ८, १० ११, १२, १३, १८, २०

यदि आपका उत्तर है—

(अ) १०·५ तो पृष्ठ ५७ पर खण्ड (६६) देखिये।
(ब) ११ पृष्ठ ५८ पर खण्ड (६७) देखिये।
(स) उपरोक्त दोनों में से कोई नहीं। पृष्ठ ५६ पर खण्ड (६८) देखिये।

बारम्बारता बंटन में मध्यांक—

एक कक्षा के कुछ छात्रों का बारम्बारता बंटन निम्न है—

प्राप्तांक (X)	बारम्बारता (f)
३	२
५	८
६	५
७	२
उपरोक्त बटन में—	N=१७

(i) २ छात्रों ने ३-३ अंक, ८ छात्रों ने ५-५ अंक, ५ छात्रों ने ६-६ अंक तथा २ छात्रों ने ७-७ अंक प्राप्त किए हैं।

(ii) प्राप्तांक ५ अथवा उससे कम ८+२=१० छात्रों ने प्राप्त किये हैं क्योंकि ८ छात्रों के प्राप्तांक ५ तथा २ छात्रों के उससे कम प्राप्तांक ३ है।

यह १० ही प्राप्तांक ५ की संचयी बारम्बारता है।

(iii) इसी प्रकार प्राप्तांक ६ या उससे कम, ५+८+२=१५ छात्रों ने प्राप्त किये है। अतः प्राप्तांक ६ की संचयी बारम्बारता=१५

(iv) प्राप्तांक ७ अथवा उससे कम २+५+८+२=१७ छात्रों ने प्राप्त किये है। अतः प्राप्तांक ७ की संचयी बारम्बारता=१७

इस प्रकार किसी प्राप्तांक की संचयी बारम्बारता उस प्राप्तांक की बारम्बारता तथा उससे पहिले सभी प्राप्तांकों की बारम्बारताओं का योग होती है।

उदाहरण—निम्न बारम्बारता बंटन में कुछ छात्रों के प्राप्तांक दिये हैं। इन प्राप्तांकों की संचयी बारम्बारतायें ज्ञात कीजिये ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
१०	३	३
११	६	१२
१२	८	२०
१३	४	२४
	N=२४	

(i) उपरोक्त बंटन में प्राप्तांक १० की बारम्बारता ३ है। यह पहला प्राप्तांक है अतः इससे पहिले सभी प्राप्तांकों की बारम्बारता ० होगी। अतः प्राप्तांक १० की संचयी बारम्बारता = ३

(ii) प्राप्तांक १० की बारम्बारता ३ है तथा प्राप्तांक ११ की बारम्बारता ६ है। अतः प्राप्तांक ११ की संचयी बारम्बारता = ६ + ३ = १२

(iii) इसी प्रकार प्राप्तांक १२ की संचयी बारम्बारता = ८ + ६ + ३ = २० तथा प्राप्तांक १३ की संचयी बारम्बारता = ४ + ८ + ६ + ३ = २३ होगी।

प्रश्न—निम्न बारम्बारता बंटन में संचयी बारम्बारता भरिए ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
३	१	....
४	२	....
५	४	....
६	६	....
७	३	....
८	१	....
	N=१७	

अपने उत्तर की जांच के लिये पृष्ठ ६० पर खण्ड (६६) देखिए।

खण्ड (१६) **खण्ड** (७८) से

निम्न बंटन में दिये प्राप्तांकों का मध्यांक ज्ञात कीजिए ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
३	१	१
४	२	३
५	४	७
६	६	१३
७	३	१६
८	१	१७
	N = १७	

यहाँ N = १७, ∴ मध्यांक $= \frac{१७ + १}{२}$ = ९ वें छात्र के प्राप्तांक

(i) संचयी बारम्बारता स्तम्भ मे हम देखते हैं कि १ वें छात्र तक का प्राप्ताँक ३ हैं तथा ३ वे छात्र तक के प्राप्ताँक ४ हैं अतः २ वें से ३ वें छात्र तक द्वारा प्राप्त अक = ४

(ii) पुनः संचयी बारम्बारता स्तम्भ से ७ वें छात्र तक के प्राप्तांक ५ है तथा १३ वें छात्र क प्राप्ताँक है। अतः ८ वें से १३ वें छात्र तक द्वारा प्राप्त अंक = ६

अतः मध्याँक = ९ वें छात्र का प्राप्तांक = ६

(क्योंकि ९ वां छात्र ८ वें और १३ वे छात्र के बीच मे स्थित है !)

उदाहरण—निम्न बारम्बारता बंटन का मध्याँक ज्ञात कीजिए ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
२०	१	१
२१	३	४
२२	५	९
२३	७	१६
२४	४	२०
२५	३	२३
	N = २३	

$$\text{मध्यांक} = \frac{N+१}{२} \text{ वें छात्र का प्राप्तांक}$$

$$= \frac{२३+१}{२} \text{ वें} \quad ,, \quad ,,$$

$$= १२ \text{ वें छात्र का प्राप्तांक}$$

उपरोक्त वंटन के संचयी बारम्बारता स्तम्भ मे हम देखते है कि १० वें से १९ वें छात्र तक के प्राप्तांक २३ हैं। इसमें $\frac{N+1}{२}$ वे छात्र अर्थात १२ वें छात्र का प्राप्तांक भी शामिल है।

अतः मध्यांक = २३

प्रश्न—एक कक्षा के कुछ छात्रों द्वारा प्राप्त अंकों का वंटन निम्नलिखित है। इनका मध्यांक ज्ञात कीजिए ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
१८	२	२
१९	९	११
२०	३	१४
२१	५	१९
२२	७	२६
२३	२	२८
२४	३	३१
	N = ३१	

यदि आपका उत्तर है—

(अ) ५ तो पृष्ठ ६३ पर खण्ड (७२) देखिये।

(ब) २१ पृष्ठ ६४ पर खण्ड (७३) देखिये।

(स) उपरोक्त दोनों में से कोई नहीं। पृष्ठ ६५ पर खण्ड (७४) देखिये।

एक कक्षा के २४ छात्रों द्वारा प्राप्त अंकों का बंटन निम्न है। इनका मध्यांक ज्ञात कीजिये ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
१०	३	३
११	१०	१३
१२	७	२०
१३	४	२४
	N=२४	

हम जानते हैं कि मध्यांक $= \frac{N+१}{२}$ वें छात्र का प्राप्तांक

$= \frac{२४+१}{२}$,, ,,

= १२'५ वें छात्र का प्राप्तांक

उपरोक्त संचयी बारम्बारता स्तम्भ से हम देखते हैं कि ४वें से १३वें छात्र तक द्वारा अंक ११ प्राप्त किये। इसी में $\frac{N+१}{२}$ अर्थात १२'५ वें छात्र का प्राप्तांक भी शामिल है। अतः मध्यांक = ११

उदाहरण— निम्नलिखित बारम्बारता बंटन का मध्यांक ज्ञात कीजिये ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
३	२	२
४	१	३
५	६	९
६	८	१७
७	३	२०
८	२	२२
	N=२२	

हम जानते हैं कि मध्यांक $= \frac{N+१}{२}$ वें छात्र के प्राप्तांक

पिछले बंटन से,

$N = २२$ अतः मध्यांक $= \frac{२२+१}{२}$ वें छात्र के प्राप्तांक

$= ११.५$ वें छात्र के प्राप्तांक

संचयी बारम्बारता स्तम्भ में [हम देखते हैं कि १० वें छात्र से १७ वें छात्र तक का प्राप्तांक ६ है जिसमें कि ११.५ वें छात्र के प्राप्तांक भी शामिल हैं।

अतः मध्यांक $= ६$

प्रश्न—कुछ छात्रों द्वारा प्राप्त अंकों का बारम्बारता बंटन निम्न है। इनका मध्यांक ज्ञात कीजिए ?

प्राप्तांक (X)	बारम्बारता (f)	संचयी बारम्बारता (cf)
७	१	१
८	३	४
९	८	१२
१४	७	१९
१५	६	२५
	$N = २५$	

यदि आपका उत्तर है—

(अ) ९ तो पृष्ठ ६६ पर खण्ड (७५) देखिये।

(ब) १३ पृष्ठ ६७ पर खण्ड (७६) देखिये।

(स) १४ पृष्ठ ६८ पर खण्ड (७७) देखिये।

वर्ग अन्तराल में मध्यांक—

निम्नलिखित वर्ग अन्तरालों में दिए प्राप्तांकों का मध्यांक वर्ग ज्ञात कीजिए ?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	सचयी बारम्बारता (Cf)
०- ५	२	२
५-१०	१०	१२
१०-१५	६	१८
१५-२०	७	२५
२०-२५	३	२७
२५-३०	२	३०
	N = ३०	

अभी तक हमने जाना है कि मध्यांक $= \frac{N+१}{२}$ छात्र का प्राप्तांक

लेकिन जब प्राप्तांक वर्ग अन्तरालों मे दिये हो तब मध्यांक ज्ञातकरने के लिए हम सर्व प्रथम मध्यांक वर्ग ज्ञात करते है यह मध्यांक वर्ग N/२ वें छात्र का वर्ग अन्तराल होता है ।

उपरोक्त बंटन में N = ३० अत: N/२ = १५

मध्यांक वर्ग = १५ वें छात्र का वर्ग अन्तराल

उपरोक्त बटन से—

(i) २ वें छात्र तक के प्राप्तांक ०- ५ वर्ग अन्तराल में है।
१२ वे छात्र तक के प्राप्तांक ५–१० वर्ग अन्तराल में हैं।
अत: वर्ग अन्तराल ५–१० में ३ वें से १२ वें छात्र तक के प्राप्तांक शामिल है।

(ii) इसी प्रकार वर्ग अन्तराल १०–१५ में १३ वें से १८ वें छात्र तक के प्राप्तांक शामिल हैं।

क्योंकि N/२ = १५ है अतः १५ वें छात्र के प्राप्तांक वर्ग अन्तराल १०–१५ में शामिल हैं। वह वर्ग अन्तराल जिसमें N/२ अर्थात १५वें छात्र का प्राप्तांक होता है मध्यांक वर्ग कहलाता हैं।

उदाहरण—एक कक्षा के कुछ छात्रों के प्राप्तांक वर्गअन्तरालों में निम्न हैं। इनका मध्यांक वर्ग ज्ञात कीजिये ?

वर्गअन्तराल (C.I.)	बारम्बारता (f)	संचयी बारम्बारता (cf)
०—१०	४	४
१०—२०	६	१०
२०—३०	८	१८
३०—४०	१०	२८
४०—५०	२	३०
	N=३०	

उपरोक्त बंटन में N=३०

अत: मध्यांक वर्ग $= \frac{३०}{२}$ वें छात्र का वर्ग अन्तराल

४ वें छात्र तक के प्राप्तांक वर्गअन्तराल ०–१० में तथा १० वें छात्र तक के प्राप्तांक वर्गअन्तराल १०–२० में हैं।

अत: वर्गअन्तराल १०–२० में ५वें से १०वें छात्र तक के प्राप्तांक शामिल हैं।

१८वें छात्र तक के प्राप्तांक वर्गअन्तराल २०–३० में हैं।

अत: वर्गअन्तराल २०–३० में ११ वें से १८ वें छात्र तक के प्राप्तांक शामिल हैं। इसमें $\frac{N}{२}$ अर्थात १५ वें छात्र के प्राप्तांक भी शामिल हैं।

अत: मध्यांक वर्ग=२०–३०

प्रश्न—निम्नलिखित बारम्बारता बंटन में मध्यांक वर्ग ज्ञात कीजिये ?

वर्गअन्तराल (C.I.)	बारम्बारता (f)	संचयीबारम्बारता (cf)
०– ५	४	४
५–१०	७	११
१०–१५	९	२०
१५–२०	३	२३
२०–२५	१	२४
	N=२४	

यदि आपका उत्तर है—

(अ) १०—१५ तो पृष्ठ ६९ पर खण्ड (७८) देखिये।

(ब) १०—१५ के अलावा कोई पृष्ठ ७० पर खण्ड (७९) देखिये।

निम्नलिखित वर्ग अन्तरालों के वंटन का मध्यांक ज्ञात कीजिए ?

वर्गअन्तराल (C.I.)	बारम्बारता (f)	संचयी बारम्बारता (cf)
०— ५	४	४
५—१०	७	११
१०—१५	९	२०
१५—२०	३	२३
२०—२५	१	२४
	N=२४	

उपरोक्त वंटन में—

N=२४, अतः मध्यांक वर्ग=N/२ वे छात्र का वर्ग अन्तराल

२४/२= १२ वें छात्र का वर्ग अन्तराल

संचयी बारम्बारता स्तम्भ से हम देखते है कि १२ वे छात्र का प्राप्तांक १०-१५ में शामिल हैं।

अतः मध्यांक वर्ग=१०-१५

इस वर्ग में कौन सा प्राप्तांक मध्यांक होगा यह निम्न सूत्र से ज्ञात करते हैं—

$$\text{मध्यांक} = l + \left(\frac{N/2 - F}{f}\right) \times २$$

जहाँ l=मध्यांक वर्ग की निम्न सीमा

F=मध्यांक वर्ग से पूर्व वर्ग की संचयी बारम्बारता

f=मध्यांक वर्ग की बारम्बारता तथा i=वर्ग विस्तार

उपरोक्त वंटन में, l=१०, F=११, f=९, i=५ है

$$\text{अतः मध्यांक} = १० + \left(\frac{१२ - ११}{९}\right) \times ५$$

$$= १० + \frac{१}{९} \times ५$$

$$= १०.५५$$

उदाहरण—निम्न प्राप्तांकों का मध्यांक ज्ञात कीजिए ?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	संचयी बारम्बारता (Cf)
१०–२०	८	८
२०–३०	४	१२
३०–४०	५	१७
४०–५०	१०	२७
५०–६०	५	३२
६०–७०	५	३७
७०–८०	३	४०
	N=४०	

उपरोक्त बंटन में—

N=४०, अत: मध्यांक वर्ग = N/२=२० वे छात्र का वर्ग अन्तराल उपरोक्त वटन के संचयी बारम्बारता स्तम्भ में हम देखते हैं कि १८ वे से २७ वें तक छात्रों के प्राप्तांक वर्ग अन्तराल ४०–५० में हैं जिसमें कि २० वें छात्र का प्राप्तांक भी है।

∴ मध्याँक वर्ग=४०–५०

अत: मध्याँक $=l + \left(\frac{N/२ - F}{f}\right) \times i$

यहाँ $l=४०$, $F=१७$, $f=१०$ तथा $i=१०$ है।

अत: मध्याँक$=४० + \left(\frac{२०-१७}{१०}\right) \times १० = ४० + \frac{३}{१०} \times १०$

$=४० + \frac{३}{१०} \times १०$

$=४३$

प्रश्न—एक कक्षा के ३३ छात्रों के प्राप्तांकों का वंटन निम्न है। इनका मध्यांक ज्ञात कीजिए ?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	संचयी बारम्बारता (Cf)
५– ७	१०	१०
७– ९	५	१५
९–११	३	१८
११–१३	७	२५
१३–१५	३	२८
१५–१७	३	३१
१७–१९	२	३३
	N=३३	

यदि आपका उत्तर है—

(अ) १० तो पृष्ठ ७१ पर खण्ड(८०) देखिये।

(ब) ८ पृष्ठ ७२ पर खण्ड (८१) देखिये।

(स) उपरोक्त दोनों में कोई नहीं पृष्ठ ७३ पर खण्ड (८२) देखिए।

अध्याय 3	समीक्षा

खण्ड (८०) से

हमने अध्याय-३ में निम्न आंकड़ों में मध्यांक की गणना करना जाना है—

(अ) अव्यवस्थित आंकड़ों में—

(i) जब छात्रों की संख्या विषम होती है तो मध्यांक $\frac{N+1}{2}$ वें छात्र का प्राप्तांक होता है।

(ii) जब छात्रों की संख्या सम हो तो मध्यांक बीच के दो प्राप्तांकों का औसत होता है।

(ब) व्यवस्थित आंकड़ों में—

(i) जब प्राप्तांक बारम्बारता बंटन मे दिये हो तब मध्यांक वह प्राप्तांक होता है जहाँ बारम्बारता $\frac{N+1}{2}$ हो जाती है।

(ii) जब प्राप्तांक वर्ग अन्तराल मे दिये हो तब हम सर्वप्रथम मध्यांक वर्ग ज्ञात करते है।

मध्यांक वर्ग $= N/2$ वें छात्र का वर्ग अन्तराल

मध्यांक वर्ग ज्ञात करने के बाद निम्न सूत्र से हम मध्यांक ज्ञात कर लेते है—

$$\text{मध्यांक} = l + \left(\frac{N/2 - F}{f}\right) \times i$$

जहां l = मध्यांक वर्ग की निम्न सीमा

F = मध्यांक वर्ग से पूर्व वर्ग की सचयी बारम्बारता

f = मध्यांक वर्ग की बारम्बारता

i = वर्ग विस्तार

अब आप पृष्ठ ३४ पर अध्याय-४ के खण्ड (२०) का अध्ययन कीजिए।

अध्याय ४

मध्यांक की गणना

खण्ड - समीक्षा अध्याय—३ से

[illegible] आंकड़ों में मध्यांक—

[illegible] किसी [illegible] निम्नलिखित अंक प्राप्त किये। इनका बहुलांक क्या होगा?

१, [illegible] २, ५, ७

हम जानते हैं कि—

बहुलांक [illegible] जो समूह में सबसे अधिक बार आया है। उपरोक्त समूह में प्राप्त [illegible] बार तथा अन्य सभी प्राप्तांक १-१ बार आया [illegible]। अतः बहुलांक [illegible]

अतः बहुलांक वह अंक है जो समूह में सबके अधिक बार आया हो।

प्रश्न—[illegible] अंक प्राप्त किये। इनका बहुलांक ज्ञात कीजिये।

[illegible] १[illegible], [illegible], २१, २७, ३०, ३०, ४[illegible], ५२

यदि आपका उत्तर है—

(अ) [illegible] तो पृष्ठ ७४ पर खण्ड (८३) देखिये।

(ब) ३० पृष्ठ ७५ पर खण्ड (८४) देखिये।

(स) ५२ पृष्ठ ७६ पर खण्ड (८५) देखिये।

बारम्बारता वंटन में बहुलांक—सामान्य विधि

एक कक्षा के कुछ छात्रा द्वारा प्राप्त अको का बारम्बारता बटन निम्न प्रकार है—

प्राप्तांक	बारम्बारता
२	१
३	५
४	३
६	४
८	२

उपरोक्त बटन मे अक २, १ छात्र ने प्राप्त किया है। इसी प्रकार प्राप्तांक ३, ५ छात्रो ने ; प्राप्ताक ४, ३ छात्रो ने; प्राप्तांक ६, ४ छात्रों ने तथा प्राप्ताक ८. २ छात्रो ने प्राप्त किया है।

हम देखते है कि प्राप्तांक ३ सबसे अधिक छात्रा अर्थात ५ छात्रो ने प्राप्त किया है। दूसरे शब्दों मे प्राप्तांक ३ की बारम्बारता सबसे अधिक है।

अतः बहुलांक = ३

इस प्रकार बहुलांक वह अंक से जिसकी बारम्बारता सबसे अधिक होती है।

प्रश्न—एक कक्षा के कुछ छात्रो द्वारा प्राप्त अंको का बारम्बारता बटन निम्न है। इनका बहुलांक क्या होगा ?

प्राप्तांक	बारम्बारता
१	२
२	२
३	१
४	७
५	३

यदि आपका उत्तर है—

(अ) २ तो पृष्ठ ७७ पर खण्ड (८६) देखिये

(ब) ४ पृष्ठ ७७ पर खण्ड (८७) देखिये ।

(स) ७ पृष्ठ ७९ पर खण्ड (८८) देखिये ।

बारम्बारता वंटन में बहुलांक—समूहन विधि

निम्नलिखित बारम्बारता वटन से बहुलांक ज्ञात कीजिए ?

प्राप्तांक	बारम्बारता
४	५
६	७
७	४
८	१६
९	१८
१०	११
११	१८
१२	७
१३	९
१४	८

उपरोक्त वटन मे सबमे अधिक बारम्बारता १८ है जो कि दो प्राप्तांकों ९ व ११ की है। अत: यह निश्चित करना कठिन है कि इनमें से कौन सा प्राप्तांक बहुलांक है। ऐसी स्थिति में हम बहुलांक ज्ञात करने के लिए समूहन विधि का प्रयोग करते हैं।

प्रश्न—निम्नलिखित बारम्बारता बंटन मे बहुलॉक किस विधि से ज्ञात करना होगा ?

प्राप्तांक	बारम्बारता
१०	५
१२	७
१६	११
१९	१३
२०	८
२१	१२
२२	४
२३	९

यदि आपका उत्तर है—

(अ) सामान्य विधि तो पृष्ठ ८० पर खण्ड (८९) देखिये ।

(ब) समूहन विधि पृष्ठ ८१ पर खण्ड (९०) देखिये ।

खण्ड (२३) **खण्ड** (९०) से

समूहन विधि से बहुलांक ज्ञात करने के लिए सर्वप्रथम समूहन सारणी बनाते हैं। समूहन सारणी बनाने में स्तम्भ—१, २ व ३ निम्न प्रकार भरे जाते हैं—

समूहन सारणी—

प्राप्तांक	१-१ की	२-२ की		३-३ की		
↓ स्तम्भ →	१	२	३	४	५	६
४	५	१२				
६	७		११			
७	४	१७				
८	१३		३१			
९	१८	२९				
१०	११		२९			
११	१८	२५				
१२	७		१६			
१३	९	१७				
१४	८					

(i) समूहन सारणी बनाने के लिए स्तम्भ-१ में प्रत्येक प्राप्तांक की बारम्बारता लिख दी जाती है। उपरोक्त सारणी से स्तम्भ-१ में प्रत्येक प्राप्तांक की बारम्बारतायें लिख दी गई हैं।

(ii) स्तम्भ-२ में प्रारम्भ से २-२ प्राप्तांकों की बारम्बारताओं का योग लिखा जाता है। उपरोक्त सारिणी के स्तम्भ-२ में प्राप्तांक ४ व ६ की बारम्बारताओं ५ व ७ का योग १२ लिख दिया है। इसी प्रकार अगले प्राप्तांकों ७ व ८ की बारम्बारताओं ४ व १३ का योग १७ लिख दिया है। इसी प्रकार इस स्तम्भ में अगले २-२ प्राप्तांकों की बारम्बारताओं का योग लिख दिया गया है।

(iii) स्तम्भ–३ मे प्रथम प्राप्तांक की बारम्बारता को छोड़कर अगले २-२ प्राप्तांको की बारम्बारताओं का योग लिख दिया जाता है। उपरोक्त बटन में प्राप्तांक ४ की बारम्बारता ५ को छोड़कर अगले प्राप्तांको ६ व ७ की बारम्बारताओं ७ व ४ का योग ११ लिख दिया है। इसी प्रकार अगले दो प्राप्तांको ८ व ९ की बारम्बारताओं १३ व १८ का योग ३१ लिख दिया है। इसी प्रकार इससे अगले २-२ प्राप्तांको की बारम्बारताओं का योग लिख दिया गया है।

नोट-- याद रखिए कि स्तम्भ [illegible] व [illegible] में बारम्बारताओं का योग लिखा जाता है प्राप्तांको का योग नहीं।

प्रश्न—निम्न समूहन सारणी में प्राप्तांक १९ व २१ की बारम्बारता सबसे अधिक है अतः बहुलांक ज्ञात करने के लिए समूहन विधि का प्रयोग करना होगा। निम्नलिखित समूहन सारणी के स्तम्भ-२ व ३ को पूर्ण कीजिए ?

समूहन सारिणी—

प्राप्तांक	१–१ की	२–२ की			३–३ की	
↓ स्तम्भ→	१	२	३	४	५	६
१०	५					
१२	७					
१६	११					
१९	१३					
२०	८					
२१	१३					
२३	४					
२८	९					

उपरोक्त सारणी की जांच के लिए पृष्ठ ५२ पर खण्ड (९१) देखिये।

समूहन सारणी —

प्राप्तांक	१–१ की	२–२ की		३–३ की		
↓ स्तम्भ→	१	२	३	४	५	६
४	५	१२		१६		
६	७		११		२४	
७	४	१७				३५
८	१३		३१	४२		
९	१८	२९			४७	
१०	११		२९			३६
११	१८	२५		३४		
१२	७		१६		२४	
१३	९	१७				
१४	८					

अब तक हमने समूहन सारणी के स्तम्भ–१, २ व ३ को भरना सीखा है। अब हम स्तम्भ–४, ५ व ६ को भरना सीखेंगे।

(i) समूहन सारणी के स्तम्भ-४ को भरने के लिए प्रारम्भ से ३–३ प्राप्तांकों की बारम्बारताओं का योग लिख दिया जाता है।

उपरोक्त समूहन सारणी के स्तम्भ–४ मे प्राप्तांक ४, ६ व ७ की बारम्बारताओं ५, ७ व ४ का योग १६ लिख दिया है। इसी प्रकार अगले ३–३ प्राप्तांकों की बारम्बारताओं का योग स्तम्भ–४ में लिख दिया है।

(ii) समूहन सारणी के स्तम्भ–५ को भरने के लिए प्रथम प्राप्तांक की बारम्बारता छोड़कर अगले ३–३ प्राप्तांकों की बारम्बारताओं का योग लिख दिया जाता है।

उपरोक्त समूहन सारणी में प्राप्तांक ४ की बारम्बारता ५ को छोड़कर अगले प्राप्तांकों ६, ७ व ८ की बारम्बारताओं ७, ४ व १३ का योग २४ लिख दिया है। इसी प्रकार उससे अगले प्राप्तांक ६, १० व ११ की बारम्बारताओं १८, ११ व १८ का योग ४७ लिखा है। इसी प्रकार उससे अगले ३—३ प्राप्तांकों की बारम्बारताओं का योग लिख दिया जाता है।

(iii) स्तम्भ-६ में प्रथम दो प्राप्तांकों की बारम्बारताओं को छोड़कर अगले ३-३ प्राप्तांकों की बारम्बारताओं का योग लिख दिया जाता है।

उपरोक्त सारणी के स्तम्भ-६ में प्राप्तांक ४ व ६ की बारम्बारताओं ५ व ७ को छोडकर उससे अगले ३-३ प्राप्तांकों ७, ८ व ६ की बारम्बारताओं ४, १२ व १८ का योग ३५ लिख दिया है। इसी प्रकार अगले ३-३ प्राप्तांकों की बारम्बारताओं का योग लिखा है।

इस प्रकार समूहन सारणी पूर्ण हो जायेगी।

नोट:—पिछले खण्ड की तरह ही यहाँ भी बारम्बारताओं का योग लिखा जाता है, प्राप्तांकों का योग नहीं।

प्रश्न—निम्नलिखित समूहन सारणी के स्तम्भ–४, ५ व ६ को पूर्ण कीजिये ?

समूहन सारिणी—

प्राप्तांक	१–१ की	२–२ की		३–३ की		
↓ स्तम्भ →	१	२	३	४	५	६
१०	५	१२				
१२	७		१८			
१६	११	२२				
१९	१३		२१			
२०	८	२१				
२१	१३		१७			
२३	४	१३				
२८	९					

अपनी समूहन सारणी की जाँच के लिए पृष्ठ ५२ पर खण्ड (९४) देखिये।

जब सबसे अधिक बारम्बारता वाले प्राप्तांक दो हों तो समूहन विधि से बहुलांक ज्ञात करते हैं। इस विधि में सर्वप्रथम समूहन सारणी बनाते हैं फिर इस समूहन सारणी के आधार पर विश्लेषण सारणी बनाई जाती है। पृष्ठ ३६ के खण्ड (२४) की समूहन सारणी पुनः प्रस्तुत की जा रही है।

समूहन सारणी—

प्राप्तांक	१–१ की	२–२ की		३–३ की		
↓ स्तम्भ→	१	२	३	४	५	६
४	५	१२		१६		
६	७		११		२४	
७	४	१७				३५
८	१३		३१	४२		
९	१८	२९			४७	
१०	११		२९			३६
११	१८	२५		३४		
१२	७		१६		२४	
१३	९	१७				
१४	८					

उपरोक्त सारणी की विश्लेषण सारणी निम्न प्रकार बनायेंगे—

(i) विश्लेषण सारणी के स्तम्भ-१ में सबसे अधिक बारम्बारता १८ है जो कि प्राप्तांक ९ व ११ की है। विश्लेषण विधि में प्राप्तांक ९ व ११ के सामने स्तम्भ-१ में टैली चिह्न लगा दिये हैं।

(ii) स्तम्भ-२ में सबसे अधिक बारम्बारता २९ है जो कि प्राप्तांक ९ व १० की बारम्बारताओं का योग है। अतः ९ व १० के सामने स्तम्भ-२ में टैली चिह्न लगा दिये हैं। इसी प्रकार स्तम्भ-३ में सबसे अधिक बारम्बारता ३१ के प्राप्तांकों के सामने टैलीचिन्ह लगा दिए हैं।

विश्लेषण सारणी —

प्राप्तांक ↓ स्तम्भ →	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह 1	२	३	४	५	६	बारम्बारता ७
४							०
६							०
७							०
८			।	।			२
९	।	।	।	।	।		५
१०		।		।	।	।	४
११	।				।	।	३
१२						।	१
१३							०
१४							०

(iii) स्तम्भ–४ में सबसे अधिक बारम्बारता ४२ है जो कि प्राप्तांक ८, ९ व १० की बारम्बारताओं का योग है। अतः विश्लेषण सारणी के स्तम्भ–४ में प्राप्तांक ८, ९ व १० के सामने टैलीचिन्ह लगा दिये हैं।

(iv) इसी प्रकार स्तम्भ–५ व ६ में अधिकतम बारम्बारता वाले प्राप्तांकों के सामने टैली चिन्ह लगा दिए जाते हैं।

टैली चिन्ह लगाने के बाद प्रत्येक प्राप्तांक के सामने के कुल टैली चिन्हों को जोड़कर प्रत्येक प्राप्तांक की बारम्बारता विश्लेषण सारणी के स्तम्भ-७ में लिखदी जाती है। उपरोक्त विश्लेषण सारणी में प्राप्तांक ४ के सामने कोई टैली चिन्ह नहीं है अतः इसकी बारम्बारता शून्य होगी। इसी प्रकार प्राप्तांक ६ व ७ की बारम्बारता शून्य होगी। प्राप्तांक ८ के सामने २ टैली चिन्ह है अतः इसकी बारम्बारता २ होगी। इसी प्रकार अन्य बारम्बारतायें लिखी हैं।

इस प्रकार हमारी विश्लेषण सारणी पूर्ण हो जायेगी।

प्रश्न—पृष्ठ ४० के खण्ड (२४) की समूहन सारणी निम्न है ।

समूहन सारणी—

प्राप्तांक	१–१ की	२–२ की		३–३ की		
↓ स्तम्भ→	१	२	३	४	५	६
१०	५	१२		२३		
१२	७		१८		३१	
१६	११	२४				३२
१९	१३		२१	३४		
२०	८	२१			२५	
२१	१३		१७			२६
२३	[illegible]	१३				
२८	९					

उपरोक्त समूहन सारणी के आधार पर निम्न विश्लेषण सारणी पूर्ण कीजिये ?

विश्लेषण सारणी—

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारबारता
↓ स्तम्भ→	१	२	३	४	५	६	७
१०							
१२							
१६							
१९	।						
२०							
२१	।						
२३							
२८							

अपनी विश्लेषण सारणी की जाँच के लिए पृष्ठ ६२ पर खण्ड (९७) देखिये ।

किसी वंटन में जब सबसे अधिक बारम्बारता वाले प्राप्तांक दो होते हैं तो हम समूहन विधि का प्रयोग करते हैं। इसमें सर्वं प्रथम हम समूहन सारणी बनाते हैं। फिर इस समूहन सारणी के आधार पर विश्लेषण सारणी में जिस प्राप्तांक की बारम्बारता सबसे अधिक होती है। वही बहुलांक होता है।

पिछले खण्ड (२५) की प्रथम विश्लेषण सारणी पुनः प्रस्तुत की जा रही है ? इस विश्लेषण सारणी से बहुलांक ज्ञात कीजिये ?

विश्लेषण सारणी —

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिह्न						बारम्बारता
↓ स्तम्भ →	१	२	३	४	५	६	७
४							०
६							०
७							०
८			|	|			२
९	|	|	|	|	|		५
१०		|		|	|	|	४
११	|				|	|	३
१२						|	१
१३							०
१४							०

उपरोक्त विश्लेषण सारणी में स्तम्भ-७ से हम देखते हैं कि सबसे अधिक बारम्बारता ५ है जो कि प्राप्तांक ९ की है।

अतः बहुलांक = ९

प्रश्न—पिछले खण्ड (२५) की विश्लेषण सारणी पुनः प्रस्तुत की जा रही है। इस विश्लेषण सारणी से बहुलांक ज्ञात कीजिये ?

विश्लेषण सारणी—

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारम्बारता
↓ समूह →	१	२	३	४	५	६	७
१०							०
१२					|		१
१६		|			|	|	३
१९	|	|	|	|	|	|	६
२०				|		|	२
२१	|		|	|			३
२३							०
२८							०

यदि आपका उत्तर है—

(अ) ६ तो पृष्ठ ६७ पर खण्ड (१००) देखिये।

(ब) १९ पृष्ठ ६८ पर खण्ड (१०१) देखिये।

(स) उपरोक्त दोनों में से कोई नहीं। पृष्ठ ६९ पर खण्ड (१०२) देखिये।

जब किसी बंटन में सबसे अधिक बारम्बारता वाले प्राप्तांक दो हों तो बहुलांक निकालने के लिए समूहन विधि का प्रयोग करते है। समूहन विधि में दो सारणी बनाते है।

अ) **समूहन सारणी**— समूहन सारणी के स्तम्भ–१ में प्रत्येक प्राप्तांक की बारम्बारता, स्तम्भ–२ व ३ में दो-दो प्राप्तांकों की बारम्बारताओं का योग तथा स्तम्भ ४, ५ व ६ में तीन-तीन प्राप्तांकों की बारम्बारताओं का योग लिखते हैं। इसी प्रकार समूहन सारणी बनाइए—

समूहन सारणी --

प्राप्तांक ↓ स्तम्भ →	१–१ की १	२–२ की २	 ३	३–३ की ४	 ५	 ६
१४	१५					
१५	२८					
१६	४०					
१७	३६					
१८	४०					
१९	३८					
२०	२०					
२१	१२					
२२	७					
२३	४					

(ब) विश्लेषण सारणी—उपरोक्त समूहन सारणी के आधार पर विश्लेषण सारणी बनाते हैं। समूहन सारणी के प्रत्येक स्तम्भ में जिन प्राप्तांकों की बारम्बारता सबसे अधिक होती है उनके सामने टैलीचिन्ह लगा देते हैं।

फिर प्रत्येक प्राप्तांक के टैली चिन्हों का योग करके हर एक प्राप्तांक की बारम्बारता ज्ञात कर लेते हैं। इसी प्रकार उपरोक्त समूहन सारणी के आधार पर निम्न विश्लेषण सारणी बनाइये।

विश्लेषण सारणी में जिस प्राप्तांक की बारम्बारता सबसे अधिक होती। वही बहुलांक होता है। उपरोक्त विश्लेषण सारणी से बहुलांक ज्ञात कीजिये।

विश्लेषण सारणी—

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टैली चिन्ह						बारम्बारता
↓ स्तम्भ →	१	२	३	४	५	६	७
१४							
१५							
१६							
१७	I						
१८							
१९	I						
२०							
२१							
२२							
२३							

क्या आपका उत्तर १८ है—

(अ) नहीं तो पृष्ठ ७५ पर खण्ड (१०३) देखिए।

(ब) हाँ पृष्ठ ७६ पर खण्ड (१०४) देखिए।

वर्ग अन्तराल में बहुलांक—

किसी कक्षा के छात्रों का अंग्रेजी विषय में प्राप्त अंकों का वंटन निम्न है। इनका बहुलांक ज्ञात कीजिये ?

वर्ग अन्तराल (C. I)	बारम्बारता (f)
२०–२५	३
२५–३०	३
३०–३५	७
३५–४०	४
४०–४५	५

हम जानते है कि बहुलांक वह अंक है जिसकी बारम्बारता सबसे अधिक हो। उपरोक्त वंटन में वर्ग अन्तराल ३०–३५ की बारम्बारता सबसे अधिक है।

अतः बहुलांक वर्ग = ३०–३५

इस वर्ग में ३० से ३५ तक अंक शामिल है। इनमें कौन सा अंक बहुलांक होगा यह हम निम्न सूत्र से ज्ञात करते हैं—

$$\text{बहुलांक} = l + \left(\frac{f-f_1}{२f-f_1-f_2}\right) \times i$$

जहां l = बहुलांक वर्ग की निम्न सीमा,
f = बहुलांक वर्ग की बारम्बारता
f_1 = बहुलांक वर्ग से पूर्व वर्ग की बारम्बारता,
f_2 = बहुलांक वर्ग से बाद के वर्ग का बारम्बारता
i = वर्ग विस्तार

उपरोक्त वंटन में l = ३०, f = ७, f_1 = ३, f_2 = ४ व i = ५ है

$$\text{अतः बहुलांक} = ३० + \left(\frac{७-३}{१४-३-४}\right) \times ५$$

$$= ३० + \frac{२०}{७}$$

$$= ३२\cdot८$$

उदाहरण—निम्न वर्गअन्तरालों में बहुलांक ज्ञात कीजिए ?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)
५–१०	५
१०–१५	७
१५–२०	१३
२०–२५	२१
२५–३०	१६
३०–३५	८

उपरोक्त वंटन में, बहुलांक वर्ग = २०–२५

$$\text{अतः बहुलांक} = l + \left(\frac{f - f_1}{2f - f_1 - f_2}\right) \times i$$

$$= २० + \left(\frac{२१ - १३}{४२ - १३ - १६}\right) \times ५$$

$$= २० + ३\cdot१ = २३\cdot१$$

प्रश्न—निम्न वंटन में छात्रों के प्राप्तांक दिये हैं। इनका बहुलांक ज्ञात कीजिये ?

वर्ग अन्तराल (C. I.)	बारम्बारता (f)
०–१०	७
१०–२०	१०
२०–३०	१९
३०–४०	१५
४०–५०	८
५०–६०	६

यदि आपका उत्तर है—

(अ) २३·१ तो पृष्ठ ७७ पर खण्ड (१०५) देखिए।

(ब) २६·६ पृष्ठ ७८ पर खण्ड (१०६) देखिए।

(स) ५१·८ पृष्ठ ७९ पर खण्ड (१०७) देखिए।

अध्याय ४

समीक्षा

खण्ड (१०६) से

पिछले अध्याय मे हमने विभिन्न प्रकार के आंकड़ों में बहुलांक की गणना करना जाना है—

(अ) **अव्यवस्थित आंकड़ों में**-जब प्राप्तांक अव्यवस्थित हो तो बहुलांक वह अंक होता है जो समूह में सबसे अधिक बार आता है।

(ब) **व्यवस्थित आंकड़ों में**—

(i) जब प्राप्तांक बारम्बारता बंटन में दिए हो तो बहुलांक वह अंक होता है जिसकी बारम्बारता सबसे अधिक हो।

(ii) जब दिए बारम्बारता बटन में बहुलांक दो अंक हो तो समूहन विधि से बहुलांक ज्ञात करते है। इस विधि में सर्वप्रथम समूहन सारणी बनाते है तथा फिर इसके आधार पर विश्लेषण सारणी बनाते है। विश्लेषण सारणी मे जिस प्राप्तांक की बारम्बारता सबसे अधिक हो वही बहुलांक होता है।

(iii) जब प्राप्तांक वर्ग अन्तरालो में दिये हो तो पहले बहुलांक वर्ग ज्ञात करते है फिर निम्न सूत्र द्वारा बहुलांक की गणना कर लेते है—

$$\text{बहुलांक} = l + \left(\frac{f - f_1}{2f - f_1 - f_2} \right) i$$

जहाँ l = बहुलांक वर्ग की निम्न सीमा,

f = बहुलांक वर्ग की बारम्बारता,

f_1 = बहुलांक वर्ग से पूर्व वर्ग की बारम्बारता तथा

f_1 = बहुलांक वर्ग के बाद के वर्ग की बारम्बारता है

इस सम्पूर्ण पुस्तक मे हमने केन्द्रीय प्रवृत्ति के तीनों मानों मध्यमान मध्यांक, बहुलांक की परिभाषा तथा उनकी विभिन्न प्रकार के प्राप्तांकों मे गणना करना सीखा है।

अब यह ज्ञात करने के लिए कि आप केन्द्रीय प्रवृत्तियों के मानों की गणना सीखने मे कितने सफल हुए है सलग्न प्रश्न पत्र के उत्तर ध्यान-पूर्वक देने का प्रयत्न कीजिये।

खण्ड (२९) **खण्ड (१) से**

आपका उत्तर गलत है।

आपने खण्ड (१) से जाना है कि

$$\text{मध्यमान} = \frac{\text{अंकों का कुल योग}}{\text{छात्रों की संख्या}}$$

दिये गये ९ छात्रों के प्राप्तांकों का मध्यमान,

$$M = \frac{१५ + १८ + २० + २२ + २५ + २७ + २७ + ३० + ३२}{९}$$

$$= \frac{२१६}{९}$$

उपरोक्त गणना से ज्ञात होता है कि आपने प्रश्न के उत्तर में अंकों का केवल योग २१६ निकाला है। उसको छात्रों की संख्या से भाग नहीं दिया है।

अब आप पृष्ठ २ पर खण्ड (१) के गहरे काले छपे भाग को पुनः समझकर प्रश्न का उत्तर दीजिये।

खण्ड (६०) **खण्ड (१२) से**

आपका उत्तर सही है।

आपने खण्ड (१२) को ठीक प्रकार समझा है। याद रखिये गणना करते समय चिह्न का अवश्य ध्यान रखते हैं।

अब तक आपने विभिन्न आँकड़ों में मध्यमान की गणना करना जाना है। अब आप पृष्ठ १९ पर समीक्षा अध्याय-२ अध्ययन कीजिये।

खण्ड (३०) खण्ड (१) से

आपका उत्तर सही है।

आपने खण्ड (१) को ठीक प्रकार समझा है।

अब आप पृष्ठ ३ पर खण्ड (२) का अध्ययन कीजिये।

खण्ड (६) खण्ड (१२) से

आपका उत्तर गलत है।

आपने सूत्र का ठीक प्रयुक्त किया है लेकिन गणना करते समय Σfd के चिन्ह का ध्यान नहीं रखा है।

$$M = \frac{-४}{१००} \times १० + ३५$$

$= -०.४ + ३५$ (यहाँ —०.४ को ३५ में से घटना चाहिये न कि जोड़ना)

अब आप Σfd के चिन्ह का ध्यान रखते हुए पृष्ठ १७ पर खण्ड (१२) को पुनः हल कीजिये।

खण्ड (६१) खण्ड (४३) से

समूहन सारणी –

प्राप्तांक	१–१ का	२–२ का			३–३ का	
↓ स्तम्भ →	१	२	३	४	५	६
१०	५	१२				
१२	७		१८			
१५	११	२४				
१९	१३		२१			
२०	८	२१				
२१	१३		१७			
२३	४	१३				
२८	९					

यदि आपकी सारणी उपरोक्त सारणी के

(अ) समान है। तो पृष्ठ ५३ पर खण्ड (६२) देखिये।

(ब) भिन्न है। पृष्ठ ५५ पर खण्ड (६३) देखिये।

खण्ड (३१) खण्ड (१) से

आपका उत्तर गलत है।

आपने खण्ड (१) का ठीक अध्ययन नहीं किया है।

हम जानते है कि मध्यमान = $\frac{\text{अंकों का कुल योग}}{\text{छात्रों की संख्या}}$

मान लो एक कक्षा के ६ छात्रों के प्राप्तांक ४, ५, ६, ८, ९ व १० है।

इन प्राप्तांकों का मध्यमान, $M = \frac{४ + ५ + ६ + ८ + ९ + १०}{६}$

$= २४$

$= ७$

इसी प्रकार खण्ड (१) में दिये प्रश्न का मध्यमान

$M = \frac{१५ + १८ + २० + २२ + २५ + २७ + २७ + ३० + ३२}{९}$

अब आप पृष्ठ २ पर खण्ड (१) का पुनः अध्ययन करके उपरोक्त उत्तर पूर्ण कीजिये।

खण्ड (६२) खण्ड (१२) से

आपका उत्तर गलत है।

आपने सम्भवतः मध्यमान ज्ञात करते समय सूत्र ठीक से नहीं समझा है। लघु विधि से मध्यमान के लिए निम्न सूत्र होता है।

$$M = \frac{\Sigma fd}{N} \times i + A$$

जहाँ Σfd = बारं० × विचलन के गुणनफल का योग

N = छात्रों की संख्या

A = कल्पित मध्यमान

i = वर्ग विस्तार

उपरोक्त सूत्र से पृष्ठ १७ पर खण्ड (१२) को पुनः हल कीजिये।

खण्ड (६२) खण्ड (६१) से

आपकी समूहन सारणी सही है।

अब आप समूहन सारणी के स्तम्भ-४, ५ व ६ को भरने के लिए पृष्ठ ३६ पर खण्ड (२८) का अध्ययन कीजिये।

खण्ड (३२) **खण्ड (२) से**

आपका उत्तर गलत है।

आपने प्राप्तांकों का मध्यमान ज्ञात किया है जबकि प्रश्न में इन प्राप्तांकों का मध्यांक ज्ञात कराया है। याद रखिये मध्यमान व मध्यांक दो अलग-अलग मान है। मध्यमान वह मान है जो छात्रों के कुल योग को उनकी संख्या से भाग देकर प्राप्त होता है जबकि मध्यांक वह अंक है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के अंक होते है।

उदाहरण के लिए प्राप्तांक ५, ८, ९, १२ व १६ का मध्यमान

$$M = \frac{५ + ८ + ९ + १२ + १६}{५} = १०$$

तथा मध्यांक = ९ है क्योंकि अंक ९ के नीचे दो छात्रों के प्राप्तांक ५ व ८ है तथा ऊपर दो छात्रों के प्राप्तांक १२ व १६ है। अतः बीच का प्राप्तांक या मध्यांक ९ होगा।

पृष्ठ ३ पर खण्ड (२) के प्रश्न में मध्यांक के ऊपर तथा नीचे क्रमशः ३-३ छात्रों के प्राप्तांक होंगे। अब इस प्रश्न का पुनः उत्तर दीजिए।

खण्ड (९३) खण्ड (१३) से

आपका उत्तर गलत है।

हम जानते है कि मध्यांक $= \frac{N+१}{२}$वें छात्र का प्राप्तांक

यहाँ $N = ९$ है अतः मध्यांक $= \frac{९+१}{२}$ वे छात्र का प्राप्तांक

$= ५$ वे छात्र का प्राप्तांक

आपने ५ वे छात्र का प्राप्तांक को मध्यांक न मानकर ५ को ही मध्यांक मान लिया है। जबकि मध्यांक ५ वे छात्र का प्राप्तांक है।

अब आप पृष्ठ २० पर खण्ड (१३) का पुनः अध्ययन करके प्रश्न का उत्तर दीजिए।

खण्ड (३३) खण्ड (२) से

आपका उत्तर बिल्कुल सही है।

क्योंकि मध्यांक १८ के ऊपर ३ तथा नीचे भी तीन छात्रों के प्राप्तांक हैं। अब आप पृष्ठ ३ पर खण्ड (३) को पढ़िये।

खण्ड (६४) खण्ड (१३) से

आपका उत्तर गलत है।

आपने मध्यांक को ठीक प्रकार नहीं समझा है। मध्यांक वह अंक है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के अंक हैं अर्थात्

$$\text{मध्यांक} = \frac{N+१}{२} \text{ वें छात्र का प्राप्तांक}$$

आपने मध्यांक के स्थान पर मध्यमान ज्ञात किया है क्योंकि खण्ड (१३) के प्रश्न का मध्यमान

$$= \frac{३+४+५+५+६+६+७+८+९}{९} = \frac{५३}{९}$$

$$= ५.८९$$

अब आप पृष्ठ २० पर खण्ड (१३) का पुनः अध्ययन करके प्रश्न का उत्तर दीजिये।

खण्ड (९३) खण्ड (९१) से

आपका उत्तर गलत है।

आपने समूह न सारणी के स्तम्भ–१, २ व ३ भरने में सम्भवतः प्राप्तांकों की बारम्बारताओं का योग करने के बजाय प्राप्तांकों का योग कर दिया है या सम्भवतः आपने स्तम्भ–१ व २ तो सही है लेकिन स्तम्भ–३ भरने की विधि को ठीक नहीं समझा है। स्तम्भ–३ भरने में पहले प्राप्तांक की बारम्बारता को छोड़कर अगले २–२ प्राप्तांकों की बारम्बारताओं का योग लिखा जाता है।

प्रश्न में पहले प्राप्तांक ४ की बारम्बारता ५ को छोड़कर अगले प्राप्तांकों की बारम्बारताओं का योग ११ लिखा है। इसी प्रकार अगले प्राप्तांकों १ व ६ की बारम्बारताओं १३ व १८ का योग ३१ स्तम्भ–३ में लिखना है। आपने इसके स्थान पर ८ की बारम्बारता १३ को छोड़कर प्राप्तांक ९ व १० की बारम्बारताओं १८ व ११ का योग २९ लिखा है जो गलत है। (शेष भाग पृष्ठ ५६ पर पढ़िये।)

खन्ड (३४) खन्ड (२) से

आपका उत्तर गलत है।

आपने खण्ड (२) को ठीक से नहीं समझा है। खण्ड (२) से हम जानते हैं कि मध्यांक वह अंक है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के प्राप्तांक हों।

उदाहरण के लिए मान लो एक कक्षा के ५ छात्रों के प्राप्तांक ३०, ३२, ३४, ३७ व ४० हैं।

उपरोक्त प्राप्तांकों में प्राप्तांक ३४ बीच का अंक है। इसके ऊपर दो छात्रों के प्राप्तांक ३७ व ४० तथा नीचे दो छात्रों के प्राप्तांक ३० व ३२ हैं। अतः मध्यांक ३४ होगा।

इसी प्रकार पृष्ठ ३ पर खण्ड (२) के प्रश्न का पुनः उत्तर दीजिये।

खन्ड (६५) खन्ड (१३) से

आपका उत्तर सही है।

आपने मध्यांक निकालने का तरीका ठीक समझा है।

क्योंकि मध्यांक $= \frac{९+१}{२}$ वें छात्र का प्राप्तांक (प्रश्न में, $N = ९$)

$= ५$

अब आप पृष्ठ २१ पर खन्ड (१४) का अध्ययन कीजिये।

खन्ड (९३) खन्ड (६३) से

शेष भाग

याद रखिये स्तम्भ-३ भरने में केवल प्रथम प्राप्तांक की बारम्बारता को छोड़कर अगले २-२ प्राप्तांकों की बारम्बारताओं के समूह बनाये जाते हैं एक समूह बनाने के बाद दूसरा समूह बनाने से पहिले एक प्राप्तांक की बारम्बारता दुबारा नहीं छोड़ते हैं।

इसी प्रकार आप पृष्ठ ३८ पर खण्ड (२३) की समूहन सारणी भरिये।

खण्ड (३५) खण्ड (३) से

आपका उत्तर सही है।

प्रश्न में २५ दो बार तथा शेष सभी प्राप्तांक १–१ बार आये हैं। अतः बहुलांक २५ होगा ।

अब आप पृष्ठ ४ पर खण्ड (४) का अध्ययन कीजिए।

खण्ड (६६)

आपका उत्तर सही है ।

आपने मध्यांक ज्ञात करना ठीक प्रकार समझा है। याद रखिए जब छात्रों की संख्या (N) सम हो तो बीच के दो प्राप्तांकों का औसतमान ही मध्यांक होता है।

अब आप पृष्ठ २२ पर खण्ड (१५) का अध्ययन कीजिये ।

खण्ड (६४) खण्ड (२४) से

समूहन सारणी—

प्राप्तांक	१–१ की	२–२ की		३–३ की		
↓ स्तम्भ→	१	२	३	४	५	६
१०	५	१२		२३		
१२	७		१८		३१	
१६	११	२४				३२
१९	१३		२१	३४		
२०	८	२१			२५	
२१	१३		१७			२६
२३	४	१७				
२८	९					

यदि आपकी सारणी उपरोक्त सारणी से

(अ) समान है। तो पृष्ठ ६० पर खण्ड (६५) देखिये ।

(ब) भिन्न है। पृष्ठ ६१ पर खण्ड (६६) देखिये ।

खण्ड (३६) 									खण्ड (३) से

आपका उत्तर गलत है।

आपने दिये प्राप्तांकों का मध्यांक निकाल दिया है। जबकि आपसे बहुलांक ज्ञात कराया है। हम जानते हैं कि मध्यांक वह अंक है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के अंक होते हैं तथा बहुलांक वह अंक है जो समूह में सबसे अधिक बार आया हो।

उदाहरण के लिए प्राप्तांकों ६, ६, १२, १४ व १६ में मध्यांक १२ है क्योंकि १२ के ऊपर २ तथा नीचे २ छात्रों के प्राप्तांक हैं तथा बहुलांक ६ है क्योंकि ६ सबसे अधिक बार आया है।

अब आप पृष्ठ ३ पर खण्ड (६) के प्रश्न का पुनः उत्तर दीजिए।

खण्ड (६७) 									खण्ड (१४) से

आपका उत्तर गलत है।

हम जानते हैं कि मध्यांक $= \frac{N+1}{२}$ वें छात्र का प्राप्तांक

$= \frac{१० + १}{२}$ वें छात्र का प्राप्तांक

$=$ ५.५ वें छात्र का प्राप्तांक

५.५ वें छात्र प्राप्तांक ५ वें व ६ वें छात्र के प्राप्तांकों का औसत होता है जबकि आपने ५.५ वें छात्र का प्राप्तांक ५ वें से अगले ६ वें छात्र का प्राप्तांक ही मान लिया है जो कि गलत है। अब आप ५ वें तथा ६ वें छात्र के प्राप्तांक का औसत ज्ञात कीजिए।

इस प्रकार पुनः पृष्ठ २१ पर खण्ड (१४) के प्रश्न का उत्तर दीजिए।

खण्ड (३७) **खण्ड (३) से**

आपका उत्तर गलत है ।

आपने दिये प्राप्तांकों का बहुलांक के स्थान पर मध्यमान निकाल दिया है । याद रखिये मध्यमान तथा बहुलांक दो अलग-अलग मान हैं । मध्यमान वह मान है जो अंकों के योग को छात्रों की संख्या से भाग देकर प्राप्त होता है जबकि बहुलांक वह अंक है जो समूह में सबसे अधिक बार आता है ।

उदाहरण के लिये प्राप्तांक ९, ९, १२, १४ व १६ का मध्यमान १२ तथा बहुलांक ९ होगा ।

इस प्रकार आप पृष्ठ ३ पर खण्ड (३) का अध्ययन करके प्रश्न का उत्तर दीजिए ।

खण्ड (६८) **खण्ड (१४) से**

आपका उत्तर गलत है ।

आपने खण्ड (१४) को ठीक प्रकार नहीं समझा है । इसे एक अन्य उदाहरण से फिर समझिये—

एक कक्षा के छात्रों के प्राप्तांक २, ३, ३, ५, ७ व ८ हैं ।

मध्यांक $=\frac{N+१}{२}$ वें छात्र का प्राप्तांक ($N=$छात्रों की संख्या)

$=\frac{६+१}{२}$ वें छात्र का प्राप्तांक $=$ ३.५ वें छात्र का प्राप्तांक

$=\frac{\text{३ वें छात्र का प्राप्तांक} + \text{४ वें छात्र का प्राप्तांक}}{२}=\frac{३+५}{२}$

$=४$

इसी प्रकार पृष्ठ २१ पर खण्ड (१४) में दिये प्रश्न को पुनः हल कीजिये ।

खण्ड (३८) खण्ड (३) से

आपका उत्तर गलत है ।

आपका उत्तर २७ है जो समूह का सबसे बड़ा अंक है । जब कि बहुलांक वह अंक होता है जिसकी बारम्बारता सबसे अधिक होती है ।

अब पृष्ठ ३ पर खण्ड (३) का पुनः अध्ययन करके प्रश्न का उत्तर दीजिए ।

खण्ड (६९) खण्ड (१५) से

प्राप्तांक (X)	बारम्बारता (f)	सञ्चित बारम्बारता (cf)
३	१	१
४	२	३
५	४	७
६	६	१३
७	३	१६
८	१	१७

यदि आपका बंटन उपरोक्त बंटन से—

(अ) भिन्न है तो पृष्ठ ६१ पर खण्ड (७०) देखिये ।

(ब) समान है पृष्ठ ६२ पर खण्ड (७१) देखिये ।

खण्ड (६५) खण्ड (६४) से

आपकी सारणी सही है ।

आपने खण्ड (२४) को ठीक प्रकार समझा है । अब बहुलांक ज्ञात करने के लिए समूहन सारणी के आधार पर विश्लेषण सारणी बनाते हैं ।

अब आप विश्लेषण सारणी बनाने के लिये पृष्ठ ४१ पर खण्ड (२५) का अध्ययन कीजिये ।

खण्ड (३६) **खण्ड (४) से**

आपका उत्तर गलत है ।

मध्यमान वह मान है जो अंकों के कुल योग को उनकी संख्या से भाग देकर प्राप्त होता है । दिये प्रश्न में वह अंक पूँछा गया है जिसके ऊपर ५०% तथा नीचे ५० छात्रों के अंक होते हैं । वह मान मध्यमान नहीं है ।

अब आप पृष्ठ ३ पर खण्ड (२) का ध्यान पूर्वक अध्ययन करके पृष्ठ ४ पर खण्ड (४) के प्रश्न का पुनः उत्तर दीजिए ।

खण्ड (७०) **खन्ड (६६) से**

आपका उत्तर गलत है

आपने संचयी बारम्बारता ठीक प्रकार से नहीं समझी है । किसी प्राप्तांक की सं० बार० इस प्राप्तांक की बारम्बारता तथा इससे पहले प्राप्तांकों की बारम्बारता का योग होती है ।

जैसे प्राप्तांक ४ की संचयी बार० = २ + १ = ३

इसी प्रकार प्राप्तांक ५ की संचयी बार० = ४ + २ + १ = ७

इसी प्रकार अन्य प्राप्तांकों की संचयी बारम्बारता पृष्ठ २० पर खण्ड (१५) के बंटन में भरिये ।

खण्ड (६६) **खण्ड (६४) से**

आपकी समूहन सारणी गलत है ।

अपनी गलती मालूम करने के लिये पृष्ठ ३६ पर खण्ड (२४) का पुनः अध्ययन कीजिये तथा फिर समूहन सारणी को पूर्ण कीजिए ।

खण्ड (४०) **खण्ड (४) से**

आपका उत्तर सही है।

मध्यमूल्य केन्द्रीय प्रवृत्ति का मान नहीं है।

अब आप पृष्ठ ५ पर समीक्षा अध्याय-१ का अध्ययन कीजिये।

खण्ड (७१) **खण्ड (६६) से**

आपका उत्तर सही है।

आपने प्राप्तांक ५ की संचयी बारम्बारता ठीक ज्ञात की है।

प्राप्तांक ५ की बारम्बारता $= ४ + २ + १ = ७$

अब आप पृष्ठ २४ पर खण्ड (१६) का अध्ययन कीजिये।

खण्ड (६७) **खण्ड (२५) से**

विश्लेषण सारणी—

प्राप्तांक	सबसे अधिक बारम्बारता वाले प्राप्तांकों के टेली चिन्ह						बारम्बारता
↓ स्तम्भ →	१	२	३	४	५	६	७
१०							०
१२					।		१
१६		।			।	।	३
१९	।	।	।	।	।	।	६
२०			।	।		।	३
२१	।			।			२
२३							०
२८							०

यदि आपकी सारणी उपरोक्त सारणी से—

(अ) समान है। तो पृष्ठ ६३ पर खण्ड (९८) देखिये।

(ब) भिन्न है। पृष्ठ ६४ पर खण्ड (९९) देखिये।

खण्ड (४१) — खण्ड (१) से

आपका उत्तर गलत है ।

बहुलाँक वह मान है जो समूह में सबसे अधिक बार आता है । दिये प्रश्न में वह अंक पूछा गया है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के अंक होते हैं । यह अंक बहुलाँक नहीं होता है ।

अब पृष्ठ ३ पर खण्ड (३) का ध्यान पूर्वक अध्ययन कीजिए तथा फिर पृष्ठ ४ पर खण्ड (४) के प्रश्न का उत्तर दीजिए ।

खण्ड (७२) — खण्ड (१६) से

आपका उत्तर गलत है ।

आपने बंटन के $\frac{N+१}{२}$ अर्थात $\frac{३१+१}{२}$ = १६ वें छात्र की बारम्बारता को ही मध्याँक लिख दिया है । जबकि $\frac{N+१}{२}$ वें छात्र का प्राप्ताँक मध्याँक होती है न कि $\frac{N+१}{२}$ वें छात्र की बारम्बारता ।

अब आप पुनः पृष्ठ २४ पर खण्ड (१६) को पढ़कर प्रश्न का उत्तर दीजिये ।

खण्ड (९८) — खण्ड (९०) से

आपका उत्तर सही है ।

आपने विश्लेषण सारणी बनाना ठीक प्रकार समझा है ।

अब आप विश्लेषण सारणी से बहुलाँक ज्ञात करने के लिए पृष्ठ ४४ पर खण्ड (२६) का अध्ययन कीजिए ।

खण्ड (४२) **खण्ड (५) से**

आपका उत्तर सही है।

आपने मध्यमान के सूत्र को ठीक प्रकार समझा है।

अब आप पृष्ठ ६ पर खण्ड (६) का अध्ययन कीजिये।

खण्ड (७३) **खण्ड (१६) से**

आपका उत्तर सही है।

क्योंकि मध्यांक = $\frac{N+1}{२}$ वें छात्र का प्राप्तांक

= $\frac{३१+१}{२}$ " "

= १६ वें छात्र का प्राप्तांक

१६ वें छात्र का प्राप्तांक २१ है क्योंकि १५ वें से २२ वें छात्र तक ने २१ अंक प्राप्त किये हैं।

अब आप पृष्ठ २६ पर खण्ड (१७) का अध्ययन कीजिए।

खण्ड (९९) **खन्ड (६९) से**

आपकी सारणी गलत है।

आपने विश्लेषण सारणी बनाने की विधि को ठीक प्रकार नहीं समझा है। विश्लेषण सारणी बनाने के लिये समूहन सारणी के प्रत्येक स्तम्भ में जिस प्राप्तांक की बारम्बारता सबसे अधिक होती है उन प्राप्तांकों के सामने प्रत्येक स्तम्भ के लिये टैली चिह्न लगा देते हैं। फिर उन टैली चिह्नों को जोड़कर प्रत्येक प्राप्तांक की बारम्बारता ज्ञात करके स्तम्भ-७ में लिख देते हैं।

उपरोक्त विधि से ही आप पृष्ठ ४३ पर खन्ड (२५) में दी सारणी बनाइये।

खण्ड (४३) खण्ड (५) से

आपका उत्तर गलत है।

खण्ड (५) से आप जानते है कि मध्यमान, $M = \frac{\Sigma X}{N}$

जहाँ N = छात्रों की संख्या, तथा ΣX = अकों का कुल योग

उपरोक्त खण्ड (५) के प्रश्न में—

ΣX = १ + ७ + ८ + ९ + ९ + ११ + १६ + १८ + २०

= ९९

आपने ΣX = ९९ को ही मध्यमान मान लिया है जब कि मध्यमान ΣX के मान को N से भाग देने पर ज्ञात होता है।

अब आप खन्ड ६ पर खन्ड (५) का अध्ययन करके प्रश्न का पुनः उत्तर दीजिये।

खन्ड (७४) खन्ड (१६) से

आपका उत्तर गलत है।

आपने बारम्बारता बंटन में मध्यांक ज्ञात करना ठीक प्रकार नहीं समझा है।

मान लो एक कक्षा के छात्रों द्वारा प्राप्त अंकों का बंटन निम्न प्रकार है।

प्राप्तांक (X)	बारम्बारता (f)	सचयी बारम्बारता (c.f.)
१०	४	४
११	५	९
१२	७	१६
१३	३	१९

उपरोक्त बंटन में N = १९ है।

अतः मध्यांक $= \frac{N + १}{२}$ वें पद का मान

$= \frac{१९ + १}{२} =$ १० वें पद का मान

उपरोक्त बंटन की संचयी बारम्बारता स्तम्भ से हम देखते हैं कि १० वें से १६ वें छात्र तक ने अंक १२ प्राप्त किये है। अतः मध्यांक = १२

इसी प्रकार पृष्ठ २४ पर खन्ड (१६) के प्रश्न को हल कीजिये।

खण्ड (८८) **खण्ड (५) से**

आपका उत्तर गलत है।

आपने खण्ड (५) को ठीक प्रकार नही समझा है। मान लीजिये एक परीक्षा में कुछ छात्रों ने ५, ६, ७, ९, १०, १५ व १६ अंक प्राप्त किये इनका मध्यमान ज्ञात कीजिये। हम जानते है कि मध्यमान,

$M = \frac{\Sigma X}{N}$ होता है जहाँ $\Sigma X =$ प्राप्त अंकों का कुल योग तथा

$N =$ छात्रों की संख्या है।

उपरोक्त प्राप्तांकों में, $\Sigma X = ५ + ६ + ७ + ९ + १० + १५ + १६$

$= ६८$

तथा $N = ७$

अतः मध्यमान $(M) = \frac{६८}{७} = ९.७$

खण्ड (५) के प्रश्न में—

$\Sigma X = १ + ७ + ८ + ९ + ९ + ११ + १६ + १८ + २० = ९९$

तथा $N = ९$

अब पृष्ठ ६ पर खण्ड (५) के प्रश्न का पुनः मध्यमान निकालिये।

खण्ड (७५) **खण्ड (१७) से**

आपका उत्तर गलत है।

आपने दिये गए प्राप्तांकों ७, ८, ९, १४, १५ में बीच के प्राप्तांक अर्थात ९ को मध्यांक मान लिया है। याद रखिये जब प्राप्तांकों के साथ उनकी बारम्बारता भी दी गई है, अर्थात प्राप्तांकों का बारम्बारता बंटन दिया हो ऐसी स्थिति में मध्यांक $\frac{N+1}{2}$ वे छात्र का प्राप्तांक

उपरोक्त गलती को ध्यान में रखते हुये पृष्ठ २६ पर खण्ड (१७) में समझाई गई विधि से पुनः मध्यांक ज्ञात कीजिये।

खण्ड (४१) खण्ड (६) से

आपका उत्तर गलत है।

आपने ΣX को N से भाग दे दिया है। आपने यह ध्यान नहीं दिया है कि यहाँ प्राप्तांकों के साथ-साथ उनकी बारम्बारता भी दी है। जब बारम्बारता दी हो तो मध्यमान, $M = \frac{\Sigma fX}{N}$ होता है न कि

$$M = \frac{\Sigma X}{N}$$

अब आप पृष्ठ ७ पर खण्ड (६) का पुनः अध्ययन करके प्रश्न का उत्तर दीजिये।

खण्ड (७६) खण्ड (१७) से

आपका उत्तर गलत है।

हम जानते हैं कि मध्यांक $= \frac{N+१}{२}$ वें छात्र का प्राप्तांक

$= \frac{२५+१}{२} =$ १३ वें छात्र का प्राप्तांक

आपने उपरोक्त १३ को ही मध्यांक मान लिया है। जब कि मध्यांक १३ वें छात्र का प्राप्तांक होता है।

अब आप पृष्ठ २४ पर खण्ड (१७) के प्रश्न का पुनः उत्तर दीजिये।

खण्ड (१००) खण्ड (२३) से

आपका उत्तर गलत है।

आपने खण्ड (२५) को ठीक प्रकार नहीं समझा है। बहुलाँक वह प्राप्तांक है जिसकी बारम्बारता विश्लेषण सारणी में सबसे अधिक होती है। यहाँ ६ सबसे अधिक बारम्बारता है जो कि प्राप्ताँक १६ की है। अतः बहुलाँक ६ नहीं होगा बल्कि वह प्राप्ताँक होगा जिसकी बारम्बारता ६ है।

अब आप पृष्ठ ४३ पर खण्ड (२५) के प्रश्न का पुनः उत्तर दीजिये।

खण्ड (४६)　　　　　　　　　　　　　　　　　　　　　खण्ड (६) से

आपका उत्तर सही है।

आपने व्यवस्थित आँकड़ों में मध्यमान निकालना $\left(M = \frac{\Sigma fX}{N}\right)$ ठीक प्रकार समझना है।

अब आप पृष्ठ ६ पर खण्ड (७) का अध्ययन कीजिये।

खण्ड (७७)　　　　　　　　　　　　　　　　　　　　　खण्ड (१७) से

आपका उत्तर सही है।

आपने म[illegible] ठीक निकाला है क्योंकि

[illegible] = $\frac{N + 1}{2}$ वें [illegible] का प्राप्तांक

= $\frac{२५ + १}{२}$ वें [illegible] का प्राप्तांक

= [illegible] का प्राप्तांक

= ५४

अब आप पृष्ठ २८ पर खण्ड (१८) का अध्ययन कीजिये।

खण्ड (९०)　　　　　　　　　　　　　　　　　　　　　खण्ड (२१) से

आपका उत्तर सही है।

[illegible] खण्ड (२१) को ठीक प्रकार [illegible] है।

अब आप पृष्ठ ४६ पर खण्ड (२७) का अध्ययन कीजिये।

खण्ड (४७) खण्ड (७) से

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	मध्यमूल्य (x)
५—१५	१५	१०
१५—२५	१२	२०
२५—३५	१०	३०
३५—४५	१३	४०
	N=५०	

यदि उपरोक्त बारम्बारता बंटन में मध्य मूल्य तथा आपके द्वारा ज्ञात मध्य मूल्य

(अ) समान है तो पृष्ठ ७० पर खण्ड (४८) देखिये ।

(ब) भिन्न है पृष्ठ ७१ पर खण्ड (४९) देखिये ।

खण्ड (७८) खण्ड (१८) से

आपका उत्तर सही है ।

खण्ड (१८) के बारम्बारता वटन से— N=२४

$$\text{अतः मध्यांक वर्ग} = \frac{N}{२} \text{ वे छात्र का वर्ग अन्तराल}$$

$$= \text{१२ वे छात्र का वर्ग अन्तराल}$$

संचयी बारम्बारता स्तम्भ से हम देखते है कि १२ वे छात्र के प्राप्तांक १२ वे से २० वे मे शामिल है जिसका वर्ग अतराल १०–१५ है। अतः मध्यांक वर्ग=१०–१५

अब आप पृष्ठ २० पर खण्ड (१९) का अध्ययन कीजिये ।

खन्ड (१०२) खन्ड (२६) से

आपका उत्तर गलत है ।

आपने खण्ड (२५) को ठीक प्रकार नही समझा है । विश्लेषण सारणी में जिस प्राप्तांक की बारम्बारता सबसे अधिक हो वही प्राप्तांक बहुलांक होता है ।

इस प्रकार आप पृष्ठ ४३ पर खण्ड (२५) के प्रश्न का सही उत्तर पुनः दीजिए ।

खण्ड (४८) खण्ड (४७) से

आपका उत्तर गलत है।

आपने मध्यमूल्य ज्ञात करने की विधि को ठीक प्रकार समझा है।

अब आप पृष्ठ १० पर खण्ड (८) का अध्ययन कीजिए।

खण्ड (७६) **खण्ड (१८) से**

आपका उत्तर गलत है।

आपने मध्यांक वर्ग निकालना ठीक प्रकार नहीं समझा है। एक अन्य उदाहरण से इसे फिर समझिये।

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	संचयी बारम्बारता (cf)
०—१०	५	५
१०—२०	६	११
२०—३०	६	२०
३०—४०	४	२४
	N=२४	

उपरोक्त बंटन में, N=२४

अतः मध्यांक वर्ग=N/२ वें छात्र का वर्ग अन्तराल

=२४/२=१२ वें छात्र का वर्ग अन्तराल

दिये बंटन से हम देखते हैं कि १२ वें छात्र तक ने वर्ग अन्तराल २०—३० में अंक प्राप्त किये हैं। इसी वर्ग अन्तराल में N/२ अर्थात १२ वें छात्र के प्राप्तांक भी शामिल हैं।

अतः मध्यांक वर्ग=२०—३०

इसी प्रकार पृष्ठ २८ पर खण्ड (१८) का पुनः अध्ययन करके प्रश्न का उत्तर दीजिए।

खण्ड (४६) खण्ड (४७) से

आपका उत्तर गलत है।

आपने खण्ड (७) को ठीक प्रकार नहीं समझा है।

किसी वर्ग अन्तराल का मध्यमूल्य ज्ञात करने के लिये निम्न सीमा तथा उच्च सीमा के योग को २ से भाग दे देते है।

उदाहरण के लिये प्रथम वर्ग अन्तराल ५ — १५

का मध्यमूल्य $= \frac{५ + १५}{२} = \frac{२०}{२} = १०$

अब आप पृष्ठ ६ पर खण्ड (७) का पुनः अध्ययन करके वर्ग अन्तराल के मध्य मूल्य ज्ञात कीजिये।

खण्ड (८०) खण्ड (१६) से

आपका उत्तर सही है

प्रश्न के अनुसार, $N = ३३$

मध्यांक वर्ग $= \left(\frac{N}{२} = \frac{३३}{२} = १६.५\right)$ वें छात्र को वर्ग अन्तराल।

अतः मध्यांक वर्ग $= ९ — ११$

इस वर्ग अन्तराल में मध्यांक निम्न सूत्र से ज्ञात किया जाता है।

$$मध्यांक = l + \left(\frac{N/२ — F}{f}\right) \times i$$

$$= ९ + \left(\frac{१६.५ — १५}{३}\right) \times २$$

$$= ९ + \left(\frac{१.५}{३}\right) \times २$$

$$= १०$$

इस प्रकार आपने मध्यांक की गणना ठीक की है। इस खण्ड तक आपने विभिन्न आँकड़ों में मध्यमान तथा मध्यांक की गणना करना जाना है।

अब आप पृष्ठ ३३ पर समीक्षा अध्याय-३ का अध्ययन कीजिये।

खण्ड (५०) **खण्ड (८) से**

आपका उत्तर गलत है।

आपने खण्ड (८) को ध्यान पूर्वक नहीं पढ़ा है। जब प्राप्तांक वर्ग अन्तराल में दिये हों तब मध्यमान $M = \frac{\Sigma fX}{N}$ होता है जहाँ X वर्ग अन्तराल का मध्यमूल्य है

आपने मध्यमान $M = \frac{\Sigma X}{N}$ से ज्ञात किया है जो कि गलत है। ΣX, मध्य मूल्यों के मानों का योग है जबकि ΣfX बारम्बारता तथा मध्यमूल्यों के गुणनफलों का योग है।

अब आप पृष्ठ १० पर खण्ड (८) का पुनः अध्ययन कीजिए तथा प्रश्न का उत्तर दीजिए।

खण्ड (८१) **खण्ड (१९) से**

आपका उत्तर गलत है।

आपने सम्भवतः मध्यांक वर्ग तो ठीक निकाला है लेकिन मध्यांक वर्ग में मध्यांक ज्ञात करते समय सूत्र में F का मान गलत रखा है। F का मान मध्यांक वर्ग से पूर्व वर्ग की संचयी बारम्बारता होती है आपने सम्भवतः F का मान मध्यांक वर्ग की संचयी बारम्बारता अर्थात १८ मान लिया है जो कि गलत है। आपके अनुसार F का मान १८ रखने पर

$$\text{मध्यांक} = ९ + \left(\frac{१६.५ - १८}{३}\right) \times २$$

$$= ९ - \frac{१.५}{३} \times २$$

$$= ९ - १ = ८ \text{ (यह उत्तर गलत है)}$$

अब आप पृष्ठ ३० पर F का मान मध्यांक वर्ग से पूर्व वर्ग की संचयी बारम्बारता अर्थात १५ रखकर खण्ड (१९) में मध्यांक की दुबारा गणना **कीजिए।**

खण्ड (५१) खण्ड (८) से

आपका उत्तर सही है।

आपने खण्ड (८) को ठीक समझा है।

आप पृष्ठ ११ पर खण्ड (९) का अययन कीजिये।

खण्ड (८२) खण्ड (१९) से

आपका उत्तर गलत है।

आपने खण्ड (१९) को ठीक प्रकार नहीं समझा है। दिये प्रश्न में, N = ३३,

अत: मध्यांक $= \frac{३३}{२}$ अर्थात १६.५ वें छात्र का वर्ग अन्तराल संचयी बारम्बारता स्तम्भ में हम देखते हैं कि १५ वें छात्र ने वर्ग अन्तराल ७–९ तक अंक पाये हैं तथा १८ वें छात्र तक ने वर्ग ९–११ तक अंक पाये हैं अर्थात १६ से १८ वें छात्र तक ने वर्ग अन्तराल ९–११ में अंक पाये हैं जिसमें कि १६.५ वां या १७ वां छात्र भी शामिल है। अत: मध्यांक वर्ग = ९–११।

हम जानते हैं कि किसी मध्यांक वर्ग में—

$$\text{मध्यांक} = l + \left(\frac{N/२ - F}{f}\right) \times i$$

जहां F = मध्यांक वर्ग से पहिले वर्ग की संचयी बारम्बारता तथा f मध्यांक वर्ग की बारम्बारता है।

अब उपरोक्त सूत्र की सहायता से पृष्ठ ३० पर खण्ड (१९) के प्रश्न का पुनः उत्तर दीजिये।

आपका उत्तर गलत है।

आपने खण्ड (८) को ठीक प्रकार नहीं समझा है। मानलो कुछ छात्रों के प्राप्तांक निम्न वर्ग अन्तरालों में दिये हैं।

वर्ग अन्तराल (C. I.)	बारम्बारता (f)	मध्यमूल्य (X)
०–१०	७	५
१०–२०	८	१५
२०–३०	५	२५

जब प्राप्तांक वर्ग अन्तराल में दिए हो तो मध्यमूल्य को ही वर्ग अन्तराल का प्रतिनिधि अंक मान लेते है अतः उपरोक्त बंटन में हम कह सकते हैं कि वर्ग अन्तराल २०–३० में ५ छात्रों ने २५–२५ अंक प्राप्त किये। इस प्रकार वर्ग अन्तराल का०-१० में ७ छात्रों द्वारा प्राप्त ५-५ अंकों का योग $= f \times x =$ ७ × ५ = ३५

इसी प्रकार वर्ग अन्तराल १०–२० में ८ छात्रों द्वारा प्राप्त १५–१५ अंकों का योग

$= f \times X =$ ८ × १५ = १२०

„ „ २०–३० में ५ छात्रों द्वारा प्राप्त २५–२५ अंकों का योग

$= f \times X =$ ५ × २५ = १२५

अतः सभी २० छात्रों द्वारा प्राप्त अंकों का कुल योग $= \Sigma fX =$ २८०

हम जानते हैं कि मध्यमान $= \dfrac{\text{सभी छात्रों द्वारा अंकों का कुल योग}}{\text{छात्रों की संख्या}}$

$$= \frac{\Sigma fx}{N} = \frac{२८०}{२०}$$

= १४

इसी प्रकार पृष्ठ १० पर खण्ड (८) के प्रश्न को पुनः हल कीजिये।

आपका उत्तर गलत है।

आपने दिये हुए समूह के प्राप्तांकों का मध्यांक ज्ञात किया है जब कि प्रश्न मे आप से बहुलांक ज्ञात कराया है। याद रखिये मध्यांक वह अंक है जिसके ऊपर ५०% तथा नीचे ५०% छात्रों के अंक हों। जब कि बहुलांक वह अंक है जो समूह में सबसे अधिक बार आया हो।

पृष्ठ ३४ पर खण्ड २० को दुबारा पढ़िये तथा बहुलांक ज्ञात कीजिये।

खण्ड (५३) खण्ड (९) से

आपका उत्तर गलत है।

आपने खण्ड (९) को ठीक प्रकार नहीं समझा है। लघु विधि का प्रयोग हम उस समय करते हैं जब पहली विधि (दीर्घ विधि) में fX के गुणा बहुत बड़े-बड़े हो जाते हैं तथा उसमें गलती होने की सम्भावनायें अधिक रहती हैं। वैसे लघु विधि के साथ-साथ हम दीर्घ विधि से भी प्रश्न को हल कर सकते हैं।

अब आप पृष्ठ ११ पर खण्ड (९) के प्रश्न का उत्तर पुनः दीजिये।

खण्ड (८४) खण्ड (२०) से

आपका उत्तर सही है।

आपने खण्ड (२०) को ठीक समझा है। दिये प्रश्न में प्राप्तांक ३० दो बार तथा शेष सभी प्राप्तांक १–१ बार ही आये है। अतः बहुलाँक ३० ही होगा।

अब आप पृष्ठ ३५ पर खण्ड (२१) को पढ़िये।

खण्ड (१०३) खण्ड (२७) से

आपका उत्तर सही है।

आपने समूह विधि से बहुलाँक ज्ञात करने की विधि को ठीक प्रकार समझा है।

अब आप वर्ग अन्तरालों में बहुलाँक ज्ञात करने की विधि का अध्ययन करने के लिये पृष्ठ ४९ पर खण्ड (२८) का अध्ययन कीजिये।

खण्ड (५४) खण्ड (६) से

आपका उत्तर सही है।

लघु विधि का प्रयोग उस समय किया जाता है जब दीर्घ विधि में fx [illegible] बड़े-बड़े हो जाते है। वैसे हम दीर्घ विधि से भी उस प्रश्न को हल कर सकते है लेकिन उसमें गणना होने की संभावनायें अधिक रहती है।

अब आप पृष्ठ २२ पर खण्ड (१०) का अध्ययन कीजिये।

खण्ड (८५) खण्ड (२०) से

आपका उत्तर गलत है।

आपने सबसे बड़े प्राप्तांक अर्थात ५२ को बहुलांक बताया है। जबकि बहुलांक वह अंक होता है जो कि समूह में सबसे अधिक बार आता है।

अब आप पृष्ठ २८ पर खण्ड (२०) का **पुनः अध्ययन करके प्रश्न का** उत्तर दीजिये।

खण्ड (१०८) खण्ड (२७) से

आपका उत्तर गलत है।

समूहित विधि में बहुलांक ज्ञात करने के लिये निम्न दो सारणी बनाते है-

(अ) समूहन सारणी

(ब) विश्लेषण सारणी

फिर विश्लेषण सारणी में जिस प्राप्तांक की बारम्बारता सबसे अधिक होती है वही प्राप्तांक बहुलांक होता है।

इसी प्रकार पृष्ठ ४१ पर खण्ड (२७) का पुनः अध्ययन करके प्रश्न हल कीजिये।

खण्ड (५५) खण्ड (१०) से

वर्ग अन्तराल (C.I.)	बारम्बारता (f)	विचलन (d)
०—१०	८	—३
१०—२०	१३	—२
२०—३०	२२	—१
३०—४०	१८	०
४०—५०	१६	+१
५०—६०	११	+२
६०—७०	६	+३

यदि आपके द्वारा ज्ञात d का मान उपरोक्त सारणी में दिये d के मानों के

(अ) भिन्न है तो पृष्ठ ७८ पर खण्ड (५६) देखिये ।

(ब) समान है पृष्ठ ७६ पर खण्ड (५७) देखिये ।

खण्ड (८६) खण्ड (२१) से

आपका उत्तर गलत है ।

आपने सबसे अधिक बार आयी बारम्बारता अर्थात दो बहुलाँक माना हैं । जबकि बहुलाँक वह अंक होता है । जिसकी बारम्बारता सबसे अधिक हो अर्थात जो अंक सबसे अधिक बार आया हो ।
उदाहरण के लिये मान लो एक बारम्बारता बंटन निम्न है—

प्राप्ताँक	बारम्बारता
४	२
६	२
१०	८
१३	६

इस बंटन में बारम्बारता २ सबसे अधिक बार आयी हैं किन्तु बहुलाँक २ नहीं होगा बहुलाँक वह अंक होता है जिसकी बारम्बारता सबसे अधिक हो । उपरोक्त बंटन में अंक १० की बारम्बारता सबसे अधिक अर्थात ८ है अतः बहुलाँक = १०

इसी प्रकार आप पृष्ठ ३५ पर खण्ड (२१) को दुबारा हल कीजिए ।

खण्ड (५६) **खण्ड (५५) से**

आपका उत्तर सही है।

आपने खण्ड (१०) को ठीक प्रकार समझा है। विचलन का मान कल्पित मध्यमान वर्ग अन्तराल मे ०, कल्पित मध्यमान से छोटे वर्ग अन्तरालों की ओर क्रमशः −१,−२,−३ आदि व बड़े वर्ग अन्तरालों की ओर क्रमशः +१, +२, +३ आदि होता है।

अब आप Σfd का मान ज्ञात करने लिए पृष्ठ १५ खण्ड (११) का अध्ययन कीजिए।

खण्ड (८७) **खण्ड (२१) से**

आपका उत्तर सही है।

दिये गये बारम्बारता बंटन में प्राप्तांक ४ की बारम्बारता सबसे अधिक अर्थात ७ है। अतः बहुलांक ४ ही होगा।

अब आप पृष्ठ ३६ पर खण्ड (२२) का अध्ययन कीजिए।

खण्ड (१०५) खण्ड (२१) से

आपका उत्तर गलत है।

दिये प्रश्न मे बहुलांक वर्ग २०—३० है क्योंकि इस वर्ग अन्तराल की बारम्बारता सबसे अधिक होती है। किसी बहुलांक वर्ग मे

$$\text{बहुलांक} = l + \left(\frac{f - f_1}{2f - f_1 - f_2}\right) \times i$$

जहा l = बहुलांक वर्ग की निम्न सीमा, f = बहुलांक वर्ग की बारम्बारता
f_1 = बहुलांक वर्ग मे पूर्व वर्ग की बारम्बारता,
f_2 = बहुलांक वर्ग के बाद वर्ग की बारम्बारता

आपका उत्तर २३.१ इसलिये गलत है क्योंकि आपने f_1 को बहुलांक वर्ग के वर्ग की बारम्बारता तथा f_2 को बहुलांक वर्ग के पूर्व की बारम्बारता माना है।

अब आप f_1 तथा f_2 के सही मान रखकर पृष्ठ ४६ पर खण्ड (२८) के प्रश्न को पुनः हल कीजिए।

खण्ड (५७) खण्ड (५२) से

आपका उत्तर गलत है।

आपने खण्ड (१०) को ठीक प्रकार नहीं समझा है। आपने विचलन का मान भरने में निम्नलिखित में से कोई एक गलती की है–

(i) आपने छोटे वर्ग अन्तराल की ओर क्रमशः +१, +२, +३, तथा बड़े वर्ग अन्तरालों की ओर क्रमशः–१, –२, –३ रख दिया है। जो कि गलत है।

(ii) आपने छोटे वर्ग अन्तरालों की ओर ऊपर से +१, +२, +३ रख दिया है जो कि गलत है।

याद रखिये विचलन (d) का मान कल्पित मध्यमान वाले वर्ग अन्तराल में ०, उससे बढ़ते क्रम वाले वर्ग अन्तरालों में क्रमशः +१, +२, +३ तथा घटते क्रम वाले वर्गअन्तरालों में क्रमशः –१,–२,–३ होता है।

अब आप पृष्ठ १३ पर खण्ड (१०) का पुनः सावधानी पूर्वक अध्ययन कीजिये फिर प्रश्न में दिये बंटन में d का मान भरिये।

खण्ड (८८) खण्ड (२१) से

आपका उत्तर गलत है।

आपने सबसे बड़ी बारम्बारता को ही बहुलांक मान लिया है। जब कि बहुलाँक सबसे बड़ी बारम्बारता वाला प्राप्तांक होता है। निम्न उदाहरण से बहुलाँक ज्ञात करना फिर से समझिये—

प्राप्तांक	बारम्बारता
४	२
६	४
९	८
१२	२
१३	६

हम जानते हैं कि बहुलाँक वह अंक है जिसकी बारम्बारता सबसे अधिक हो। उपरोक्त बारम्बारता बंटन में प्राप्ताँक ८ सबसे बड़ी बारम्बारता है किन्तु यह बहुलांक नहीं होगा। बहुलाँक सबसे अधिक बारम्बारता वाला प्राप्ताँक होता है। प्राप्ताँक ९ की बारम्बारता सबसे अधिक है

अतः बहुलाँक = ९

उपरोक्त तरीके से ही पृष्ठ ३५ पर खण्ड (२१) में दिये प्रश्न का बहुलांक ज्ञात कीजिये।

खण्ड (५८) **खण्ड (११) से**

आपका उत्तर सही है।

आपने खण्ड (११) को ठीक प्रकार समझा है। याद रखिये Σfd का मान निकालते समय उसे चिन्ह सहित लिखते हैं।

अब आप Σfd के मान से मध्यमान (M) ज्ञात करने के लिये पृष्ठ १७ पर खण्ड (१२) पढ़िये।

खन्ड (८९) **खन्ड (२२) से**

आपका उत्तर गलत है।

दिये हुये प्रश्न में सबसे अधिक बारम्बारता १३ है **जो कि दो** प्राप्तांकों क्रमशः १९ व २१ की है। **१९** *या* २१ में से कौन सा बहुलाँक है। यह सामान्य विधि से नहीं निश्चित कर सकते हैं।

अब आप पृष्ठ ३६ पर खण्ड (२२) को पुनः पढ़िये तथा प्रश्न का सही उत्तर दीजिये।

खण्ड (१०६) **खण्ड (२८) से**

आपका उत्तर सही है।

आपने बहुलाँक ज्ञात करना ठीक समझा है। इस प्रकार इस अध्याय में आपने बहुलाँक की गणना करना जाना।

अब आप पृष्ठ ५० पर समीक्षा अध्याय-४ का अध्ययन कीजिये।

खण्ड (५६) **खण्ड (११) से**

आपका उत्तर गलत है।

आपने fd का गुणा करते समय चिन्ह का ध्यान नहीं रखा है। या आपने Σfd का मान ज्ञात करते समय चिन्ह का ध्यान नहीं रखा है तथा सभी को जोड़ दिया है। जो कि गलत है। याद रखिये Σfd का मान चिन्ह सहित ज्ञात करते है जैसे—

–२४, –१०, ०, +१२, +८ का योग –१४ होगा न कि +५४।

अब आप पृष्ठ १५ पर खण्ड (११) को पुनः हल कीजिए।

खण्ड (६०) **खण्ड (२२) से**

आपका उत्तर सही है।

दिये हुये प्रश्न में सबसे अधिक बारम्बारता **१३ है** जो कि प्राप्तांक १६ व २१ की है। यह निश्चित करने के लिये कि इन दोनों में से कौनसा अंक बहुलाँक है हम समूहन की विधि का प्रयोग करते हैं।

अब आप समूहन विधि से बहुलांक ज्ञात करने की विधि का अध्ययन करने के लिये पृष्ठ ३७ पर खण्ड (२३) पढ़िये।

खण्ड (१०७) **खण्ड (२८) से**

आपका उत्तर गलत है।

आपने बहुलांक वर्ग अन्तिम तथा सबसे बड़े वर्ग अन्तराल को माना है जो कि गलत है। बहुलाँक वर्ग वह वर्ग होता है जिसकी बारम्बारता सबसे अधिक होती है।

पृष्ठ ४६ पर दिये खण्ड (२८) के प्रश्न में बहुलांक वर्ग २०—३० है।

अब आप इस वर्ग अन्तराल में बहुलांक ज्ञात कीजिये।